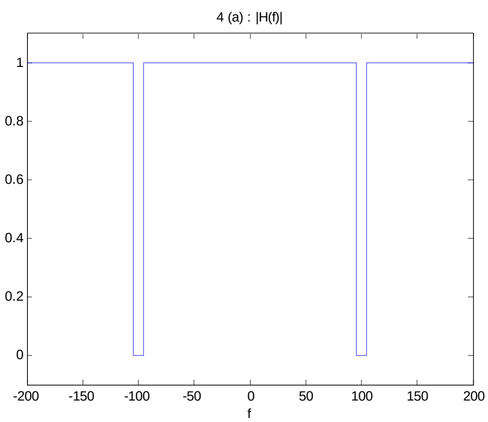
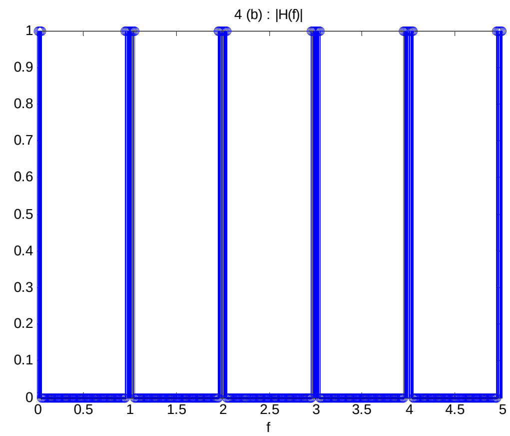
2704: Signals and Systems

Homework #7
Solutions

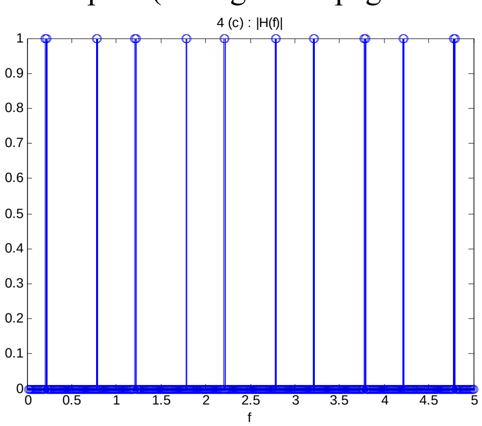
4) (a) Bandstop (see below)



(b) Lowpass (see figure on page 399 in text for explanation)



(c) Bandpass (see figure on page 399 in text for explanation)



7) (a) $H(f) = \text{sinc}(f)$ → $h(t) = \text{rect}(t)$ Not Causal

(b) $H(f) = \text{sinc}(f)\, e^{-j\pi f}$ → $h(t) = \text{rect}(t-0.5)$ Causal

(c) $H(j\omega) = \text{rect}(\omega)$ → $h(t) = \dfrac{1}{2\pi}\,\text{sinc}\left(\dfrac{t}{2\pi}\right)$ Not Causal

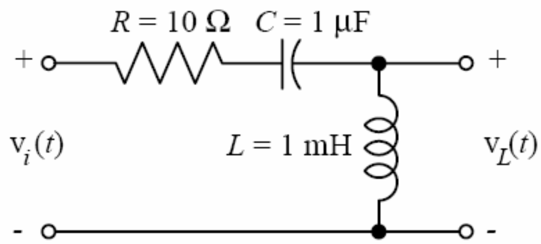
(d) $H(j\omega) = \text{rect}(\omega)\; e^{-j\omega}$ → $h(t) = \dfrac{1}{2\pi}\,\text{sinc}\left(\dfrac{t-1}{2\pi}\right)$ Not Causal

(e) $H(f) = A$ → $h(t) = A\delta(t)$ Causal

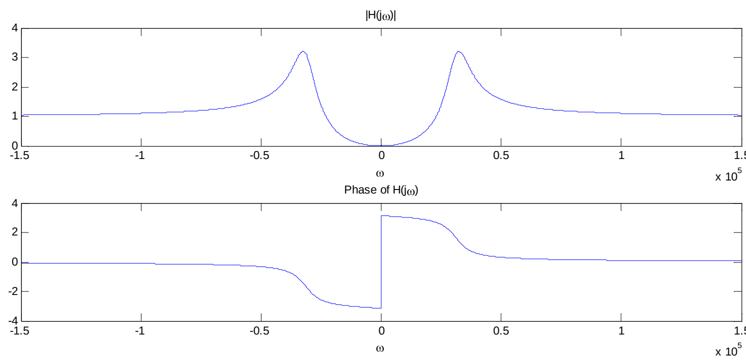
(e) $H(f) = A\, e^{j2\pi f}$ → $h(t) = A\delta(t+1)$ Not Causal

9) (a)

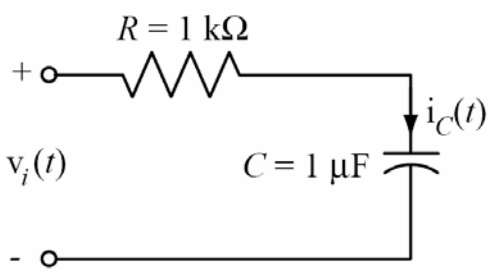
Excitation, $v_i(t)$ - Response, $v_L(t)$



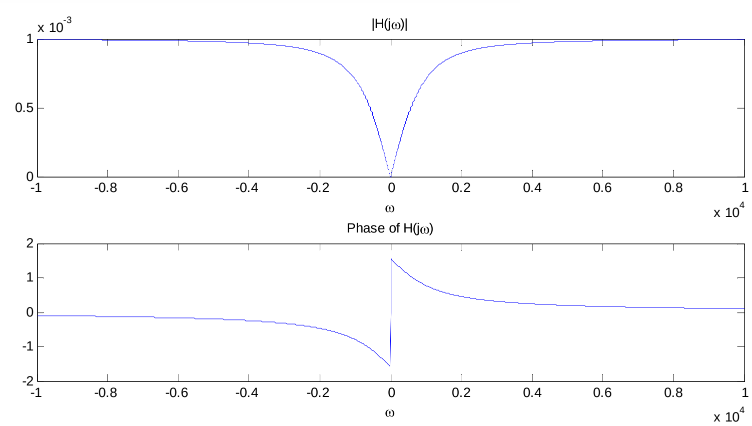
$$H(j\omega)=\frac{V_L(j\omega)}{V_i(j\omega)}=\frac{j\omega L}{j\omega L+\frac{1}{j\omega C}+R}=\frac{-\omega^2 LC}{1-\omega^2 LC+j\omega RC}$$



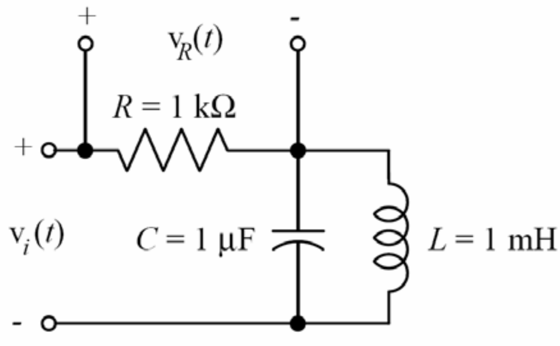
(b) Excitation, $v_i(t)$ - Response, $i_C(t)$



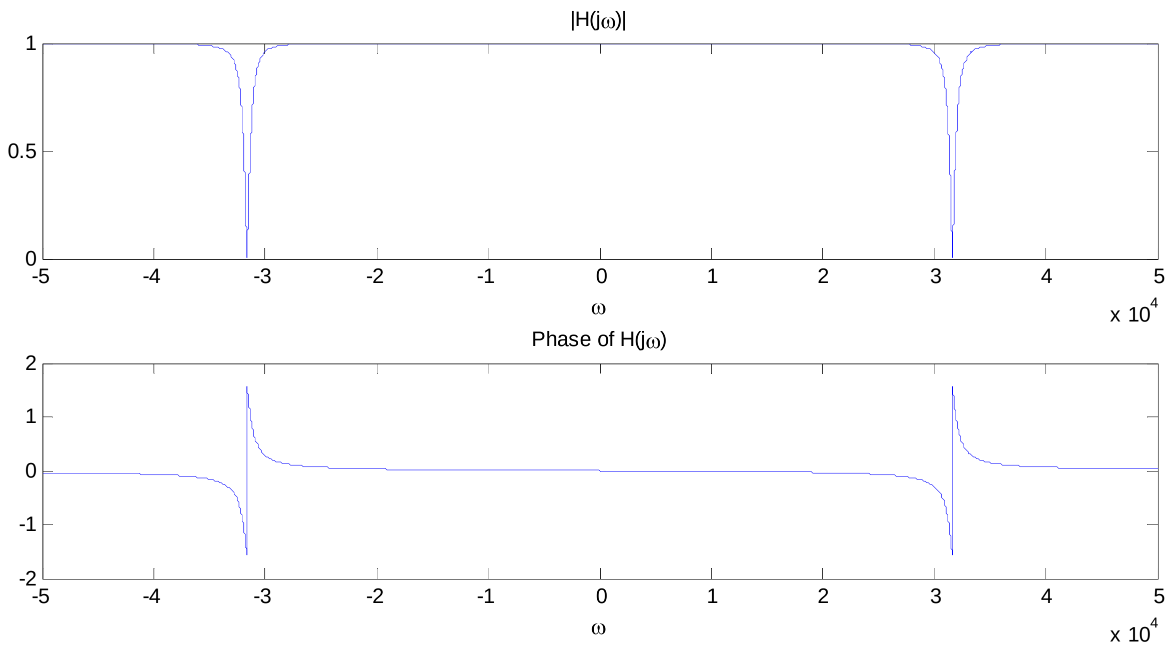
$$H(j\omega)=\frac{I_C(j\omega)}{V_i(j\omega)}=\frac{1}{R+\frac{1}{j\omega C}}=\frac{j\omega C}{j\omega RC+1}$$



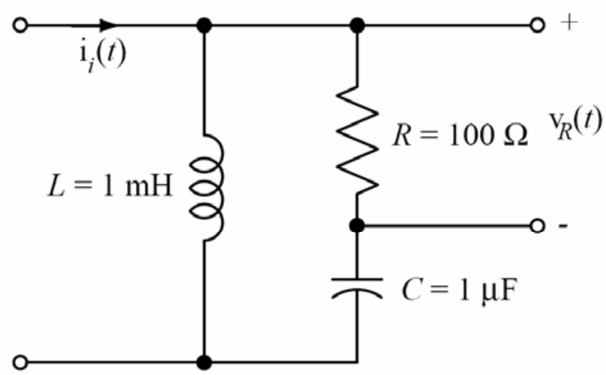
(c) Excitation, $v_i(t)$ – Response, $V_R(t)$



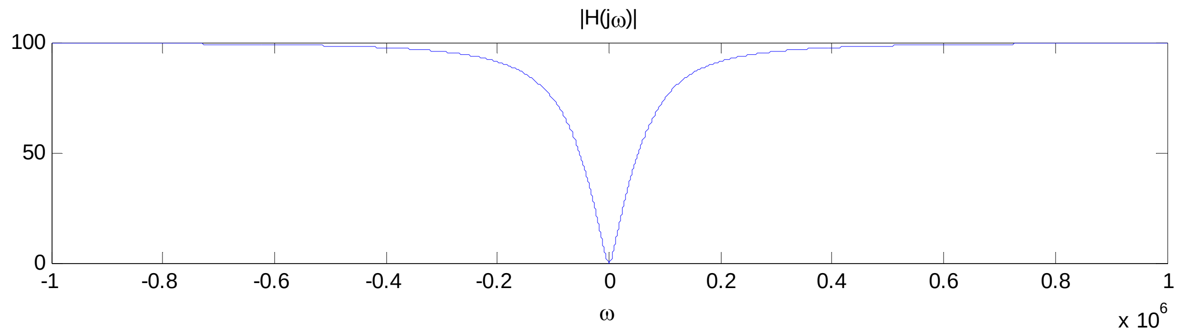
$$\mathrm{H}(j\omega) = \frac{\mathrm{V}_R(j\omega)}{\mathrm{V}_i(j\omega)} = \frac{R}{R + \dfrac{\dfrac{j\omega L}{j\omega C}}{j\omega L + \dfrac{1}{j\omega C}}} = \frac{R}{R + \dfrac{j\omega L}{1-\omega^2 LC}} = \frac{R\left(1-\omega^2 LC\right)}{R\left(1-\omega^2 LC\right) + j\omega L}$$

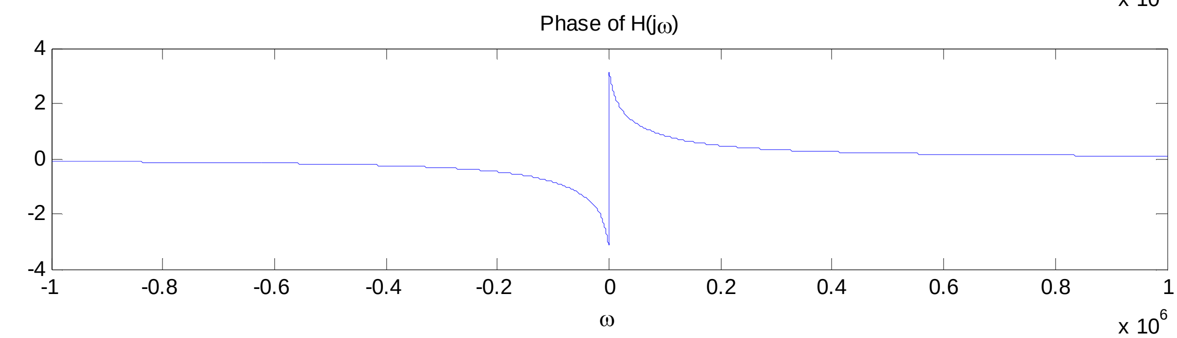


(d) Excitation, $i_i(t)$ - Response, $v_R(t)$



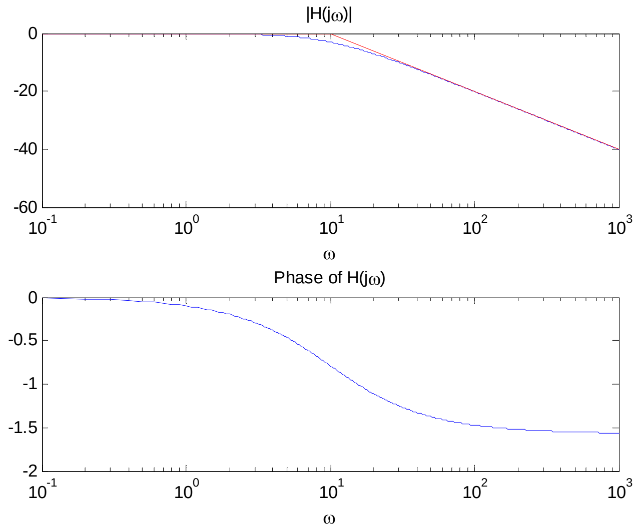
$$H(j\omega) = \frac{V_R(j\omega)}{I_i(j\omega)} = R\frac{I_{RC}(j\omega)}{I_i(j\omega)} = R\frac{j\omega L}{j\omega L + R + \frac{1}{j\omega C}} = -R\frac{\omega^2 LC}{1-\omega^2 LC + j\omega RC}$$



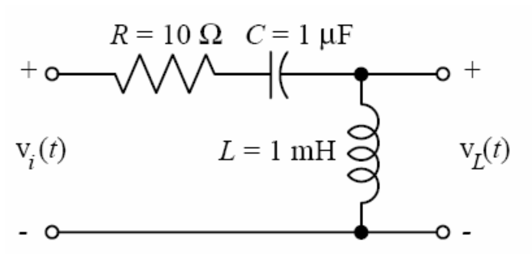


12. (a) An RC lowpass filter with R = 1MΩ and C = 0.1 μF. From the notes,

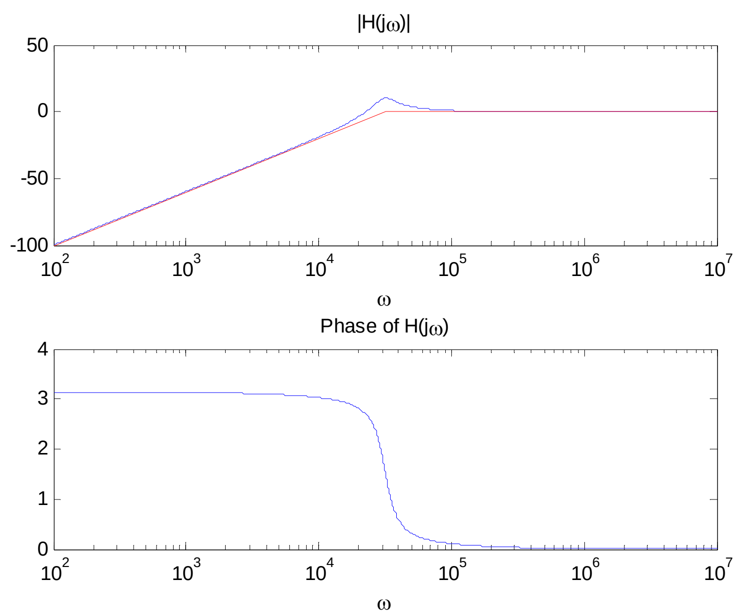
$$H(j\omega) = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j0.1\omega}$$



(b) From exercise 9a



Therefore $H(j\omega) = \dfrac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC} = \dfrac{-10^{-9}\omega^2}{1 - 10^{-9}\omega^2 + j\omega 10^{-5}}$



24) (a) $H(j\omega)=\dfrac{2}{j\omega}\rightarrow h(t)=\operatorname{sgn}(t)$ Not Causal

(b) $H(j\omega)=\dfrac{10}{6+j4\omega}\rightarrow h(t)=\dfrac{5}{2}e^{-\frac{3}{2}t}u(t)$ Causal

(c) $H(j\omega)=\dfrac{4}{25-\omega^2+j6\omega}=\dfrac{4}{(j\omega+3)^2+16}\rightarrow h(t)=e^{-3t}\sin(4t)u(t)$ Causal

(d) $H(j\omega)=\dfrac{4}{25-\omega^2+j6\omega}e^{j\omega}=\dfrac{4}{(j\omega+3)^2+16}e^{j\omega}\rightarrow h(t)=e^{-3t}\sin[4(t+1)]u(t+1)$ Not Causal

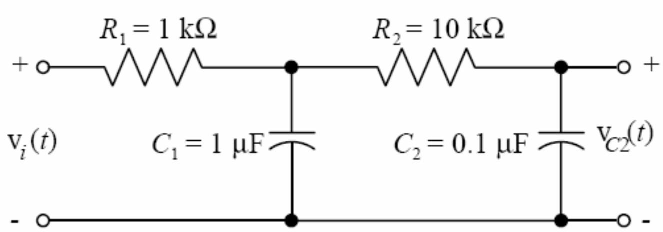
(e) $H(j\omega)=\dfrac{4}{25-\omega^2+j6\omega}e^{-j\omega}=\dfrac{4}{(j\omega+3)^2+16}e^{-j\omega}\rightarrow h(t)=e^{-3t}\sin[4(t-1)]u(t-1)$ Causal

(f) $H(j\omega)=\dfrac{j\omega+9}{45-\omega^2+j6\omega}=\dfrac{j\omega+3}{(j\omega+3)^2+36}+\dfrac{6}{(j\omega+3)^2+36}$ Causal

$\rightarrow h(t)=e^{-3t}[\cos(6t)+\sin(6t)]u(t)$

(g) $H(j\omega)=\dfrac{49}{49+\omega^2}=\dfrac{49}{14}\left(\dfrac{2(7)}{7^2+\omega^2}\right)\rightarrow h(t)=\dfrac{49}{14}e^{-7|t|}$ Not Causal

26) (a)



The transfer function can be found in multiple ways. One way is to think of this circuit as two voltage dividers. The first voltage division is from the excitation, $v_i(t)$, to the voltage across the first capacitor. The second voltage division is from that voltage to the response voltage, $v_{C2}(t)$.

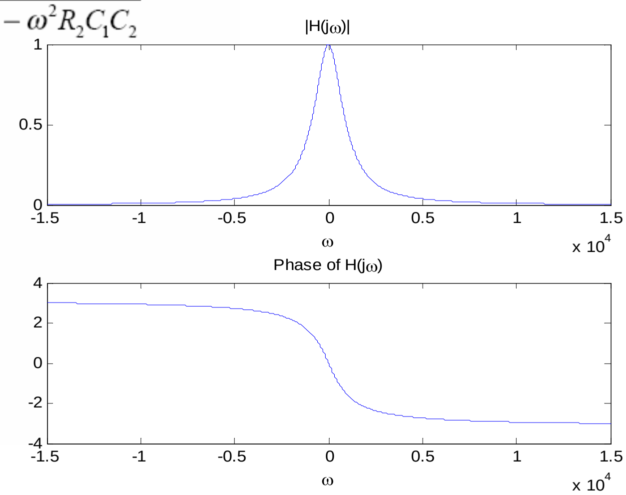
$$\mathrm{H}(j\omega)=\frac{V_{C2}(j\omega)}{V_i(j\omega)}=\underbrace{\frac{Z_\pi(j\omega)}{R_1+Z_\pi(j\omega)}}_{\text{first voltage division}}\underbrace{\frac{\frac{1}{j\omega C_2}}{R_2+\frac{1}{j\omega C_2}}}_{\text{second voltage division}}=\frac{Z_\pi(j\omega)}{R_1+Z_\pi(j\omega)}\frac{1}{j\omega R_2C_2+1}$$

$$Z_\pi(j\omega)=\frac{\frac{1}{j\omega C_1}\left(R_2+\frac{1}{j\omega C_2}\right)}{\frac{1}{j\omega C_1}+R_2+\frac{1}{j\omega C_2}}=\frac{R_2+\frac{1}{j\omega C_2}}{1+j\omega C_1R_2+\frac{C_1}{C_2}}=\frac{j\omega R_2C_2+1}{j\omega(C_1+C_2)-\omega^2R_2C_1C_2}$$

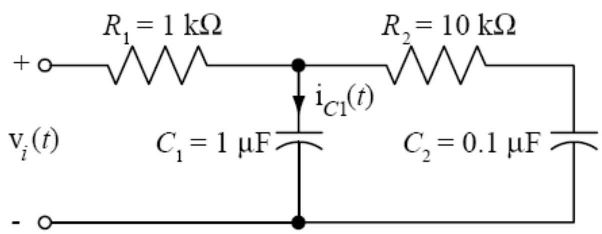
$$\mathrm{H}(j\omega)=\frac{\frac{j\omega R_2C_2+1}{j\omega(C_1+C_2)-\omega^2R_2C_1C_2}}{R_1+\frac{j\omega R_2C_2+1}{j\omega(C_1+C_2)-\omega^2R_2C_1C_2}}\frac{1}{j\omega R_2C_2+1}$$

$$\mathrm{H}(j\omega)=\frac{1}{(j\omega(C_1+C_2)-\omega^2R_2C_1C_2)R_1+j\omega R_2C_2+1}$$

$$\mathrm{H}(j\omega)=\frac{1}{1-\omega^2R_1R_2C_1C_2+j\omega[(C_1+C_2)R_1+R_2C_2]}$$



(b)



Think of the transfer function as the transfer function from the excitation to the current in R_1 times the transfer function from the current in R_1 to $i_{C1}(t)$.

$$\mathrm{H}(j\omega)=\frac{\mathrm{I}_{C1}(j\omega)}{\mathrm{V}_i(j\omega)}=\underbrace{\frac{\mathrm{I}_{R1}(j\omega)}{\mathrm{V}_i(j\omega)}}_{\text{first transfer function}}\underbrace{\frac{R_2+\dfrac{1}{j\omega C_2}}{\dfrac{1}{j\omega C_1}+R_2+\dfrac{1}{j\omega C_2}}}_{\text{second transfer function}}=\underbrace{\frac{1}{Z_i(j\omega)}}_{\text{input impedance}}\underbrace{\frac{j\omega R_2C_2+1}{j\omega R_2C_2+1+\dfrac{C_2}{C_1}}}_{\text{second transfer function}}$$

$$Z_i(j\omega)=R_1+Z_\pi(j\omega)=R_1+\frac{j\omega R_2C_2+1}{j\omega(C_1+C_2)-\omega^2R_2C_1C_2}$$

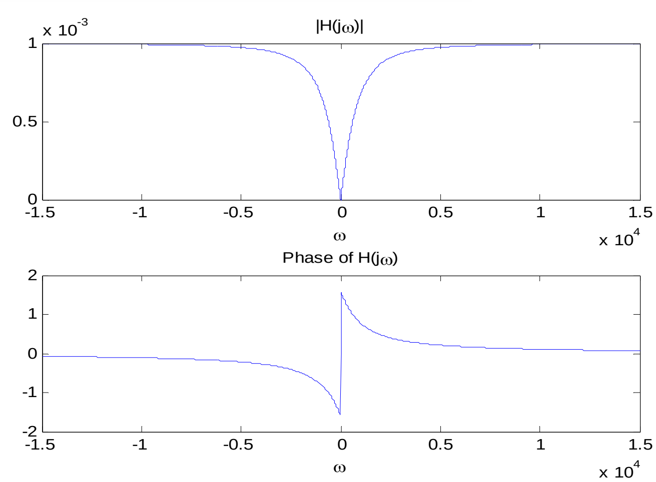
$$Z_i(j\omega)=\frac{R_1\left[j\omega(C_1+C_2)-\omega^2R_2C_1C_2\right]+j\omega R_2C_2+1}{j\omega(C_1+C_2)-\omega^2R_2C_1C_2}$$

$$\mathrm{H}(j\omega)=\frac{j\omega(C_1+C_2)-\omega^2R_2C_1C_2}{R_1\left[j\omega(C_1+C_2)-\omega^2R_2C_1C_2\right]+j\omega R_2C_2+1}\frac{j\omega R_2C_2+1}{j\omega R_2C_2+1+\dfrac{C_2}{C_1}}$$

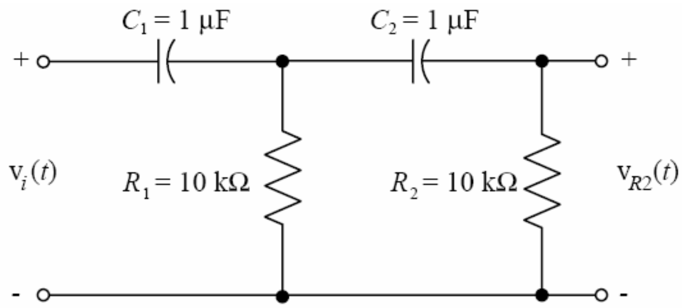
$$\mathrm{H}(j\omega)=\frac{j\omega C_1\left[\left(1+\dfrac{C_2}{C_1}\right)+j\omega R_2C_2\right]}{R_1\left[j\omega(C_1+C_2)-\omega^2R_2C_1C_2\right]+j\omega R_2C_2+1}\frac{j\omega R_2C_2+1}{j\omega R_2C_2+1+\dfrac{C_2}{C_1}}$$

$$\mathrm{H}(j\omega)=\frac{j\omega C_1(j\omega R_2C_2+1)}{R_1\left[j\omega(C_1+C_2)-\omega^2R_2C_1C_2\right]+j\omega R_2C_2+1}$$

$$\mathrm{H}(j\omega)=\frac{j\omega C_1(j\omega R_2C_2+1)}{1-\omega^2R_1R_2C_1C_2+j\omega\left[R_1(C_1+C_2)+R_2C_2\right]}$$



(c)



$$\mathrm{H}(j\omega) = \frac{\mathrm{V}_{R2}(j\omega)}{\mathrm{V}_i(j\omega)} = \frac{Z_\pi(j\omega)}{\frac{1}{j\omega C_1} + Z_\pi(j\omega)} \frac{R_2}{\frac{1}{j\omega C_2} + R_2} = \frac{j\omega C_1 Z_\pi(j\omega)}{1 + j\omega C_1 Z_\pi(j\omega)} \frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2}$$

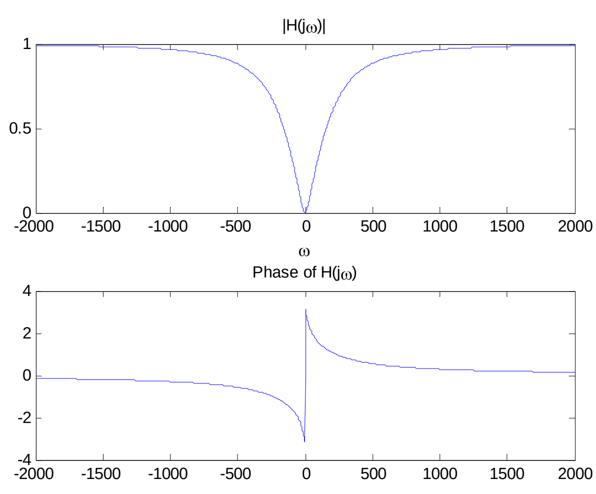
$$Z_\pi(j\omega) = \frac{R_1\left(\frac{1}{j\omega C_2} + R_2\right)}{R_1 + \frac{1}{j\omega C_2} + R_2} = R_1 \frac{1 + j\omega R_2 C_2}{j\omega C_2(R_1 + R_2) + 1}$$

$$\mathrm{H}(j\omega) = \frac{j\omega C_1 R_1 \frac{1 + j\omega R_2 C_2}{j\omega C_2(R_1 + R_2) + 1}}{1 + j\omega C_1 R_1 \frac{1 + j\omega R_2 C_2}{j\omega C_2(R_1 + R_2) + 1}} \frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2}$$

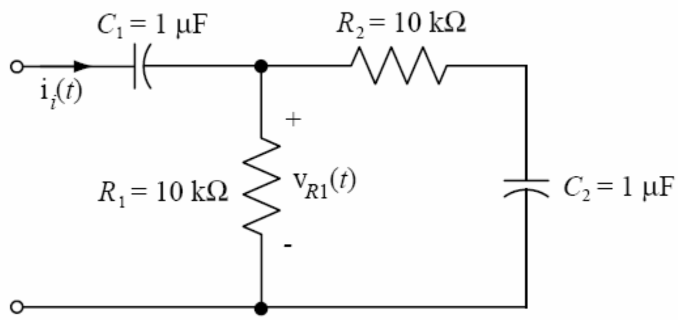
$$\mathrm{H}(j\omega) = \frac{-\frac{\omega^2 R_1 R_2 C_1 C_2}{j\omega C_2(R_1 + R_2) + 1}}{1 + j\omega C_1 R_1 \frac{1 + j\omega R_2 C_2}{j\omega C_2(R_1 + R_2) + 1}}$$

$$\mathrm{H}(j\omega) = -\frac{\omega^2 R_1 R_2 C_1 C_2}{j\omega C_2(R_1 + R_2) + 1 + j\omega C_1 R_1(1 + j\omega R_2 C_2)}$$

$$\mathrm{H}(j\omega) = -\frac{\omega^2 R_1 R_2 C_1 C_2}{1 - \omega^2 R_1 R_2 C_1 C_2 + j\omega\left(C_2(R_1 + R_2) + C_1 R_1\right)}$$

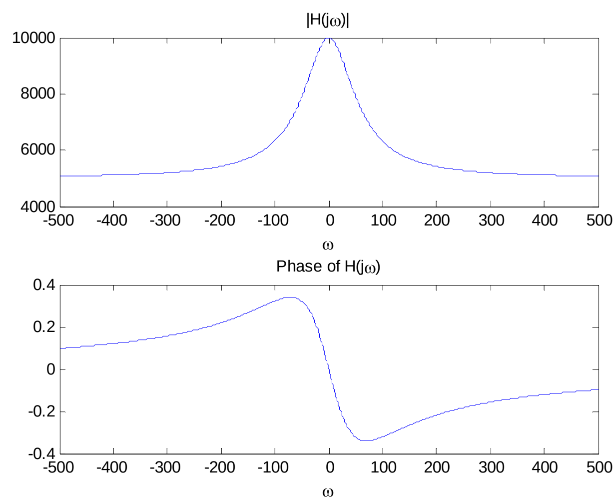


(d)

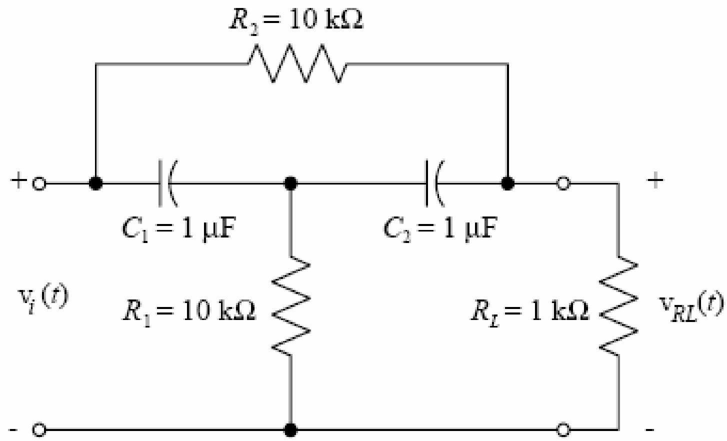


$$H(j\omega) = \frac{V_{R1}(j\omega)}{I_i(j\omega)} = R_1 \frac{I_{R1}(j\omega)}{I_i(j\omega)} = R_1 \frac{R_2 + \frac{1}{j\omega C_2}}{R_1 + R_2 + \frac{1}{j\omega C_2}}$$

$$H(j\omega) = R_1 \frac{j\omega R_2 C_2 + 1}{j\omega (R_1 + R_2) C_2 + 1}$$



(e)



Write two nodal equations and solve for the transfer function.

Summing currents to zero at the middle node and the right-hand node,

$$V_{R_1}(j\omega)[j\omega C_1 + j\omega C_2 + G_1] - V_i(j\omega)j\omega C_1 - V_{RL}(j\omega)j\omega C_2 = 0$$

$$V_{RL}(j\omega)[j\omega C_2 + G_L + G_2] - V_i(j\omega)G_2 - V_{R_1}(j\omega)j\omega C_2 = 0$$

$$\begin{bmatrix} j\omega(C_1+C_2)+G_1 & -j\omega C_2 \\ -j\omega C_2 & j\omega C_2 + (G_L+G_2) \end{bmatrix} \begin{bmatrix} V_{R_1}(j\omega) \\ V_{RL}(j\omega) \end{bmatrix} = \begin{bmatrix} j\omega C_1 \\ G_2 \end{bmatrix} V_i(j\omega)$$

$$\Delta = [j\omega(C_1+C_2)+G_1][j\omega C_2 + (G_L+G_2)] + \omega^2 C_2^2$$

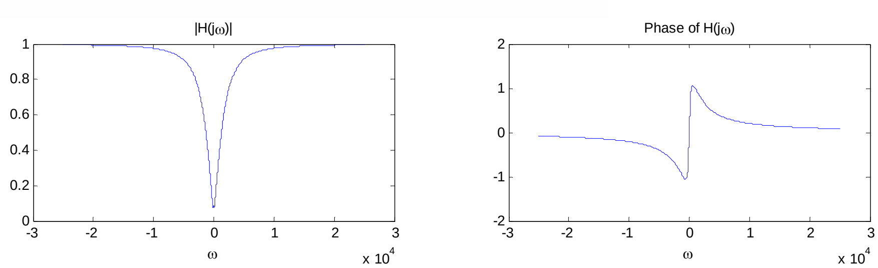
$$\Delta = -\omega^2 C_1 C_2 + j\omega[(C_1+C_2)(G_L+G_2) + G_1 C_2] + G_1(G_L+G_2)$$

$$V_{RL}(j\omega) = \frac{1}{\Delta}\begin{vmatrix} j\omega(C_1+C_2)+G_1 & j\omega C_1 \\ -j\omega C_2 & G_2 \end{vmatrix} V_i(j\omega)$$

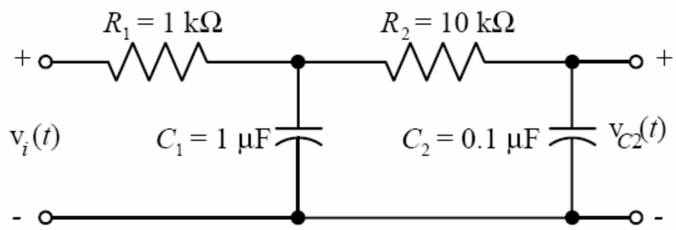
$$H(j\omega) = \frac{V_{RL}(j\omega)}{V_i(j\omega)} = \frac{-\omega^2 C_1 C_2 + j\omega(C_1+C_2)G_2 + G_1 G_2}{-\omega^2 C_1 C_2 + j\omega[(C_1+C_2)(G_L+G_2) + G_1 C_2] + G_1(G_L+G_2)}$$

$$H(j\omega) = \frac{-\omega^2 C_1 C_2 + j\omega\frac{C_1+C_2}{R_2} + \frac{1}{R_1 R_2}}{-\omega^2 C_1 C_2 + j\omega\left[(C_1+C_2)\left(\frac{1}{R_L}+\frac{1}{R_2}\right) + \frac{C_2}{R_1}\right] + \frac{1}{R_1}\left(\frac{1}{R_L}+\frac{1}{R_2}\right)}$$

$$H(j\omega) = \frac{-\omega^2 R_1 R_2 C_1 C_2 + j\omega R_1(C_1+C_2) + 1}{-\omega^2 R_1 R_2 C_1 C_2 + j\omega\left[(C_1+C_2)\left(1+\frac{R_2}{R_L}\right)R_1 + R_2 C_2\right] + \frac{R_L+R_2}{R_L}}$$



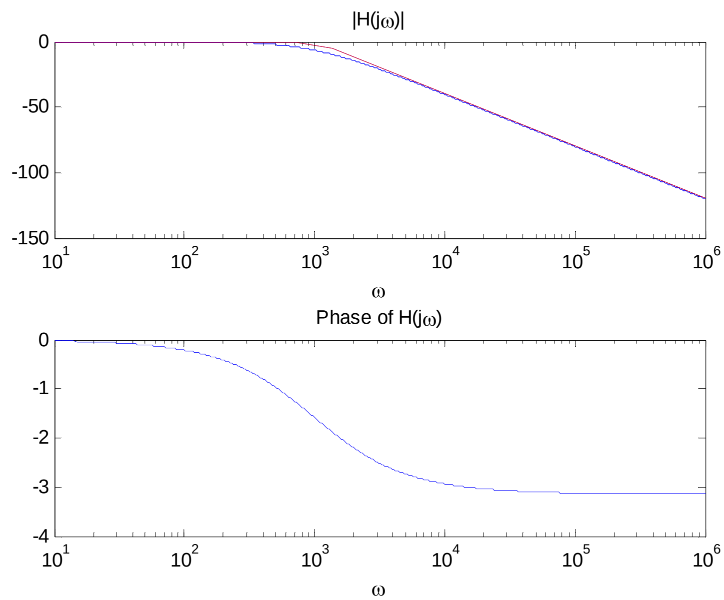
29) (a)



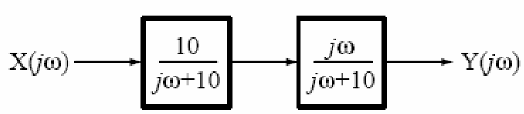
From Exercise 26(b)

$$H(j\omega) = \frac{1}{1 - \omega^2 R_1 R_2 C_1 C_2 + j\omega\left[(C_1 + C_2)R_1 + R_2 C_2\right]}$$

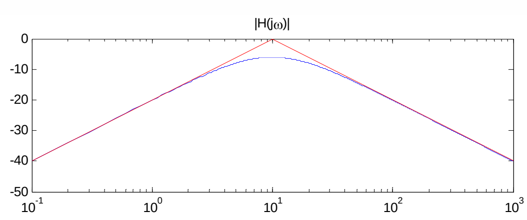
$$H(j\omega) = \frac{1}{1 - 10^{-6}\omega^2 + j2.1 \times 10^{-3}\omega}$$

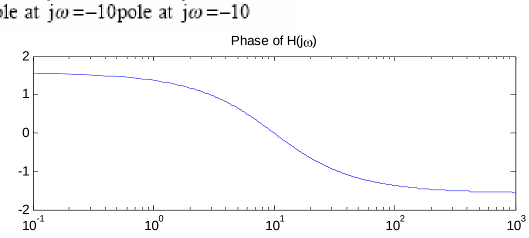


(b)



$$H(j\omega) = \frac{10}{j\omega + 10}\frac{j\omega}{j\omega + 10} = \frac{j10\omega}{(j\omega + 10)^2} = \underbrace{\frac{1}{10}}_{\substack{\text{frequency}\\\text{independent}\\\text{gain}}} \frac{\overbrace{j\omega}^{\text{zero at } j\omega = 0}}{\underbrace{\left(j\frac{\omega}{10} + 1\right)}_{\text{pole at } j\omega = -10}\underbrace{\left(j\frac{\omega}{10} + 1\right)}_{\text{pole at } j\omega = -10}}$$





(c) $\mathrm{H}(j\omega)=\dfrac{j20\omega}{10{,}000-\omega^2+j20\omega}$

$$\mathrm{H}(j\omega)=\frac{j20\omega}{(j\omega+10-j99.5)(j\omega+10+j99.5)}=\frac{j20\omega}{10000\left[1+j\dfrac{\omega}{500}-\dfrac{\omega^2}{10000}\right]}$$

$$\mathrm{H}(j\omega)=\frac{1}{500}\frac{j\omega}{1+j\dfrac{\omega}{500}-\dfrac{\omega^2}{10000}}$$