2704: Signals and Systems

Midterm Exam I
February 13, 2006

I pledge that I have neither given nor received any assistance on this exam.

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(signed)

______________________________ Name (print)

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ECE 2704 Midterm I – Test A

1. (20 points) Multiple Choice – Choose the answer which best completes the sentence

1.1 Let $g(t) = ramp(t)$. The function $h(t) = g\left(\frac{t}{5}\right)$ will increase

(a) ~~faster than~~ *g(t)*
(b) slower than *g(t)*
(c) ~~at the same~~ rate as *g(t)*
(d) None of the above

To see this, g(10) = 10 whereas h(10) = g(2) = 2. Thus, h(t) is time scaled so that it is changing more slowly than g(t).

1.2 Let $x_3(t) = x_1(t) * x_2(t)$. Then we know that

(a) $x_3(t) = \int_{-\infty}^{t} x_1(\lambda)x_2(z-\lambda)d\lambda$

(b) $x_3(t) = \int_{-\infty}^{\infty} x_1(\lambda)x_2(\lambda - t)d\lambda$

(c) $x_3(t) = \int_{-\infty}^{\infty} x_1(\lambda)x_2(t-\lambda)d\lambda$

(d) $x_3(t) = \int_{-\infty}^{t} x_1(\lambda)x_2(\lambda)d\lambda$

This is clear by the definition of convolution.

1.3 The function $rect(t)$ can be re-written as

(a) $u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right)$

(b) $u\left(t+\frac{1}{2}\right)u\left(-\left(t-\frac{1}{2}\right)\right)$

(c) $u\left(t+\frac{1}{2}\right)u\left(-t+\frac{1}{2}\right)$

(d) All of the above
(e) None of the above

All three expressions will result in a unit rectangle function. Note that (b) and (c) are identical expressions. Further u(t+1/2) turns on at t = -1/2 whereas u(-t+1/2) is on for all values of t < ½.

1.4 $x(t) * \delta(t - t_o) =$

(a) $x(t - t_o)$
(b) $x(t_o)$
(c) $x(t)$
(d) None of the above

Convolving a function with an impulse simply shifts the function in time.

2. (20 points) Complete the following properties of convolution:

2.1 The commutative property of convolution states that $x_1(t)*x_2(t)=$

$$x_2(t)*x_1(t)$$

2.2 The distributive property of convolution states that $x_1(t)*\left[x_2(t)+x_3(t)\right]=$

$$x_1(t)*x_2(t)+x_1(t)*x_3(t)=$$

2.3 The associative property of convolution states that $x_1(t)*\left[x_2(t)*x_3(t)\right]=$

$$\left[x_1(t)*x_2(t)\right]*x_3(t)$$

2.4 The derivative of convolution $\frac{d}{dt}\left[x_1(t)*x_2(t)\right]=$

$$\frac{d}{dt}x_1(t)*x_2(t)$$

3. (30 points) The properties of systems.

3.1 (15 points) A system can be defined as $y(t) = 2x^2(t)$

For each of the following the output for function $x_1(t)$ is $y_1(t) = 2x_1^{\,2}(t)$

(a) Is the system homogeneous?

No. If $x_2(t) = Kx_1(t)$ we get $y_2(t) = 2\left[Kx_1(t)\right]^2 = 2K^2x_1^{\,2}(t) \neq Ky_1(t) = 2Kx_1^{\,2}(t)$

(b) Is the system time-invariant?

Yes. If $x_2(t) = x_1(t-t_o)$ we get

$$y_2(t) = 2\left[x_1(t-t_o)\right]^2 = 2x_1^{\,2}(t-t_o) = y_1(t-t_o) = 2x_1^{\,2}(t-t_o)$$

(c) Is the system linear?

No. No. If $x_3(t) = K_1x_1(t) + K_2x_2(t)$ *we get*

$$y_3(t) = 2\left[K_1x_1(t) + K_2x_2(t)\right]^2 \neq K_1y_1(t) + K_2y_2(t) = 2K_1x_1^{\,2}(t) + 2K_2x_2^{\,2}(t)$$

(d) Does the system have memory?

No. The output at time t depends only on the input at time t.

(e) Is the system causal?

Yes. The output at time t depends only on the input at time t and not on future values of t.

3.2 (15 points) A system can be defined as $y(t) = x\left(\frac{t}{2}\right) - x(t+4)$

For each of the following the output for function $x_1(t)$ is $y_1(t) = x_1\left(\frac{t}{2}\right) - x_1(t+4)$

(a) Is the system homogeneous?

Yes. If $x_2(t) = Kx_1(t)$ we get $y_2(t) = Kx_1\left(\frac{t}{2}\right) - Kx_1(t+4) = Ky_1(t)$

(b) Is the system time-invariant?

No.

(c) Is the system linear?

Yes.

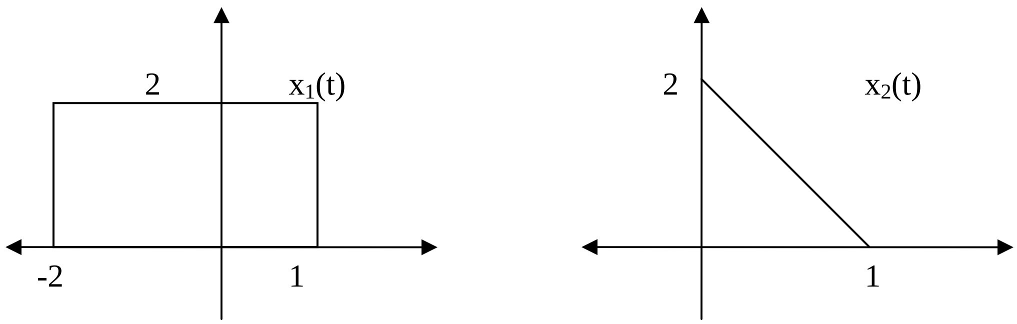
(d) Does the system have memory?

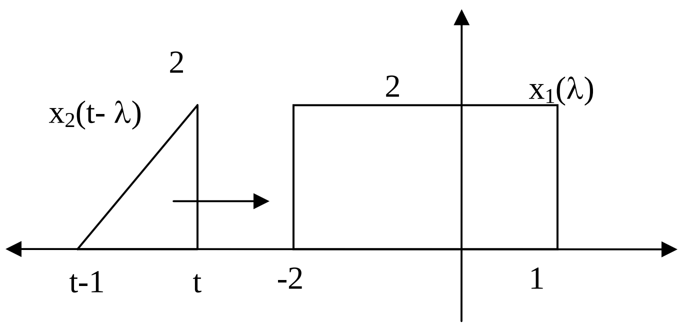
Yes.

(e) Is the system causal?

No.

4. (30 points) Determine the convolution of $x_1(t)$ with $x_2(t)$ when





First interval: $-\infty < t \le -2$

$$x_3(t) = 0$$

Second interval: $-2 \le t \le -1$

$$\begin{aligned} x_3(t) &= \int_{-2}^{t} 2(2\lambda + 2 - 2t)\, d\lambda \\ &= \frac{4\lambda^2}{2} + (4 - 4t)\lambda \Big|_{-2}^{t} \\ &= -2t^2 - 4t \end{aligned}$$

Third interval: : $-1 \le t \le 1$

$$x_3(t) = \int_{t-1}^{t} 2(2\lambda + 2 - 2t)\,d\lambda$$
$$= \frac{4\lambda^2}{2} + (4-4t)\lambda \Big|_{t-1}^{t}$$
$$= 2$$

Fourth interval: : $1 \le t \le 2$

$$x_3(t) = \int_{t-1}^{1} 2(2\lambda + 2 - 2t)\,d\lambda$$
$$= \frac{4\lambda^2}{2} + (4-4t)\lambda \Big|_{t-1}^{1}$$
$$= 2t^2 - 8t + 8$$

Fifth interval: $2 \le t < \infty$

$$x_3(t) = 0$$