3614: Introduction to Communication Systems

Midterm Exam I

September 21, 2006

I pledge that I have neither given nor received any assistance on this exam.

(signed)

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Name (print) Student Number

ECE 3614 Midterm I – Test A

1. (20 points) Multiple Choice – Choose the answer which best completes the sentence

1.1 [5 points] The Fourier Transform of a periodic time-domain signal is

(a) periodic in the frequency domain
(b) discrete in the frequency domain
(c) continuous in the frequency domain
(d) None of the above

1.2 [5 points] The Fourier Transform of a signal which is of finite duration in the time domain is

(a) periodic in the frequency domain
(b) of infinite duration in the frequency domain
(c) discrete in the frequency domain
(d) None of the above

1.3 [5 points] $\delta(t-10)\sin\left(\frac{\pi}{20}t\right)=$

(a) $\delta(t-10)$
(b) 1
(c) $\delta(t)\sin\left(\frac{\pi}{20}t\right)$
(d) $\sin\left(\frac{\pi}{20}t\right)$
(e) None of the above

1.4 [5 points] Which of the following represents the relationship between the autocorrelation functions of the input, *x(t)*, and output, *y(t)*, of a linear time-invariant system?

(a) $R_y(\tau)=h^2(\tau)\otimes R_x(\tau)$
(b) $R_y(\tau)=h(\tau)\otimes h(-\tau)\otimes R_x(\tau)$
(c) $R_y(\tau)=h(\tau)h(-\tau)R_x(\tau)$
(d) None of the above

2. (25 points) Fourier Transform

(a) [10 points] Determine the Fourier Transform of the signal

$$x(t) = rect\left(\frac{t-1}{10}\right)$$

(b) [10 points] Determine the Fourier Transform of

$$x(t) = rect\left(\frac{t}{10}\right)\cos(2000\pi t)$$

(c) [5 points] Determine the Fourier Transform of

$$x(t) = rect(t) \otimes \sum_{k=-\infty}^{\infty} \delta(t - 10k)$$

where $\otimes$ is the convolution operation.

3. (25 points) System Response

Consider the signal $x(t) = rect(10t)\exp(j2000\pi t)$.

(a) [10 points] Determine the Energy Spectral Density.

(b) [10 points] Determine the auto-correlation function.

(c) [5 points] What is the bandwidth of the signal if bandwidth is defined according as the location of the first null after the center frequency minus the location of the first null before the center frequency ? (This is termed the first-null-to-null bandwidth).

4. (30 points) Filtering

Consider an ideal low pass filter with a gain of one and a bandwidth of *B* Hz

(a) [15 points] If a power signal with autocorrelation function $R_x(\tau) = \delta(\tau)$ is passed through this filter, what is the autocorrelation function of the output?

(b) [15 points] Consider a signal *x(t)* that has a power spectral density equal to $S_x(f)=\frac{A_c^2}{4}\delta(f-10)+\frac{A_c^2}{4}\delta(f+10)$. Further, consider a second signal *y(t)* which has a power spectral density equal to $S_y(f)=\frac{1}{2}rect\left(\frac{f-20}{5}\right)+\frac{1}{2}rect\left(\frac{f+20}{5}\right)$. Determine an appropriate value for *B*, the bandwidth of the ideal LPF, such that *x(t)* is within the passband of the ideal filter, while *y(t)* is rejected (i.e., *y(t)* is within the stopband of the ideal filter).

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Useful Info:

Exponential Fourier Series coefficients of a square pulse train with amplitude *A*, pulse duration *T* and period T_o:

$$c_n = \frac{AT}{T_o}\text{sinc}\left(\frac{nT}{T_o}\right)$$