

# 2704: Signals and Systems

## Quiz #3

February 22, 2006

Given the function

$$\text{rect}(t) = \begin{cases} 1 & |t| \leq 1/2 \\ 0 & |t| > 1/2 \end{cases}$$

If  $h(t) = \text{rect}(t/T)$  is the impulse response of a system  $H$ , determine the output when the input is  $x(t) = \text{rect}(t)$ .

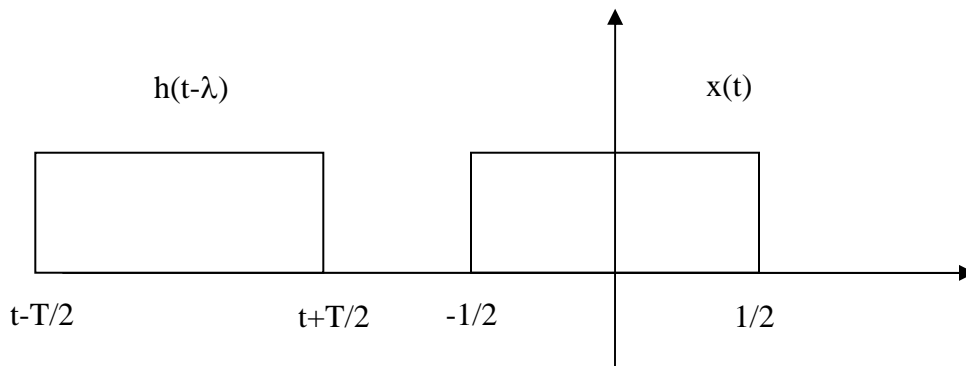
*First, we must recall that if  $h(t)$  is the impulse response of a LTI system, the output  $y(t)$  corresponding to any input  $x(t)$  can be written as*

$$y(t) = x(t) * h(t)$$

*In our case  $h(t) = \text{rect}(t/T)$  and  $x(t) = \text{rect}(t)$ . Thus,*

$$y(t) = \text{rect}(t) * \text{rect}(t/T)$$

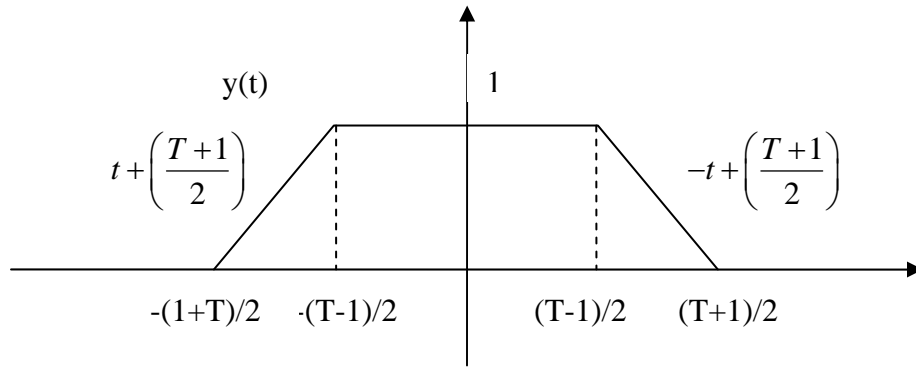
*We can determine the output using graphical convolution*



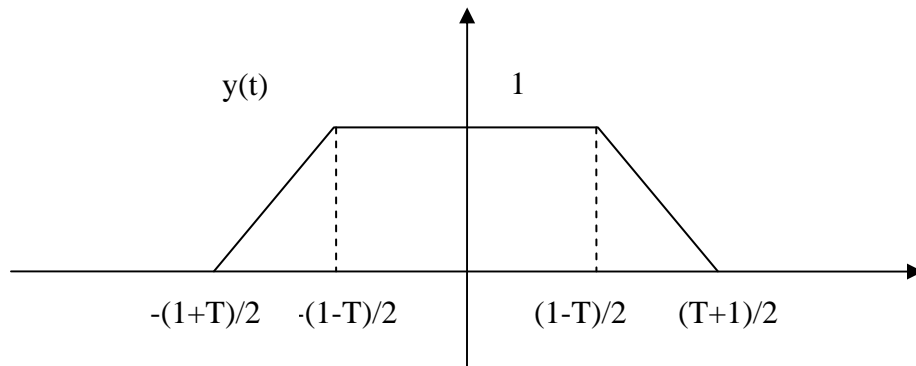
*Since  $T$  is not specified, let's assume that  $T > 1$ :*

*Then we have  $y(t) = 0$   $t < -(T+1)/2$ . During the interval  $-(T+1)/2 \leq t \leq -(T-1)/2$  the area increases linearly until the two rectangles are fully overlapped. At this point, the convolution is equal to one. The convolution will*

remain one while  $-(T-1)/2 \leq t \leq (T-1)/2$ . Afterwards the area decreases linearly during the interval  $(T-1)/2 \leq t \leq (T+1)/2$ . This is drawn below.



Now if  $T < 1$ , the shape doesn't change, although the intervals change slightly. Specifically, the interval where the area remains constant is  $-(1-T)/2 \leq t \leq (1-T)/2$



If  $T = 1$ , the result is a triangle:

