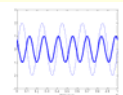



ECE 2704
 Signals and Systems
 Spring 2006

Instructor: Dr. R. Michael Buehrer
 Lecture #3: Description and Analysis of Systems

Introduction

- What to read:
 - Chapter 3 – Sections 3.1-3.3
- Purpose of today's lecture
 - To define the concept of a system
 - To describe some important properties of systems that will be useful in this course
 - To provide examples which illustrate these properties

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What is a System?

- "System" is an engineering term that covers a very broad range of things
- Definitions
 - Anything that takes an input, processes it and produces an output
 - Anything that responds when stimulated
- Types of systems
 - Electrical systems
 - Biological systems
 - Mechanical systems
 - Computer systems
 - Economic systems

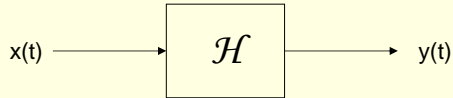
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System Representation



- \mathcal{H} typically used to represent the system
- $x(t)$ typically used to represent the excitation or input to the system
- $y(t)$ typically used to represent the response or output of the system
- Systems can have multiple inputs and/or multiple outputs
- Example of a Single-Input Single Output system:

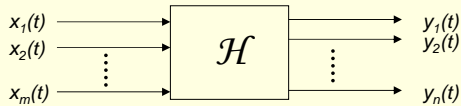


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General System



- In general, a system can have many inputs and many outputs
- This is often referred to as a Multi-Input Multi-Output (MIMO) system



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Goal



- Our goal is to mathematically describe
 - System Input
 - System components and how the components interact
- Using this mathematical representation we can predict how the system will respond to a given input
 - This allows us to design a system based on expected inputs
- We will use the following notation

$$x(t) \xrightarrow{\mathcal{H}} y(t)$$

to represent "the excitation $x(t)$ of system \mathcal{H} produces the response $y(t)$ ":

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Input-Output Relationships



- There are many ways to relate the system output to the system input.
- Instantaneous Relationships
 - The output at time t is simply an algebraic equation of the input at time t
 - For example

or
$$y(t) = Ax(t) + B$$

$$y(t) = Ax(t) + Bx^3(t)$$

Input-Output Relationships (cont.)



- Linear, Constant Coefficient, Ordinary Differential Equations
 - The output is related to the input through integrals and derivatives. Removing all integrals through repeated derivatives:

$$\begin{aligned} a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t) \\ = b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + b_0 x(t) \end{aligned}$$

Input-Output Relationships (cont.)



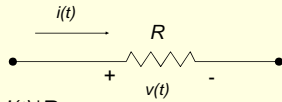
- Integral Relationships

$$y(t) = \int_{-\infty}^{\infty} h(\lambda) x(t - \lambda) d\lambda$$

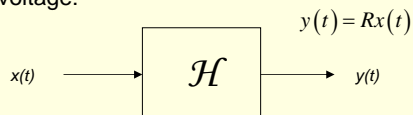
- This is known as the *superposition integral*
- $x(t)$ is input, $y(t)$ is the output and $h(t)$ is called the *impulse response* of the system
 - We will talk more about this later in the course

Example

- Voltage drop across a resistor



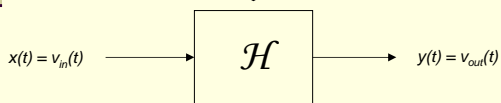
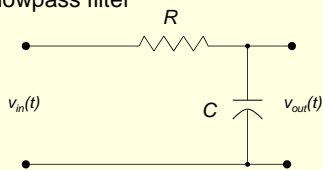
- $v(t) = i(t) \cdot R$
- If the input is the current and output is the voltage:



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Example

- RC lowpass filter



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Example

- Using Kirchhoff's Voltage law, the sum of the voltages around the circuit is zero:

$$v_{in}(t) = Ri(t) + v_{out}(t)$$

- The voltage across the capacitor is related to the current through the resistor

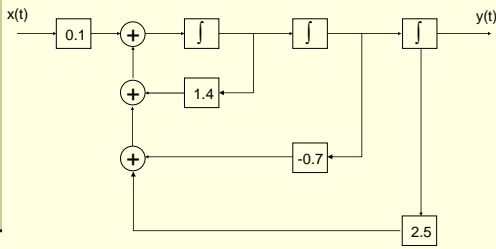
$$i(t) = C \frac{dv_{out}(t)}{dt}$$

- Eliminating $i(t)$ from the previous equation

$$v_{in}(t) = RC \frac{dv_{out}(t)}{dt} + v_{out}(t)$$

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Example



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System Properties

- There are several properties of systems that are important to understand
- Many properties allow us to make simplifications in our analysis
- Specific properties
 - Homogeneity
 - Time Invariance
 - Additivity
 - Linearity
 - Stability
 - Causality
 - Memory
 - Invertibility

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Homogeneity

- A system is said to be *homogeneous* (i.e., exhibit the property of homogeneity) if multiplying the input by a constant results in the multiplication of the output by that same constant

$$x(t) \xrightarrow{\mathcal{H}} y(t) \Rightarrow Kx(t) \xrightarrow{\mathcal{H}} Ky(t)$$

- Note that K could be *complex*

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Example

■ Assume $y(t) = 4x(t)$

■ Let $x_1(t) = 2u(t)$

then $y_1(t) = 4x(t)$
 $= 8u(t)$

Now, let $x_2(t) = Kx_1(t)$
 $= 2Ku(t)$

$y_2(t) = 4x_2(t)$
 $= 8Ku(t)$

→ $y_2(t) = Ky_1(t)$ when
 $x_2(t) = Kx_1(t)$. Thus,
the system is
homogeneous

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Example

■ Assume $y(t) = 4x(t) + B$

■ Let $x_1(t) = 2u(t)$

then $y_1(t) = 4x(t) + B$
 $= 8u(t) + B$

Now, let $x_2(t) = Kx_1(t)$
 $= 2Ku(t)$

$y_2(t) = 4x_2(t) + B$
 $= 8Ku(t) + B$

→ $y_2(t) \neq Ky_1(t)$ when
 $x_2(t) = Kx_1(t)$. Thus,
the system is not
homogeneous

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Time Invariance

- If an arbitrary excitation $x(t)$ causes a response $y(t)$ and an excitation $x(t-t_0)$ causes a response $y(t-t_0)$, for any t_0 , the system is said to be time-invariant.

$$x(t) \xrightarrow{\mathcal{H}} y(t) \Rightarrow x(t-t_0) \xrightarrow{\mathcal{H}} y(t-t_0)$$

- In other words, a time-invariant system is one whose output doesn't depend on when the input is applied

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Example



Assume $y(t) = \{250 + 25t\}x(t)$

- Let signal $x_1(t) = \cos(2\pi t)u(t)$

$$y_1(t) = \{250 + 25t\}x_1(t) \\ = \{250 + 25t\}\cos(2\pi t)u(t)$$

- Let $x_2(t)$ be a time shifted version of $x_1(t)$:

$$y_2(t) = \{250 + 25t\}x_2(t) \\ = \{250 + 25t\}x_1(t - t_o) \\ = \{250 + 25t\}\cos(2\pi(t - t_o))u(t - t_o) \\ = \{250 + 25t\}\cos(2\pi t - 2\pi t_o)u(t - t_o) \\ = \{250 + 25t\}\cos(2\pi t - \theta_o)u(t - t_o)$$

Example (cont.)



- For time invariance $y_2(t) = y_1(t - t_o)$ when $x_2(t) = x_1(t - t_o)$

$$y_1(t - t_o) = \{250 + 25(t - t_o)\}\cos(2\pi(t - t_o))u(t - t_o) \\ = \{(250 - 25t_o) + 25t\}\cos(2\pi t - 2\pi t_o)u(t - t_o) \\ = \{(250 - 25t_o) + 25t\}\cos(2\pi t - \theta_o)u(t - t_o)$$

$$y_2(t) = \{250 + 25t\}\cos(2\pi t - \theta_o)u(t - t_o)$$

NOTE: We could have guessed this from the system definition. If any coefficients of the system are functions of time, then we have a time-varying system.

$$y_2(t) \neq y_1(t - t_o)$$

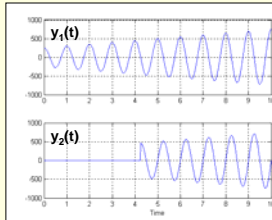
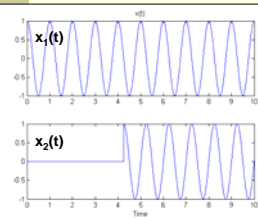
Thus, the system is time-varying

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Example (cont.)



- Input and output plots for $t_o = 4.25$



We can see that while $x_1(t)$ is simply a time-shifted version of $x_2(t)$, $y_1(t)$ is not a time-shifted version of $y_2(t)$

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Example 2



However, if $y(t) = 250x(t)$

Then if signal $x_1(t) = \cos(2\pi t)u(t)$

$$y_1(t) = 250x_1(t) \\ = 250\cos(2\pi t)u(t)$$

- Now if $x_2(t)$ is simply a time shifted version of $x_1(t)$:

$$y_2(t) = 250x_2(t) \\ = 250x_1(t - t_o) \\ = 250\cos(2\pi(t - t_o))u(t - t_o) \\ = 250\cos(2\pi t - \theta_o)u(t - t_o)$$

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Example 2 (cont.)



- For time invariance $y_2(t) = y_1(t - t_o)$ when $x_2(t) = x_1(t - t_o)$

$$y_1(t - t_o) = 250\cos(2\pi(t - t_o))u(t - t_o) \\ = 250\cos(2\pi t - \theta_o)u(t - t_o)$$

$$y_2(t) = 250\cos(2\pi t - \theta_o)u(t - t_o)$$

$$y_2(t) = y_1(t - t_o)$$

Thus, the system is time-invariant

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Additivity



- Assume that a system produces the output $y_1(t)$ when excited by input $x_1(t)$ and the output $y_2(t)$ when excited by input $x_2(t)$. If the input $x_1(t) + x_2(t)$ produces the output $y_1(t) + y_2(t)$ the system is said to be *additive*.

$$x_1(t) \xrightarrow{\mathcal{H}} y_1(t) \quad \text{and} \quad x_2(t) \xrightarrow{\mathcal{H}} y_2(t) \Rightarrow x_1(t) + x_2(t) \xrightarrow{\mathcal{H}} y_1(t) + y_2(t)$$

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Linearity



- Any system which is both *homogeneous* and *additive* is said to be *linear*:

$$x_1(t) \xrightarrow{\mathcal{H}} y_1(t) \quad \text{and} \quad x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$$
$$\Rightarrow Ax_1(t) + Bx_2(t) \xrightarrow{\mathcal{H}} Ay_1(t) + By_2(t)$$

- This characteristic is also termed *superposition* since if we superpose two inputs (put them on top of one another) the result is that the corresponding outputs are superposed.

Example



- Determine whether or not the following system is linear

$$\frac{dy(t)}{dt} + ty(t) = x(t)$$

- The responses due to a specific pair of inputs $x_1(t)$ and $x_2(t)$ are

$$\frac{dy_1(t)}{dt} + ty_1(t) = x_1(t)$$

$$\frac{dy_2(t)}{dt} + ty_2(t) = x_2(t)$$

Example (cont.)



- Multiplying these two equations by α_1 and α_2 respectively and adding them gives

$$\alpha_1 \frac{dy_1(t)}{dt} + \alpha_2 \frac{dy_2(t)}{dt} + \alpha_1 ty_1(t) + \alpha_2 ty_2(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$$

$$\frac{d}{dt} \{ \alpha_1 y_1(t) + \alpha_2 y_2(t) \} + t \{ \alpha_1 y_1(t) + \alpha_2 y_2(t) \} = \alpha_1 x_1(t) + \alpha_2 x_2(t)$$

- Thus, superposition holds and the system is linear.

Example



- Determine whether or not the following system is linear

$$\frac{dy(t)}{dt} + 10y(t) + 5 = x(t)$$

- The responses due to a specific pair of inputs $x_1(t)$ and $x_2(t)$ are

$$\frac{dy_1(t)}{dt} + 10y_1(t) + 5 = x_1(t)$$

$$\frac{dy_2(t)}{dt} + 10y_2(t) + 5 = x_2(t)$$

Example (cont.)



- Multiplying these two equations by α_1 and α_2 respectively and adding them gives

$$\alpha_1 \frac{dy_1(t)}{dt} + \alpha_2 \frac{dy_2(t)}{dt} + \alpha_1 10y_1(t) + \alpha_2 10y_2(t) + 5\alpha_1 + 5\alpha_2 = \alpha_1 x_1(t) + \alpha_2 x_2(t)$$

$$\frac{d}{dt} \{ \alpha_1 y_1(t) + \alpha_2 y_2(t) \} + 10 \{ \alpha_1 y_1(t) + \alpha_2 y_2(t) \}$$

This term prevents the system from being linear

$$+ 5(\alpha_1 + \alpha_2) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$$

- Thus, superposition does not hold and the system is *not* linear.

LTI Systems



- The most common systems analyzed in practical system design and analysis are Linear Time-Invariant (LTI) systems
- LTI systems are linear and time-invariant
- The vast majority of systems analyzed in this course will be LTI
- In the coming weeks we will show that if a system is LTI, we can exploit this property to find the system response to an arbitrary input using the concept of the *impulse response*

Stability



- A signal is bounded if it is less than some finite value for all time:

$$|x(t)| < \infty \quad -\infty < t < \infty$$

- Any system for which the output is bounded when the input is bounded is called a bounded-input bounded-output (BIBO) stable system
- An example of an *unstable* system is

$$y(t) = A(1 + x(t))^t$$

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Causality



- Any system for which the response occurs only during or after the time in which the excitation is applied is called a *causal* system.
- All physical systems are causal
- However, some useful signal processing techniques are superficially non-causal. These techniques can only be used by storing the input signal in memory thus having the entire history of the input available for processing.
 - This inserts a delay into the system

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Memory

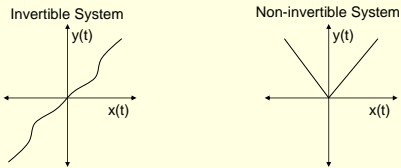


- If any system's output $y(t)$ at an arbitrary time t_0 , $y(t_0)$, depends only on the input at t_0 , $x(t_0)$, then the system is said to be *static*.
- However, if a system $y(t_0)$ depends on past inputs or past outputs, the system is said to be *dynamic* or which also means to have *memory*.
- Circuits with energy storage elements (inductors or capacitors) exhibit memory since their output depends on past values as well as current values.

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Invertibility

- A system is said to be invertible if unique inputs produces unique outputs
- In such a case given an output, theoretically, we can determine the input which caused it

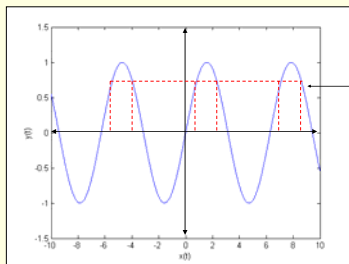


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Example – Non-invertible System

$$y(t) = \sin(x(t))$$



One output corresponds to multiple inputs. Thus, there is no unique mapping between a given output and possible inputs.

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Overview

- In this lecture we have discussed the definition of a system and several system properties
 - Homogeneity
 - Additivity
 - Linearity
 - Time-invariance
 - Causality
 - Memory
 - Stability

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