2704: Signals and Systems

Homework #1
Solutions

1. g(t) = $7e^{-2t-3}$

   (a) g(3) = $7e^{-2(3)-3} = 7e^{-9}$

   (b) g(2-t) = $7e^{-2(2-t)-3} = 7e^{2t-7}$

   (c) $g\left(\frac{t}{10}+4\right) = 7e^{-2\left(\frac{t}{10}+4\right)-3} = 7e^{-t/5-11}$

   (d) g(jt) = $7e^{-2(jt)-3} = 7e^{-j2t-3}$

   (e) $\frac{g(jt)+g(-jt)}{2} = 7e^{-3}\frac{e^{-j2t}+e^{j2t}}{2} = 7e^{-3}\cos(2t)$

   (f) $\frac{g((jt-3)/2)+g((-jt-3)/2)}{2} = 7\frac{e^{-jt}+e^{jt}}{2} = 7\cos(t)$

2. (a) >> t = 3; g = sin(t)

```
g =
   0.1411
```

   (b) >> x = 1:5; g = cos(pi*x)

```
g =
   -1     1    -1     1    -1
```

   (c) >> f = -1:0.5:1 ; w = 2*pi*f ; g = 1./(1+j*w')

```
g =
  0.0247 + 0.1552i
  0.0920 + 0.2890i
  1.0000
  0.0920 - 0.2890i
  0.0247 - 0.1552i
```

4. This method has two solutions. Method 1 is the “easiest” approach however method 2 is the most efficient.

Solution method 1:

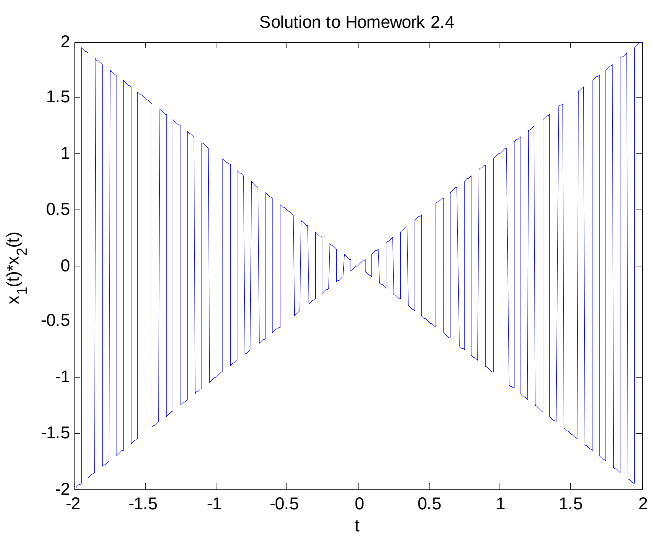
Matlab listing for x1.m

```
function y = x1(t)

y = zeros(1, length(t));

for i = 1:length(t)
   if sin(20*pi*t(i)) >= 0
      y(i) = 1;
   else
      y(i) = -1;
   end
end
```

Matlab listing for x2.m

```
function y = x2(t)

y = zeros(1, length(t));

for i = 1:length(t)
   if sin(2*pi*t(i)) >= 0
      y(i) = t(i);
   else
      y(i) = -t(i);
   end
end
```

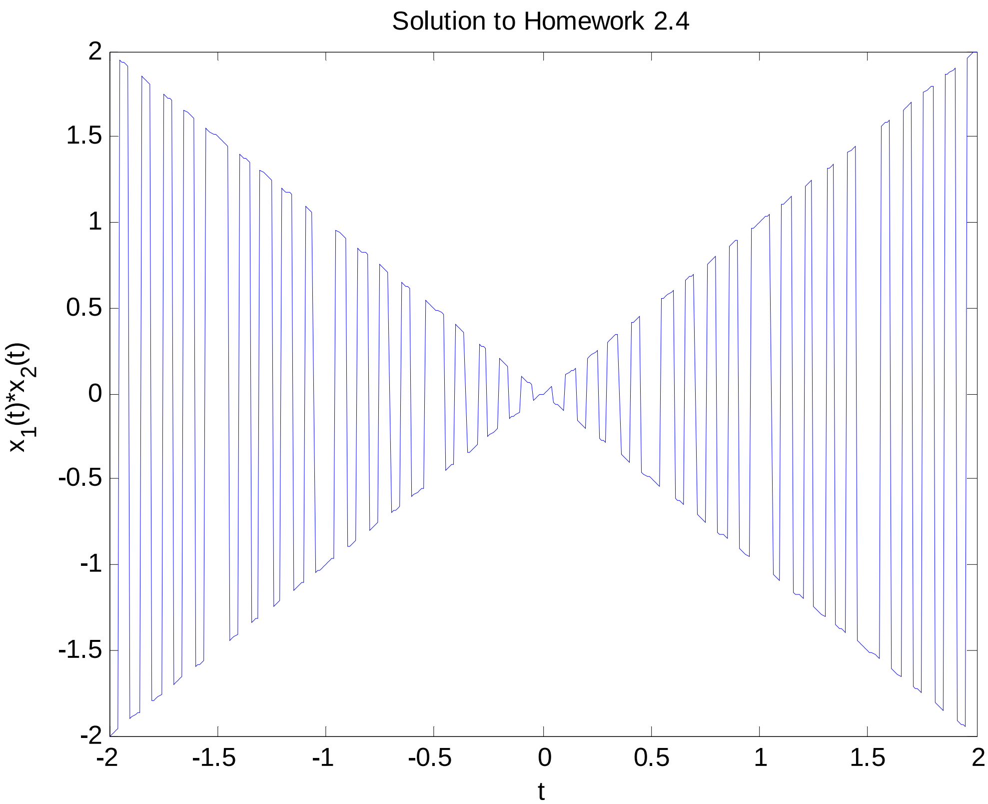
```
>> t = -2:.001:2;
>> plot(t, x1(t).*x2(t) );
>> xlabel( 't' );
>> ylabel('x_1(t)*x_2(t)');
>> title('Solution to Homework 2.4');
```



Solution method 2:

```
t = -2:0.01:2;
ind = find(sin(20*pi*t)>=0);
x1 = -1*ones(1,length(t));
```

```
x1(ind) = 1;
x2 = t;
ind = find(sin(2*pi*t)<0);
x2(ind) = -t(ind);
figure
plot(t, x1.*x2)
xlabel( 't' );
ylabel('x_1(t)*x_2(t)');
title('Solution to Homework 2.4');
```



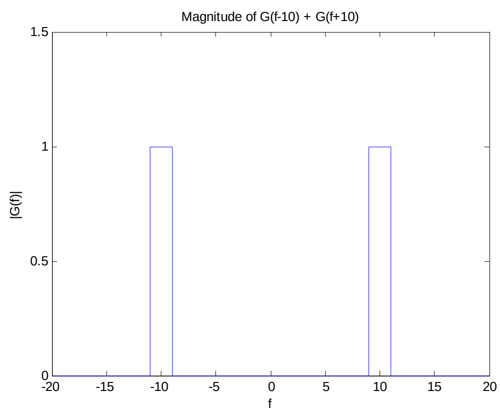
6. $G(f) = e^{-j2\pi f} rect\left(\frac{f}{2}\right)$

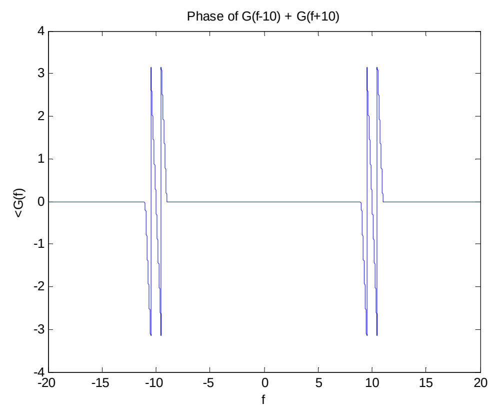
Matlab listing for G.m

```
function y = G(f)

y = exp(-j*2*pi.*f).*rect(f./2);
```

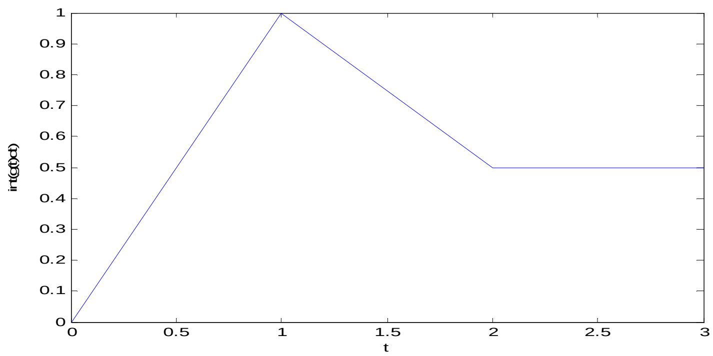
Matlab commands:

```
>> f = -20:.001:20;
>> Y = G(f - 10 ) + G(f + 10 ) ;
>> plot(f, abs(Y) );
>> axis( [-20 20 0 1.5] );
>> xlabel('f');
>> ylabel('|G(f)|');
>> title( 'Magnitude of G(f-10) + G(f+10)' );
>> figure;
>> plot(f, angle(Y) );
>> xlabel('f');
>> ylabel('<G(f)');
>> title( 'Phase of G(f-10) + G(f+10)' );
```

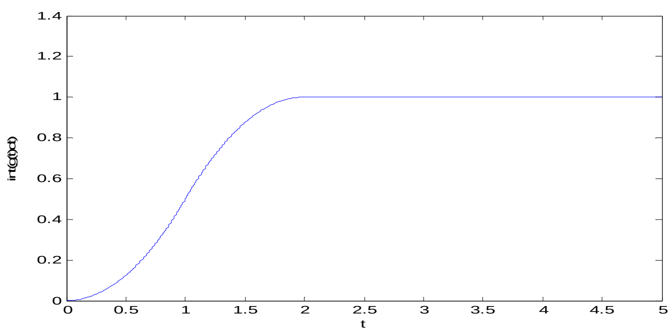




8. (a)
```
>> plot([0,1,2,3], [0, 1, 0.5, 0.5] );
>> xlabel( 't' );
>> ylabel( 'int(g(t)dt)' );
```



(b)
```
>> t = 0:.001:5;
>> y = zeros(1,length(t) );
>> y(1:find(t==1)) = t(1:find(t==1));
>> y(find(t==1):find(t==2)) = t(find(t==1):-1:1);
>> plot(t,cumtrapz(t,y))
>> xlabel('t');
>> ylabel('int(g(t)dt)')
```



12.

(a) $$\int_{-1}^{1}(2+t)dt=\int_{-1}^{1}\underbrace{2}_{\text{even}}dt+\int_{-1}^{1}\underbrace{t}_{\text{odd}}dt=2\int_{0}^{1}2dt=4$$

(b)
$$\int_{-\frac{1}{20}}^{\frac{1}{20}}\left[4\cos(10\pi t)+8\sin(5\pi t)\right]dt=\int_{-\frac{1}{20}}^{\frac{1}{20}}\underbrace{4\cos(10\pi t)}_{\text{even}}dt+\int_{-\frac{1}{20}}^{\frac{1}{20}}\underbrace{8\sin(5\pi t)}_{\text{odd}}dt=8\int_{0}^{\frac{1}{20}}\cos(10\pi t)dt=\frac{8}{10\pi}$$

(c) $$\int_{-\frac{1}{20}}^{\frac{1}{20}}4\underbrace{\underbrace{t}_{\text{odd}}\underbrace{\cos(10\pi t)}_{\text{even}}}_{\text{odd}}dt=0$$

(d) $$\int_{-\frac{1}{10}}^{\frac{1}{10}}\underbrace{\underbrace{t}_{\text{odd}}\underbrace{\sin(10\pi t)}_{\text{odd}}}_{\text{even}}dt=2\int_{0}^{\frac{1}{10}}t\sin(10\pi t)dt=2\left[-t\frac{\cos(10\pi t)}{10\pi}\bigg|_{0}^{\frac{1}{10}}+\int_{0}^{\frac{1}{10}}\frac{\cos(10\pi t)}{10\pi}dt\right]$$

$$\int_{-\frac{1}{10}}^{\frac{1}{10}}\underbrace{\underbrace{t}_{\text{odd}}\underbrace{\sin(10\pi t)}_{\text{odd}}}_{\text{even}}dt==2\left[\frac{1}{100\pi}+\frac{\sin(10\pi t)}{(10\pi)^2}\bigg|_{0}^{\frac{1}{10}}\right]=\frac{1}{50\pi}$$

(e) $$\int_{-1}^{1}\underbrace{e^{-|t|}}_{\text{even}}dt=2\int_{0}^{1}e^{-|t|}dt=2\int_{0}^{1}e^{-t}dt=2\left[-e^{-t}\right]_0^1=2\left(1-e^{-1}\right)\approx 1.264$$

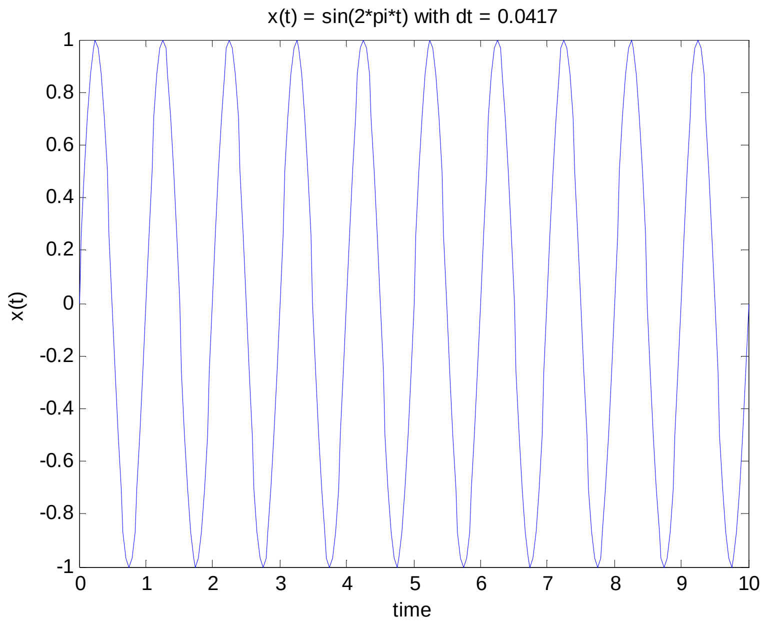
(f) $$\int_{-1}^{1}\underbrace{\underbrace{t}_{\text{odd}}\underbrace{e^{-|t|}}_{\text{even}}}_{\text{odd}}dt=0$$

26.

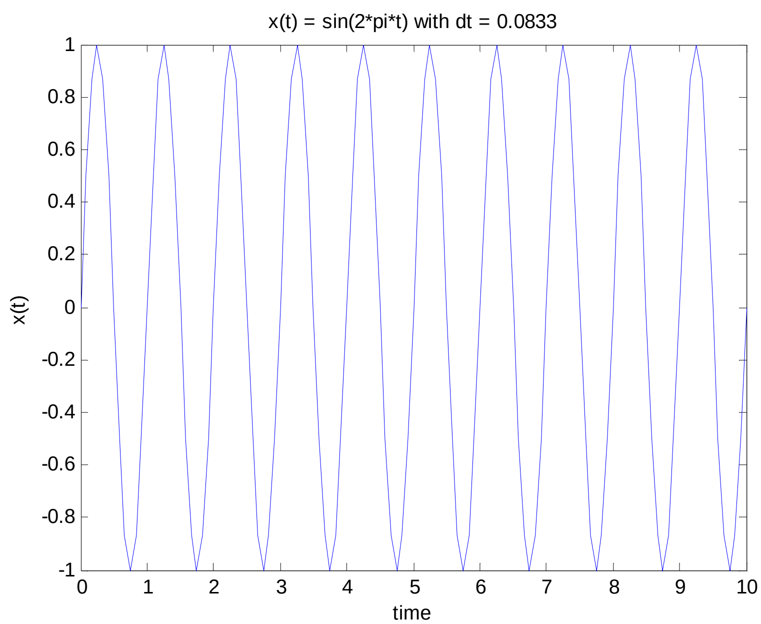
For this problem, we will assume t is in seconds. Therefore, $\Delta t = ¼$ is equivalent to a 4Hz sampling rate. This also means that $\sin(2\pi t)$ is a 1Hz sine-wave. Sampling theory tells us to accurately represent this signal, we will require a sampling rate greater than 2 Hz.

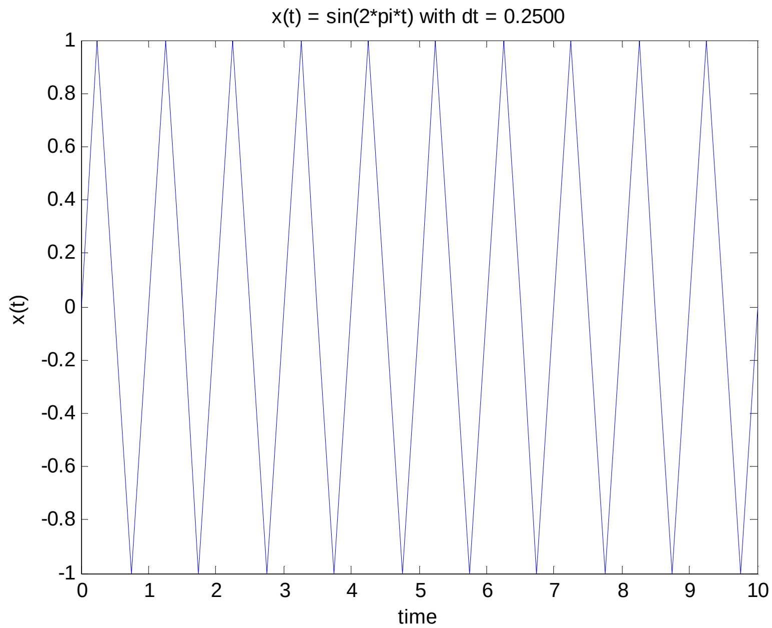
MATLAB listing for ece2704_ch2_26.m

```
dt = [1/24, 1/12, 1/4, 1/2, 2/3, 5/6, 1];

for i = 1:length(dt),
    t = 0:dt(i):10;
    x = sin(2*pi*t);
    figure;
    plot(t,x);
    xlabel('time' );
    ylabel('x(t)' );
    title( sprintf('x(t) = sin(2*pi*t) with dt = %6.4f', dt(i) ) );
end
```



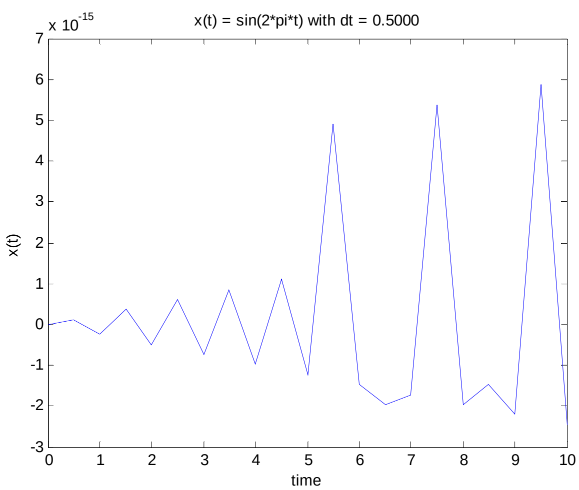
High sampling rate provides smooth curve (above). General shape is still recognizable below, but due to the sampling rate being cut in half, distortion is evident.

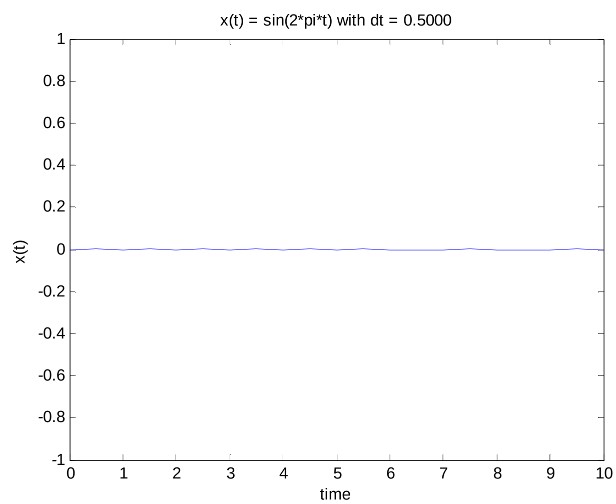


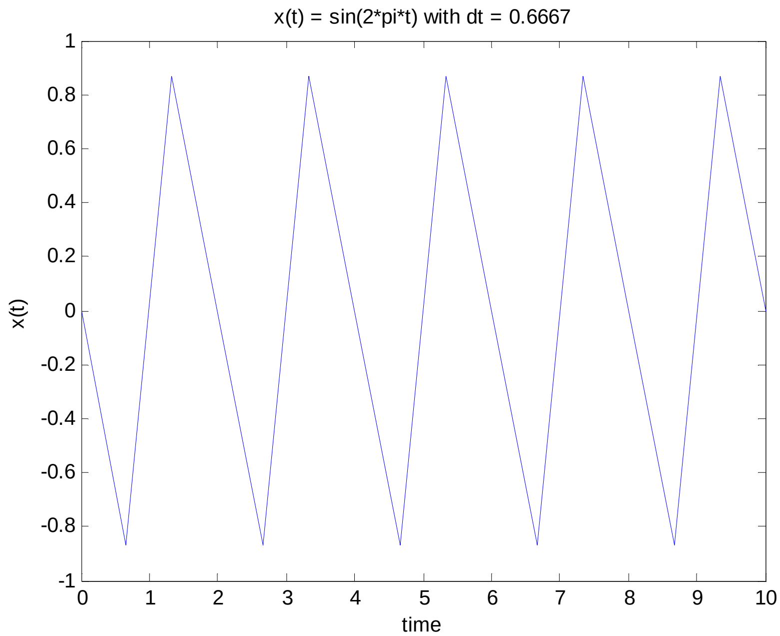


As before, general "sinusoidal" shape still evident and frequency of original signal is visually determinable (above).

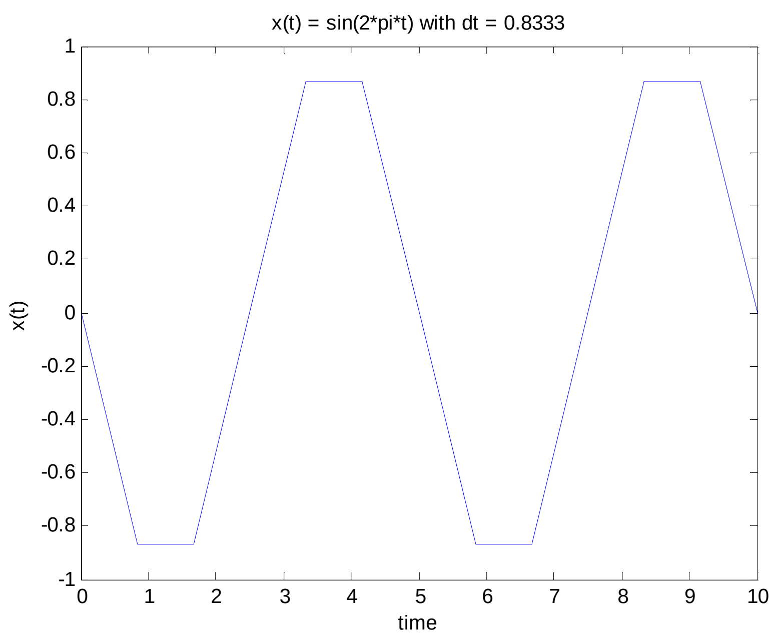
The signal below and to the left is sampled exactly at the Nyquist frequency which is just below the required rate. If we look at the amplitude multiplier and realize the typical accuracy of MATLAB's floating point numbers, we can see that we are sampling exactly at the zeros of the sin function. If we zoom out and have the same axis as the other plots, we see that we are actually just seeing zeros all the way across (right).

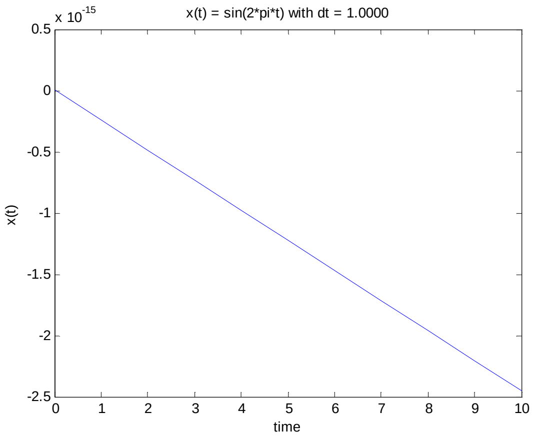


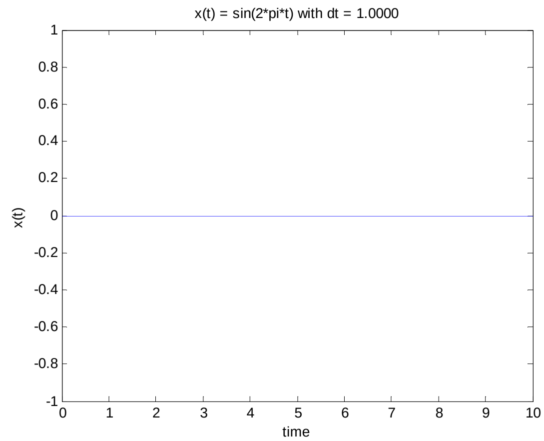




The sampled functions are periodic, however the low sampling rate reveal the wrong frequencies.







As when the sampling rate was 2 Hz, we are sampling again exactly at the zeros (above left), which is made a little clearer when we zoom out (above right).

One thing to notice with all of the above signals is that in typical communications signals they are not what would be seen in a real-world system as a low-pass filter (or band-pass filter depending on the application) would be used to remove all extraneous information. In the cases of the 2/3 and 5/6 time resolution, such a "nice" looking signal would not be detected as all the "sharp" angles would be removed and the resulting signal would be "junky". In the case of the 1/24, 1/12, and 1/4 time resolutions, all three signals would look identical at the output of an appropriate filter as the "sharpness" in the time-domain is simply additive high-frequency components that would be removed.

27. (a) $g(t) = 100\sin\left(200\pi t + \frac{\pi}{4}\right)$, $g(0.001) = 100\sin\left(0.2\pi + \frac{\pi}{4}\right) = 100\sin\left(\frac{9\pi}{20}\right) = 98.7688$

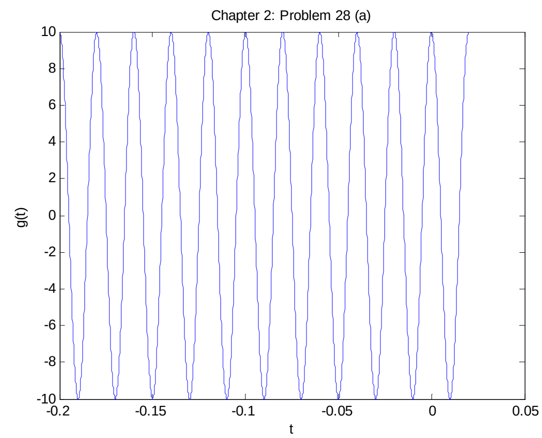
(b) $g(t) = 13 - 4t + 6t^2$, $g(2) = 13 - 4(2) + 6(4) = 29$

(c) $g(t) = -5e^{-2t}e^{-j2\pi t} = -5e^{-2t}\left(\cos(2\pi t) - j\sin(2\pi t)\right)$

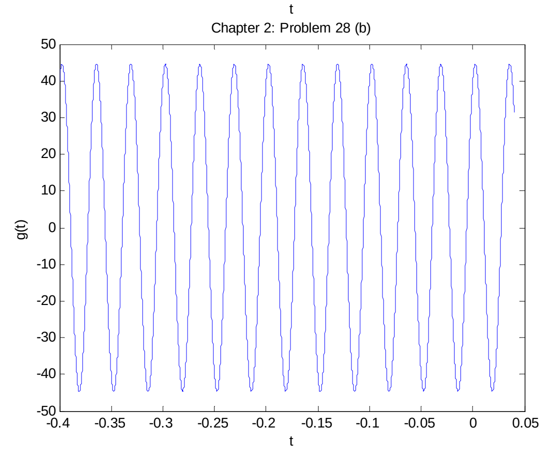
$$g(1/4) = -5e^{-\frac{1}{2}}\left(\cos(\pi/2) - j\sin(\pi/2)\right) = j5e^{-\frac{1}{2}}$$

28.

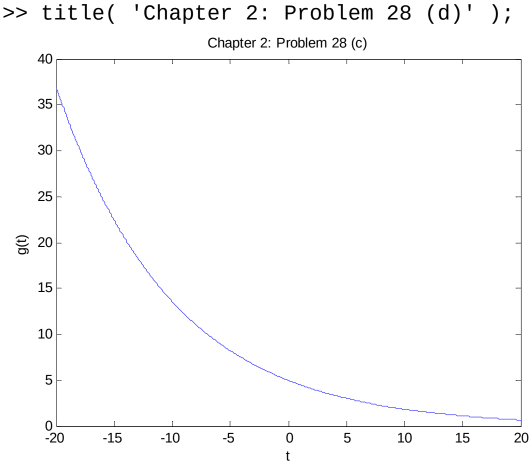
```
>> t = -0.2:.0001:.02;
>> g = 10*cos(100*pi*t);
>> plot(t,g);
>> xlabel('t');
>> ylabel('g(t)');
>> title( 'Chapter 2: Problem 28 (a)' );
```

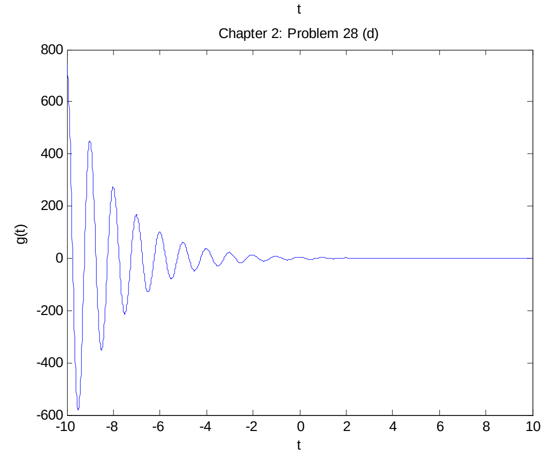



```
>> t = -0.4:.0001:.04;
>> g = 40*cos(60*pi*t)+20*sin(60*pi*t);
>> plot(t,g);
>> xlabel('t');
>> ylabel('g(t)');
>> title( 'Chapter 2: Problem 28 (b)' );
```




```
>> t = -20:.01:20; g = 5*exp(-t/10);
>> plot(t,g);
>> xlabel('t');
>> ylabel('g(t)');
>> title( 'Chapter 2: Problem 28 (c)' );
>>
>> t = -10:.01:10;
>> g = 5*exp(-t/2).*cos(2*pi*t);
>> plot(t,g);
>> xlabel('t');
>> ylabel('g(t)');
>> title( 'Chapter 2: Problem 28 (d)' );
```





29.

```
>> t=-10:.001:10;
>> g = 2*u(4-t);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (a)' );
>> axis( [-10, 10, -0.5, 2.5] );
```

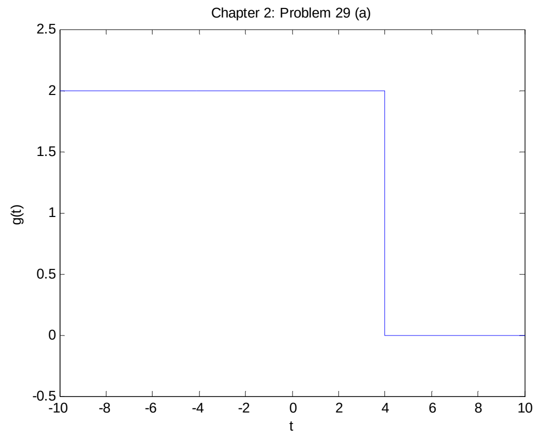
```
>> t=-10:.001:10;
>> g = 5*sign(t-4);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (c)' );
>> axis( [-10, 10, -5.5, 5.5] );
```

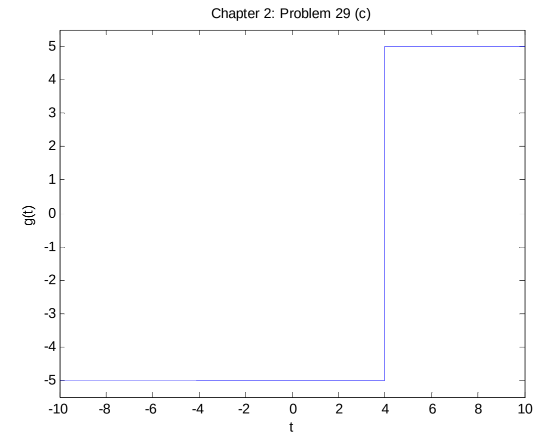
```
>> t=-10:.001:10;
>> g = 5*ramp(t+1);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (e)' );
>> axis( [-10, 10, -0.5, 60.5] );
```

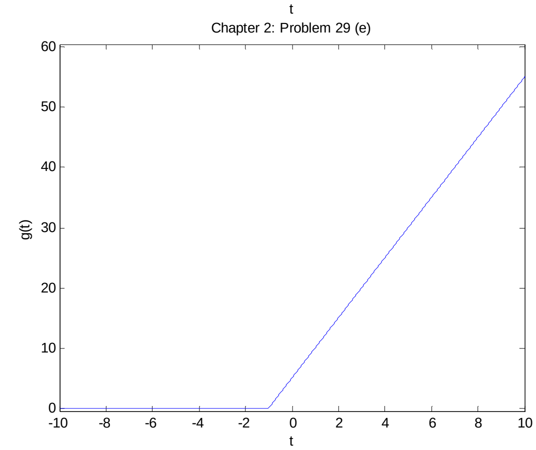
```
>> t=-10:.001:10;
>> g = zeros(1,length(t));
>> g(find(t==-3)) = 2;
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (g)' );
>> axis( [-10, 10, -0.5, 2.5] );
```

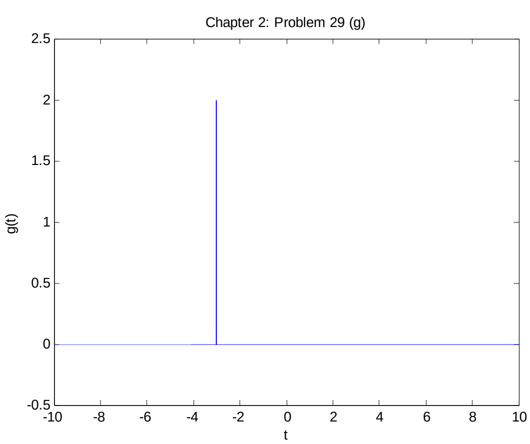
```
>> t=-10:.001:10;
>> g = 2*rect(t/3);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (m)' );
>> axis( [-10, 10, -0.5, 2.5] );
```

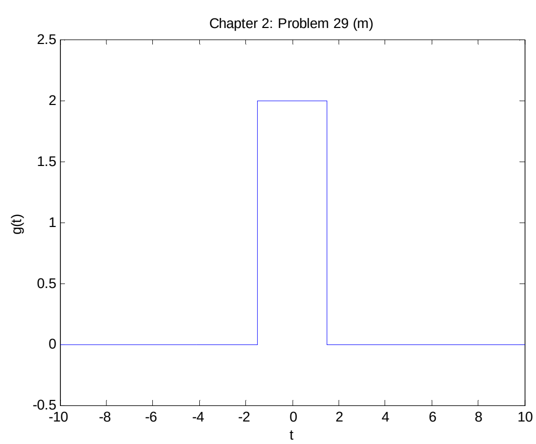
```
>> t=-1:.001:1;
>> g = tri(4*t);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (o)' );
>> axis( [-1, 1, -0.5, 1.5] );
```



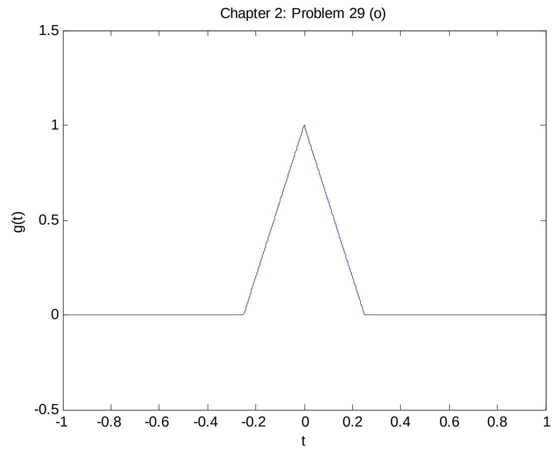




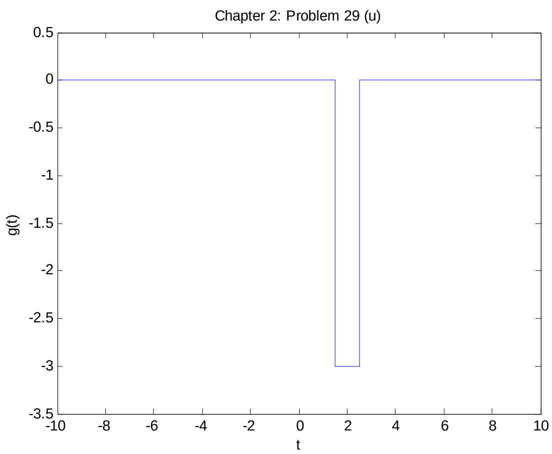


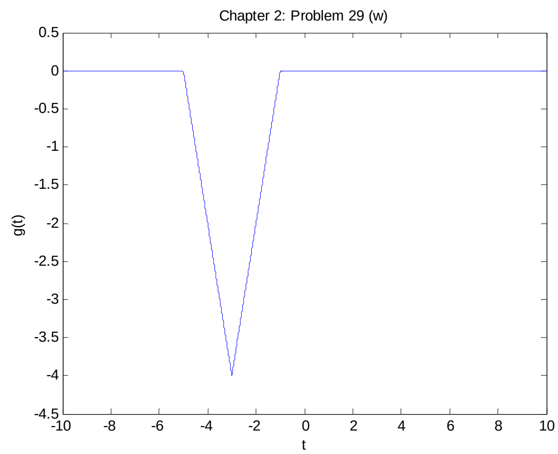


```
>> t=-10:.001:10;
>> g = -3*rect(t-2);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (u)' );
>> axis( [-10, 10, -3.5, 0.5] );
```




```
>> t=-10:.001:10;
>> g = -4*tri((3+t)/2);
>> plot( t, g );
>> xlabel( 't' );
>> ylabel('g(t)' );
>> title('Chapter 2: Problem 29 (w)' );
>> axis( [-10, 10, -4.5, 0.5] );
```





34. (a) We see that the figure is shifted the left by 2 and flipped about the *g(t)*-axis and. Therefore:

$$h(t) = g(-(t - 2)) = g(-t + 2)$$

(b) Initially, we see the figure is compressed by a factor of two in amplitude.
To provide the rest of transformation is two options:
Flip about t-axis and shift by 1

(a) To the right: $h(t) = -\frac{g(t-1)}{2}$ (b) To the left: $h(t) = -\frac{g(t+1)}{2}$

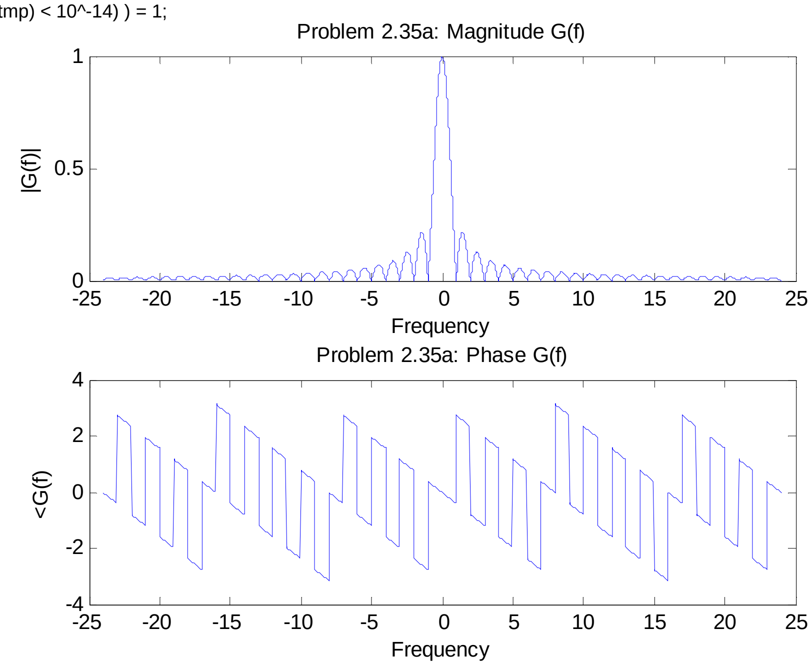
35. MATLAB listing for ece2704_ch2_35.m

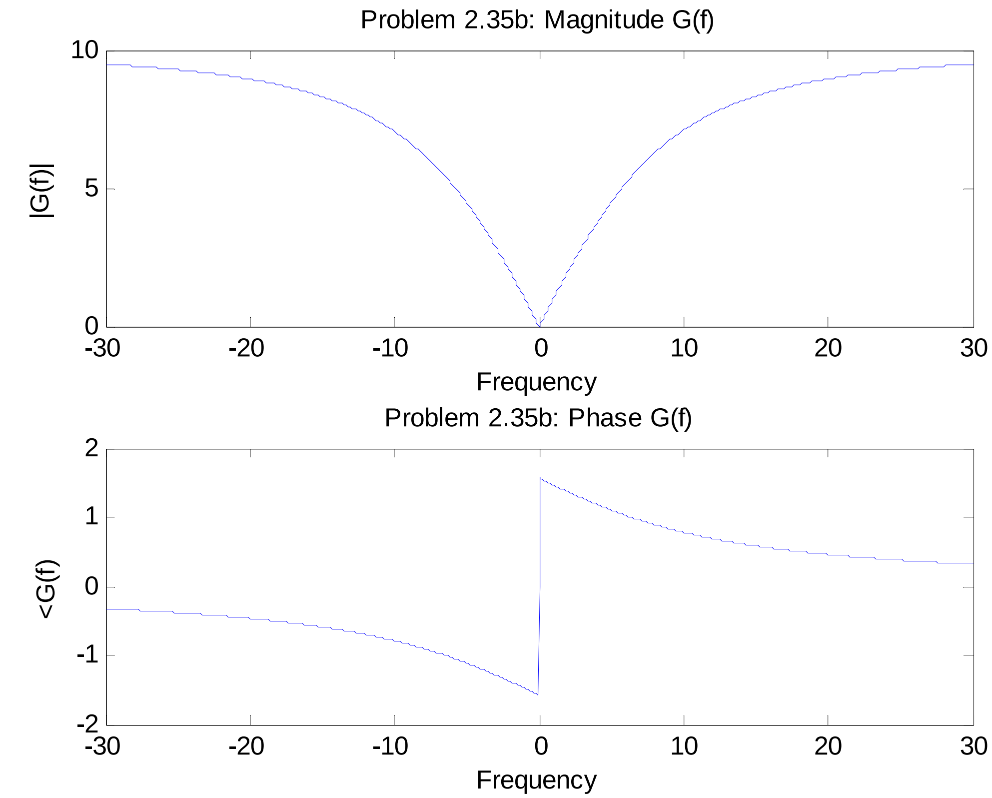
```
%Part a
figure;
f = -24:.001:24;
G = sinc(f).*exp(-j*(pi.*f./8));
subplot(2,1,1);
plot(f,abs(G) );
xlabel('Frequency');
ylabel('|G(f)|');
title ('Problem 2.35a: Magnitude G(f)' );
subplot(2,1,2);
plot(f,angle(G) );
xlabel('Frequency' );
ylabel('<G(f)');
title( 'Problem 2.35a: Phase G(f)' );

%Part b
figure;
f = -30:.001:30;
G = j.*f./(1+j.*(f./10));
subplot(2,1,1);
plot(f,abs(G) );
xlabel('Frequency');
ylabel('|G(f)|');
title ('Problem 2.35b: Magnitude G(f)' );
subplot(2,1,2);
plot(f,angle(G) );
```

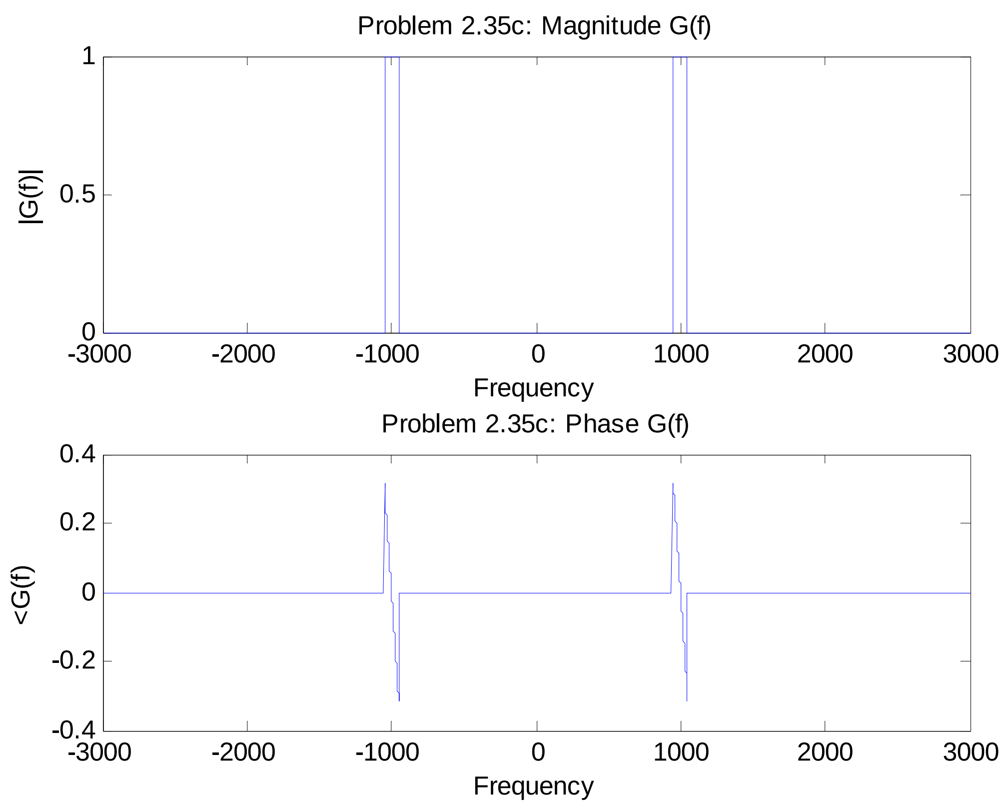
```
xlabel('Frequency' );
ylabel('<G(f)');
title( 'Problem 2.35b: Phase G(f)' );

%Part c
figure;
f = -3000:.1:3000;
G = exp(-j*pi/500.*f).*(rect( (f-1000)./100 ) + rect( (f+1000)./100) );
subplot(2,1,1);
plot(f,abs(G) );
xlabel('Frequency');
ylabel('|G(f)|');
title ('Problem 2.35c: Magnitude G(f)' );
subplot(2,1,2);
plot(f,angle(G) );
xlabel('Frequency' );
ylabel('<G(f)');
title( 'Problem 2.35c: Phase G(f)' );

%Part d
figure;
f = -300:.001:300;
G = 1./(250-f.^2+j*3.*f);
subplot(2,1,1);
plot(f,abs(G) );
xlabel('Frequency');
ylabel('|G(f)|');
title ('Problem 2.35d: Magnitude G(f)' );
subplot(2,1,2);
plot(f,angle(G) );
xlabel('Frequency' );
ylabel('<G(f)');
title( 'Problem 2.35d: Phase G(f)' );

%Part e
figure;
f = -0.2:.0001:0.2;
G = comb(100.*f).*sinc(25.*f).*exp(j*(pi.*f./50));
subplot(2,1,1);
stem(f,abs(G) );
xlabel('Frequency');
ylabel('|G(f)|');
title ('Problem 2.35e: Magnitude G(f)' );
subplot(2,1,2);
stem(f,angle(G) );
xlabel('Frequency' );
ylabel('<G(f)');
title( 'Problem 2.35e: Phase G(f)' );
```

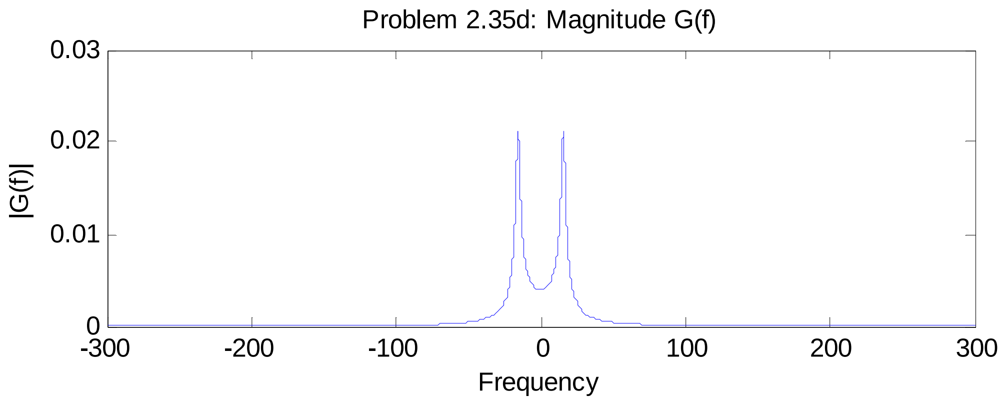
MATLAB listing for comb.m

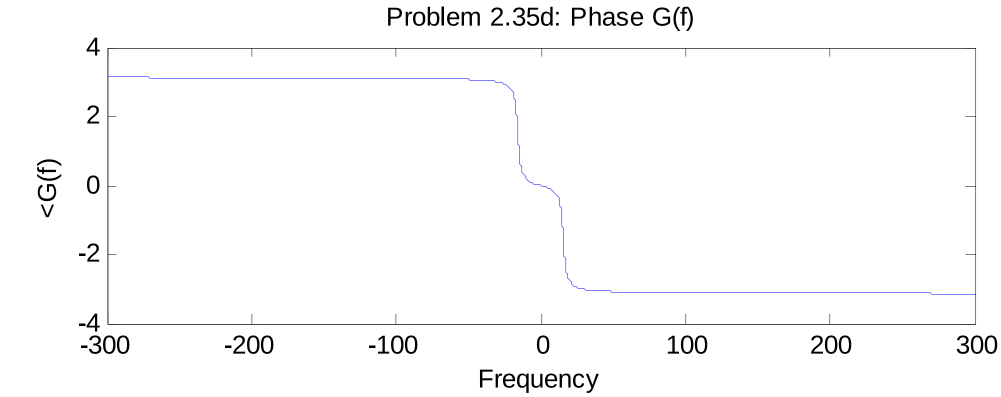
```
function y = comb( f )

y = zeros(1,length(f));
tmp = f - round(f);
%Can't test on '0' due to floating point inexactness
y( find( abs(tmp) < 10^-14) ) = 1;
```

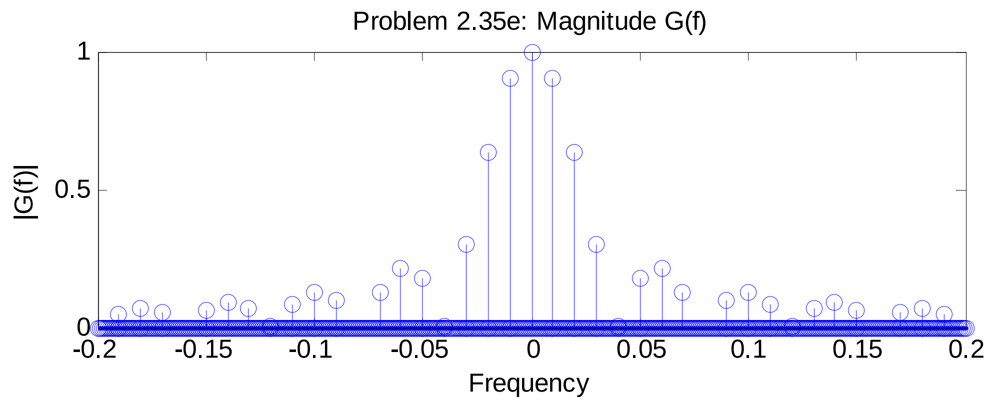


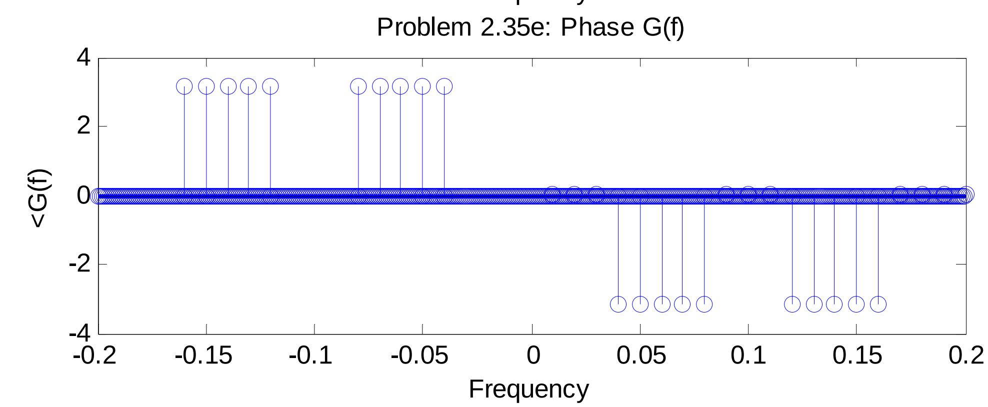

Problem 2.35b: Magnitude G(f)
|G(f)|
10
5
0
-30
-20
-10
0
10
20
30
Frequency
Problem 2.35b: Phase G(f)
<G(f)
2
1
0
-1
-2
-30
-20
-10
0
10
20
30
Frequency


Problem 2.35c: Magnitude G(f)
|G(f)|
1
0.5
0
-3000
-2000
-1000
0
1000
2000
3000
Frequency
Problem 2.35c: Phase G(f)
<G(f)
0.4
0.2
0
-0.2
-0.4
-3000
-2000
-1000
0
1000
2000
3000
Frequency









43. (a) $\int_{-\infty}^{\infty} \delta(t)\cos(48\pi t)dt = \cos(48\pi \cdot 0) = \cos(0) = 1$

(b) $\int_{-\infty}^{\infty} \delta(t-5)\cos(\pi t)dt = \cos(5\pi) = \cos(\pi) = -1$

(c) $\int_0^{20} \delta(t-8) tri\left(\frac{t}{32}\right) dt = tri\left(\frac{8}{32}\right) = \frac{3}{4}$

(d) $\int_0^{20} \delta(t-8) rect\left(\frac{t}{16}\right) dt = rect\left(\frac{8}{16}\right) = 1$

(e) $\int_{-2}^{2} \delta(t-1.5)\,\text{sinc}(t) dt = \text{sinc}\,(1.5) = \frac{\sin(\pi 1.5)}{\pi 1.5} = -\frac{2}{3\pi} = -0.2122$

(f) $\int_{-2}^{2} \delta(t-1.5)\,\text{sinc}(4t) dt = \text{sinc}\,(6) = \frac{\sin(\pi 6)}{\pi 6} = \frac{0}{\pi 6} = 0$

47. Using the hints given, we will first construct a piece-wise function.

Hint #1: g(t) = 0 for t <= -5 provides the first piece.
Hint #2: d/dt(g(t)) = -2 tells us that the second piece has the form -2t + m.
We find m by knowing that g(t) = 0 when t = -5 → m = -10
Hint #3: Unit sinusoid with a frequency of ¼ Hz plus a constant.
We know that ¼ Hz is π/2 in radians, therefore we need something of the form $\sin\left(\frac{\pi}{2}t\right)$+C. Using the sine function in this case is convenient since $\sin\left(\frac{\pi}{2}t\right)$ when t = -2 equates to 0. From the previous step, -2(-2)-10 = -6, therefore C must equal -6.
Hint #4: A decaying exponential with a time constant of 2 s tells us we must be in the form $e^{-\frac{t+C}{2}}$. From before, $\sin\left(\frac{\pi}{2}t\right)-6$ = -6 at t = 2. Therefore we must find C such that $-e^{-\frac{t+C}{2}}$ = -6 when t = 2. Through simple algebra, we find C = -2ln(6)-2

$$g(t) = \begin{cases} 0 & t <= -5 \\ -2t-10 & -5 < t <= -2 \\ \sin\left(\frac{\pi}{2}t\right)-6 & -2 < t <= 2 \\ -e^{-\frac{t-2\ln(6)-2}{2}} & t > 2 \end{cases}$$

$$\therefore g(t) = (-2t-10) rect\left(\frac{t+3.5}{3}\right) + \left(\sin\left(\frac{\pi}{2}t\right)-6\right) rect\left(\frac{t}{4}\right) - e^{-\frac{t-2\ln(6)-2}{2}} u(t-2)$$

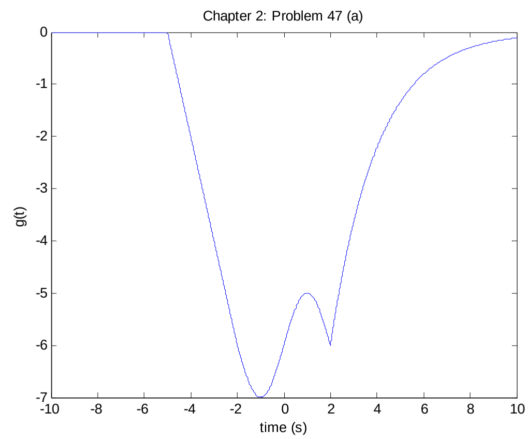
MATLAB listing for G.m

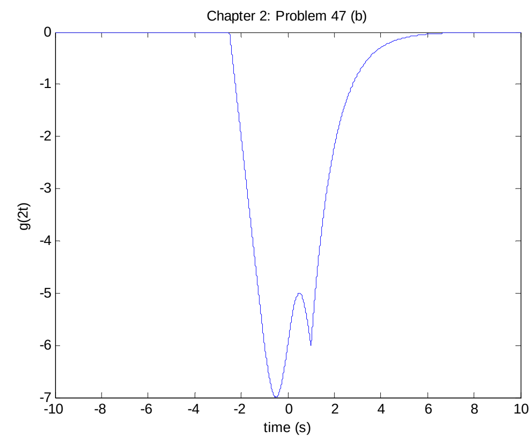
```
function y = G( t )
y = ( -2.*t-10).*rect( (t+3.5)./3) + ( sin(pi/2.*t)-6).*(rect(t./4)) - exp(-(t-2*log(6)-2)./2).*u(t-2);
```

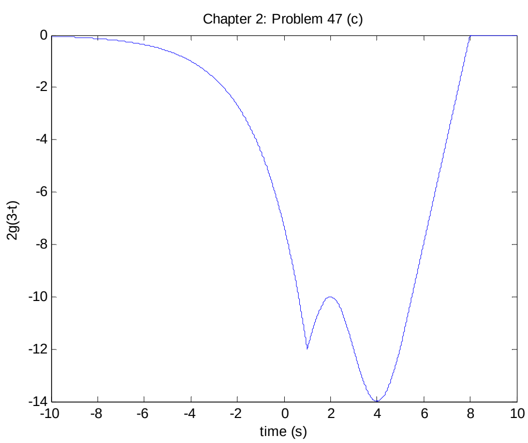
MATLAB listing for ece3274_ch2_47.m

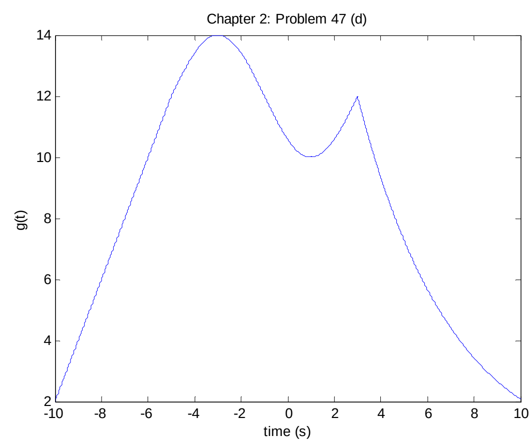
```
t = -10:.001:10;
g = G(t);
figure; plot(t,g);
xlabel('time (s)' );
ylabel('g(t)');
title('Chapter 2: Problem 47 (a)');
g = G(2.*t);
figure; plot(t,g);
xlabel('time (s)' );
ylabel('g(2t)');
title('Chapter 2: Problem 47 (b)');
```

```
g= 2.*G(3-t);
figure; plot(t,g);
xlabel('time (s)' );
ylabel('2g(3-t)');
title('Chapter 2: Problem 47 (c)');
g = -2.*G( (t+1)./2 );
figure; plot(t,g);
xlabel('time (s)' );
ylabel('g(t)');
title('Chapter 2: Problem 47 (d)');
```









53. The signal is an inverted rectangle with a height of 10 and a width of 5 μs centered at t = 2.5μs

$$x(t) = -10\,rect\left(\frac{t-2.5\mu s}{5\mu s}\right) \text{with } t \text{ in microseconds.}$$

56. Use $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

(a) x(t) = 2 *rect*(-t) $\therefore E_x = \int_{-\infty}^{\infty} |2rect(-t)|^2 dt = \int_{-1/2}^{1/2} |2|^2 dt = 4t\Big|_{-1/2}^{1/2} = 4$

(c) x(t) = $3rect\left(\frac{t}{4}\right)$ $\therefore E_x = \int_{-\infty}^{\infty} \left|3rect\left(\frac{t}{4}\right)\right|^2 dt = \int_{-2}^{2} |3|^2 dt = 9t\Big|_{-2}^{2} = 36$

(e) x(t) = $3tri\left(\frac{t}{4}\right)$

$$\therefore E_x = \int_{-\infty}^{\infty} \left|3tri\left(\frac{t}{4}\right)\right|^2 dt = 9\int_{-4}^{4} \left|-\frac{t}{4}+1\right|^2 dt = 18\int_{0}^{4} \left|-\frac{t}{4}+1\right|^2 dt = 18\int_{0}^{4} \left(\frac{t^2}{16}-\frac{t}{2}+1\right) dt$$

$$= 18\left(\frac{t^3}{48}-\frac{t^2}{4}+t\right)\Bigg|_0^4 = 24$$

(g) x(t) = $\delta(t) \approx \lim_{a\to 0} \frac{1}{a} rect\left(\frac{t}{a}\right)$

$$\therefore E_x = \int_{-\infty}^{\infty} \left|\lim_{a\to 0} \frac{1}{a} rect\left(\frac{t}{a}\right)\right|^2 dt = \lim_{a\to 0} \frac{1}{a^2} \int_{-a/2}^{a/2} \left|rect\left(\frac{t}{a}\right)\right| dt = \lim_{a\to 0} \frac{1}{a^2} \int_{-a/2}^{a/2} \left|rect\left(\frac{t}{a}\right)\right| dt$$

$$= \lim_{a\to 0} \frac{1}{a^2} \int_{-a/2}^{a/2} dt = \lim_{a\to 0} \frac{a}{a^2} \to \infty \text{ by L'Hopital's rule}$$