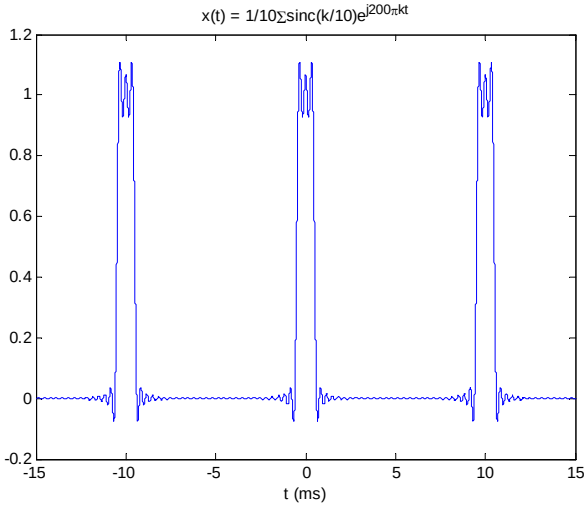
2704: Signals and Systems

Homework #5
Solutions

1)

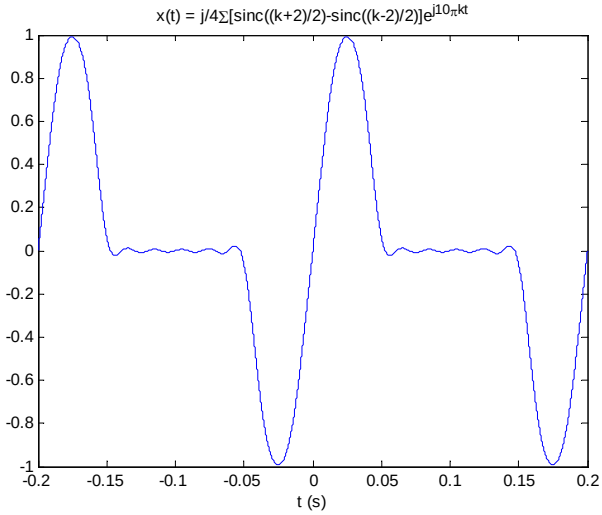
(a)

```
>> t = ( -15:.001:15) .* 10^-3;
>> x = 0;
>> for k = -30:30,
     x = x + sinc(k./10).*exp(j*200*pi*k.*t);
   end
>> x = x ./ 10;
>> plot( t.*10^3, real(x) );
```



(b)

```
>> t = (-200:.1:200).*10^-3;
>> x = 0;
>> for k = -9:9,
     x = x + (sinc( (k+2)/2 ) - sinc( (k-2)/2 )).*exp(j*10*pi*k.*t);
   end;
>> x = j./4.*x;
>> plot(t, real(x) );
```



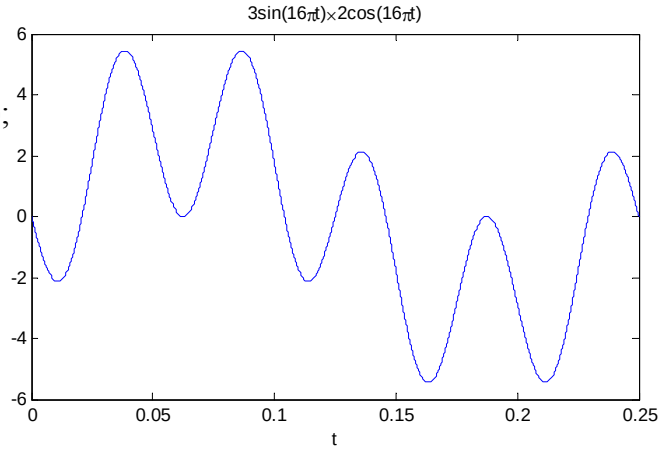
4)

(a)

```
>> t = 0:.0001:.25;
>> x = -3.*sin(16*pi.*t).*2.*cos(24*pi.*t);
>> plot(t,x);
>> xlabel( 't' );
>> title('3sin(16\pit)\times2cos(16\pit)')
>> sum(x)

        ans =

        -4.3960e-013 ≈ 0
```

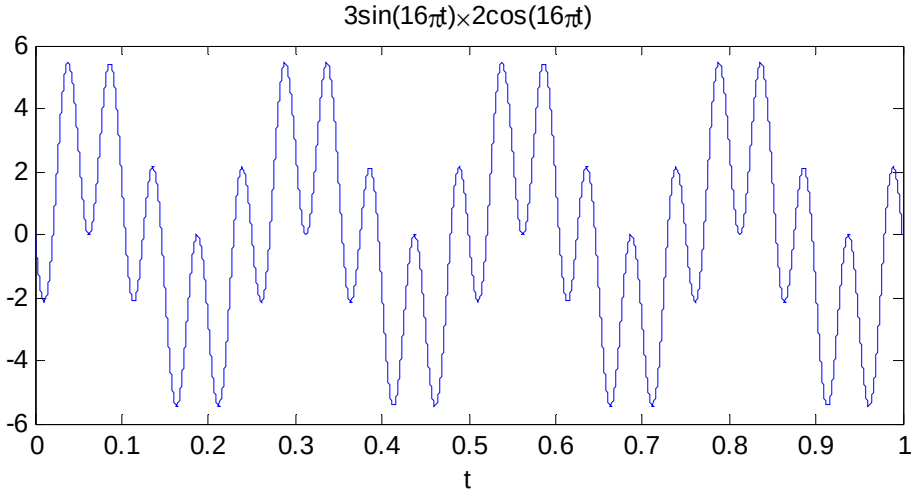


(b)

```
>> t = 0:.0001:1;
>> x = -3.*sin(16*pi.*t).*2.*cos(24*pi.*t);
>> plot(t,x);
>> xlabel( 't' );
>> title('3sin(16\pit)\times2cos(16\pit)')
>> sum(x)
```

ans =

7.4513e-013 ≈ 0

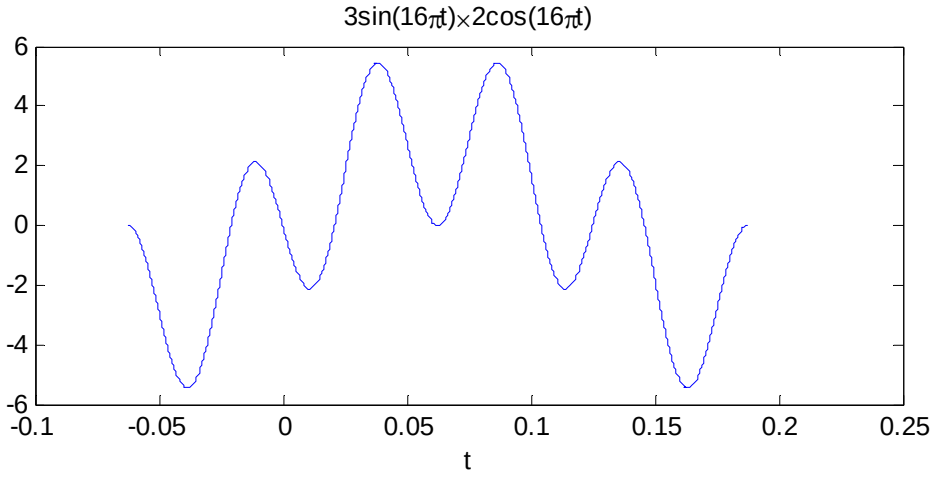


(c)

```
>> t = -1/16:.0001:3/16;
>> x = -3.*sin(16*pi.*t).*2.*cos(24*pi.*t);
>> plot(t,x);
>> xlabel( 't' );
>> title('3sin(16\pit)\times2cos(16\pit)')
>> sum(x)
```

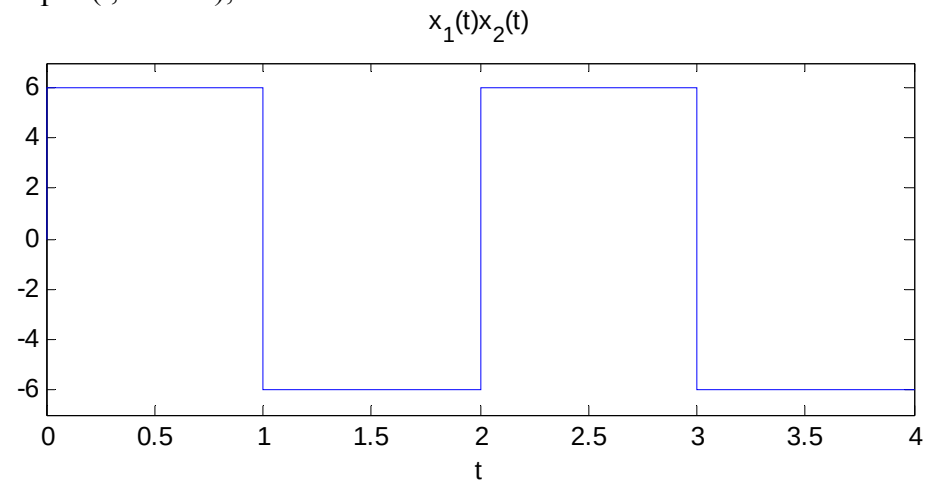
ans =

-5.0249e-013 ≈ 0



(d)

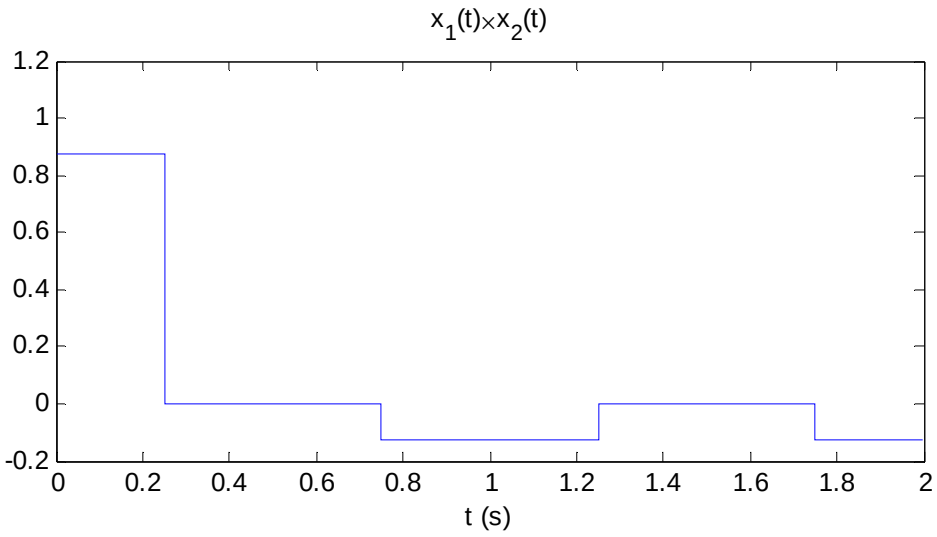
```
>> t = 0:.0001:4;
>> x1 = 2.*sign( cos(2*pi*1/4.*t) );
>> x2 = 3.*sign( sin(2*pi*1/4.*t) );
>> plot(t, x1.*x2);
```



NOTE: The sqrt(6) was used above to simply match the result with the books graph. There is no hint or clue given as to the magnitude of x1 or x2, so any input/output magnitude is acceptable.

(e)

```
>> t = -2:.001:4;
>> x1 = conv( rect(2.*t), comb(t) );
>> %Note the 1/2 has been removed from infront of the comb
>> %due to the limitations of the comb function supplied.
>> x2 = conv(rect( 4.*(t-1/8) ), comb(t./2) ) - 1/8;
>> x = x1.*x2;
>> idx = find( t > 0 & t < 2);
>> plot(t(idx),x(idx));
```



6)

(a) $x(t) = 4rect(4t) * comb(t)$

$$X[k] = \frac{1}{T_0}\int_{T_0} x(t)e^{-j2\pi(kf_0)t}dt = \int_{-1/2}^{1/2} 4rect(4t)e^{-j2\pi kt}dt = 4\int_{-1/8}^{1/8} e^{-j2\pi kt}dt = 4\left[\frac{e^{-j2\pi kt}}{-j2\pi k}\right]_{-1/8}^{1/8}$$

$$\therefore X[k] = \frac{4}{-j2\pi k}\left[e^{-j\frac{\pi}{4}k} - e^{j\frac{\pi}{4}k}\right] = \frac{4}{\pi k}\left[\frac{e^{-j\frac{\pi}{4}k} - e^{j\frac{\pi}{4}k}}{-j2}\right] \underset{identity}{\overset{euler's}{\rightarrow}} \frac{4}{\pi k}\sin\left(\frac{\pi k}{4}\right) = \text{sinc}\left(\frac{k}{4}\right)$$

To satisfy the books solution:

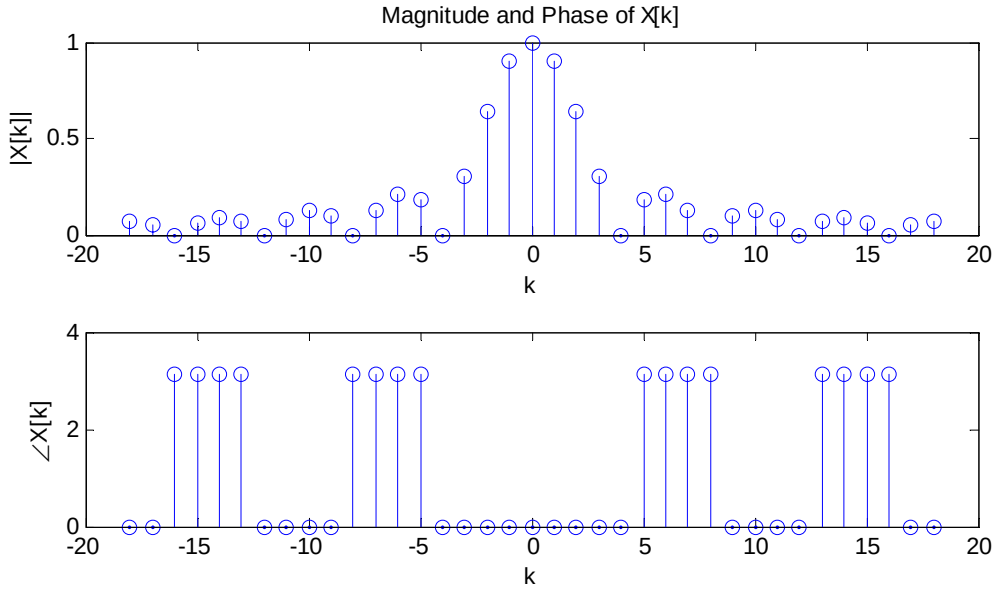
$$X_c[k] = X[k] + X^*[k] = 2\text{sinc}\left(\frac{k}{4}\right),\ X_s[k] = j(X[k] - X^*[k]) = 0$$

From appendix E, we know

$$rect(4t) * comb(t) \overset{FS}{\leftrightarrow} \frac{1}{4}\text{sinc}\left(\frac{k}{4}\right) \therefore 4rect(4t) * comb(t) \overset{FS}{\leftrightarrow} \text{sinc}\left(\frac{k}{4}\right)$$

Check with CTFT:

$$X(f) = \text{sinc}\left(\frac{f}{4}\right) comb(f) = \text{sinc}\left(\frac{f}{4}\right)\sum_{k=-\infty}^{\infty}\delta(f-k) = \sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{k}{4}\right)\delta(f-k)$$



(b) $x(t) = 4rect(4t) * \frac{1}{4} comb\left(\frac{t}{4}\right)$

$$X[k] = \frac{1}{T_0}\int_{T_0} x(t)e^{-j2\pi(kf_0)t}dt = \frac{1}{4}\int_{-1/2}^{1/2} \frac{4}{4} rect(4t)e^{-j\frac{2\pi kt}{4}}dt = \int_{-1/8}^{1/8} e^{-j\frac{\pi kt}{2}}dt$$

$$\therefore X[k] = \left[\int_{-1/8}^{1/8} \underbrace{\cos\left(\frac{\pi k}{2}t\right)}_{even}dt - j\int_{-1/8}^{1/8} \underbrace{\sin\left(\frac{\pi k}{2}t\right)}_{odd}dt\right] = 2\int_{0}^{1/8} \cos\left(\frac{\pi k}{2}t\right)dt = 2\left[\frac{\sin\left(\frac{\pi k}{2}t\right)}{\frac{\pi k}{2}}\right]_0^{1/8}$$

$$\therefore X[k] = 2\frac{\sin\left(\frac{\pi k}{16}\right)}{\frac{\pi k}{2}} = \frac{1}{4}\frac{\sin\left(\frac{\pi k}{16}\right)}{\frac{\pi k}{16}} = \frac{1}{4}\text{sinc}\left(\frac{k}{16}\right)$$

$$X_c[k] = X[k] + X^*[k] = \frac{1}{2}\text{sinc}\left(\frac{k}{16}\right),\ X_s[k] = j(X[k] - X^*[k]) = 0$$

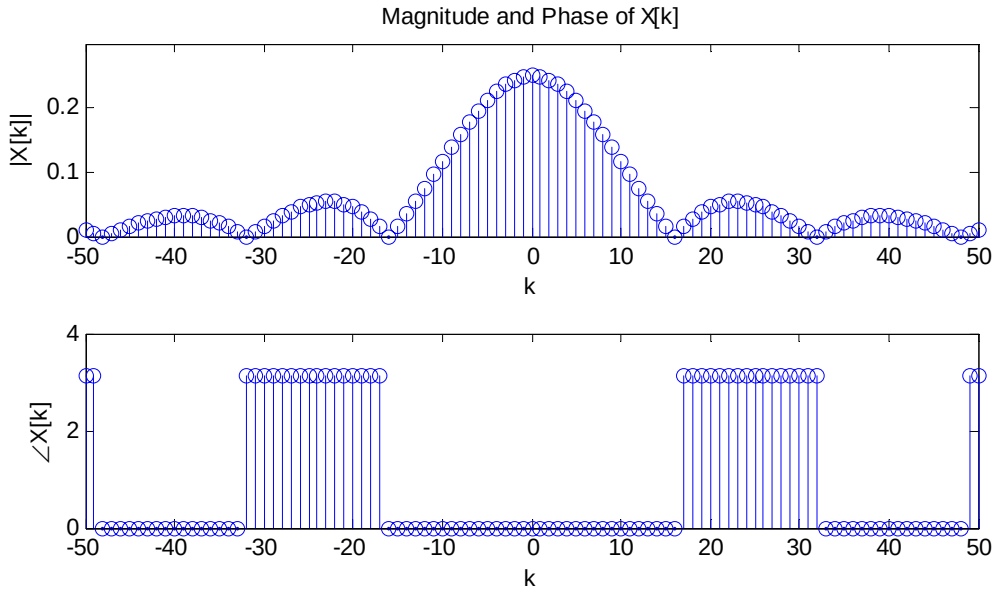
From appendix E, we know

$$rect(4t) * comb(t) \overset{FS}{\leftrightarrow} \frac{1}{4}\text{sinc}\left(\frac{k}{4}\right)$$

$$\therefore 4rect(4t) * \frac{1}{4} comb\left(\frac{t}{4}\right) \overset{FS}{\leftrightarrow} \frac{4}{16}\text{sinc}\left(\frac{k}{16}\right) = \frac{1}{4}\text{sinc}\left(\frac{k}{16}\right)$$

Check with CTFT:

$$X(f) = \text{sinc}\left(\frac{f}{4}\right) comb(4f) = \frac{1}{4}\sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k}{16}\right)\delta\left(f - \frac{k}{4}\right)$$



(c) $x(t)=\begin{cases}\text{sgn}(t) & |t|<1\\ 0 & 1<|t|<2\end{cases}$

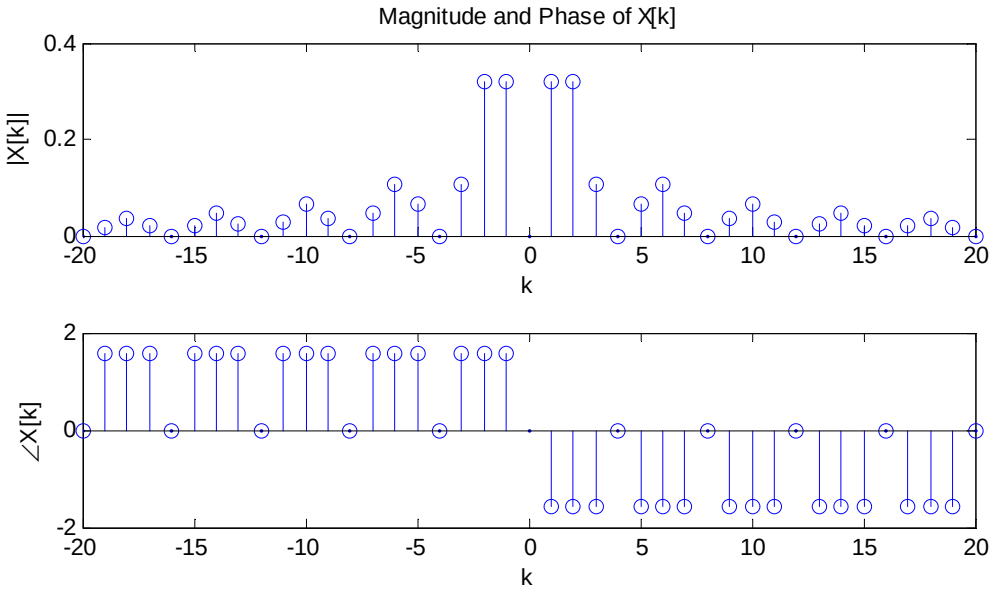
From the definition above, we know that $T_0=4$. Therefore,

$$X[k]=\frac{1}{T_0}\int_{T_0}x(t)e^{-j2\pi(kf_0)t}dt$$

$$=\frac{1}{4}\int_{-1}^{1}\text{sgn}(t)e^{-j\frac{\pi k}{2}t}dt=\frac{1}{4}\left[\int_{-1}^{1}\underbrace{\underbrace{\text{sgn}(t)}_{odd}\underbrace{\cos\left(\frac{\pi k}{2}t\right)}_{even}}_{odd}dt-j\int_{-1}^{1}\underbrace{\underbrace{\text{sgn}(t)}_{odd}\underbrace{\sin\left(\frac{\pi k}{2}t\right)}_{odd}}_{even}dt\right]$$

$$=\frac{1}{4}\left[-2j\int_{0}^{1}\underbrace{\text{sgn(t)}}_{=1}\sin\left(\frac{\pi k}{2}t\right)dt\right]=\frac{-j}{2}\int_{0}^{1}\sin\left(\frac{\pi k}{2}t\right)dt=j\frac{\cos\left(\frac{\pi k}{2}\right)-1}{\pi k}$$

$$X_c[k]=X[k]+X^*[k]=0,\ X_s[k]=j(X[k]-X^*[k])=-2\frac{\cos\left(\frac{\pi k}{2}\right)-1}{\pi k}$$



7)

(a) $x(t) = 10\sin(20\pi t), \quad T_F = \frac{1}{10}$

$$\sin(2\pi f_0 t) \xleftrightarrow{FS} \frac{j}{2}\left(\delta[k+1] - \delta[k-1]\right), \quad T_F = T_0$$

$$10\sin(20\pi t) \xleftrightarrow{FS} j5\left(\delta[k+1] - \delta[k-1]\right), \quad T_F = T_0 = \frac{1}{10}$$

(b) $x(t) = 2\cos(100\pi(t-0.005)), \quad T_F = \frac{1}{50}$

$$\cos(2\pi f_0 t) \xleftrightarrow{FS} \frac{1}{2}\left(\delta[k+1] + \delta[k-1]\right), \quad T_F = T_0$$

$$2\cos(100\pi t) \xleftrightarrow{FS} \delta[k+1] + \delta[k-1], \quad T_F = T_0 = \frac{1}{50}$$

$$2\cos(100\pi(t-0.005)) \xleftrightarrow{FS} \left(\delta[k+1] + \delta[k-1]\right)e^{-j2\pi k f_0 t_0}, \quad T_F = T_0 = \frac{1}{50}$$

$$2\cos(100\pi(t-0.005)) \xleftrightarrow{FS} \left(\delta[k+1] + \delta[k-1]\right)e^{-j0.5\pi k}, \quad T_F = T_0 = \frac{1}{50}$$

$$2\cos(100\pi(t-0.005)) \xleftrightarrow{FS} j\left(\delta[k+1] - \delta[k-1]\right), \quad T_F = T_0 = \frac{1}{50}$$

(c) $x(t) = -4\cos(500\pi t), \quad T_F = \frac{1}{50}$

$f_0 = 250 \rightarrow T_0 = \frac{1}{250} \Rightarrow T_F = 5T_0$

Using the table entry,

$$T_F = mT_0$$

$$\cos(2\pi f_0 t) \xleftrightarrow{FS} \frac{1}{2}\left(\delta[k-m] + \delta[k+m]\right)$$

We get,

$$\cos(500\pi t) \xleftrightarrow{FS} \frac{1}{2}\left(\delta[k-5] + \delta[k+5]\right), \quad T_F = 5T_0$$

$$-4\cos(500\pi t) \xleftrightarrow{FS} -2\left(\delta[k-5] + \delta[k+5]\right), \quad T_F = 5T_0$$

(d) $x(t)\frac{d}{dt}\left(e^{-j10\pi t}\right), \quad T_F = \frac{1}{5} = T_0$

$$e^{j2\pi f_0 t} \xleftrightarrow{FS} \delta[k-1], \quad T_F = T_0 = \frac{1}{5}$$

$$e^{-j10\pi t} \xleftrightarrow{FS} \delta[k+1], \quad T_F = T_0 = \frac{1}{5}$$

$$\frac{d}{dt}\left(e^{-j10\pi t}\right) \xleftrightarrow{FS} j10\pi k\delta[k+1], \quad T_F = T_0 = \frac{1}{5}$$

$$\frac{d}{dt}\left(e^{-j10\pi t}\right) \xleftrightarrow{FS} -j10\pi\delta[k+1], \quad T_F = T_0 = \frac{1}{5}$$

(e) $x(t) = rect(t) * comb\left(\frac{t}{4}\right), \quad T_F = 4$

$$rect\left(\frac{t}{w}\right) * \frac{1}{T_0} comb\left(\frac{t}{T_0}\right) \xleftrightarrow{FS} \frac{w}{T_0} \text{sinc}\left(\frac{w}{T_0} k\right), \quad T_F = T_0 = 4$$

$$rect(t) * \frac{1}{4} comb\left(\frac{t}{4}\right) \xleftrightarrow{FS} \frac{1}{4} \text{sinc}\left(\frac{k}{4}\right), \quad T_F = T_0 = 4$$

$$rect(t) * comb\left(\frac{t}{4}\right) \xleftrightarrow{FS} \text{sinc}\left(\frac{k}{4}\right), \quad T_F = T_0 = 4$$

(f) $x(t) = rect(t) * comb(t), \quad T_F = 1$

$$rect(t) * comb(t) \xleftrightarrow{FS} \text{sinc}(k), \quad T_F = T_0 = 1$$

The only non-zero value of sinc(k) (k an integer harmonic number) is at $k = 0$. Therefore sinc(k) = $\delta[k]$, giving us:

$$rect(t) * comb(t) \xleftrightarrow{FS} \delta[k]$$

(g) $x(t) = tri(t) * comb(t), \quad T_F = 1$

$$tri\left(\frac{t}{w}\right) * \frac{1}{T_0} comb\left(\frac{t}{T_0}\right) \xleftrightarrow{FS} \frac{w}{T_0} \text{sinc}^2\left(\frac{w}{T_0} k\right), \quad w = 1, T_0 = 1$$

$$tri(t) * comb(t) \xleftrightarrow{FS} \text{sinc}^2(k) = \delta[k]$$