3614: Introduction to Communication Systems

Midterm Exam II

November 2, 2006

SOLUTION

I pledge that I have neither given nor received any assistance on this exam.

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(signed)

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Name (print) Student Number

ECE 3614 Midterm II – Test A

1. (20 points) Short answer and multiple choice

1.1 [5 points] Rank the following modulation schemes in terms of required bandwidth (least bandwidth = 1, most bandwidth = 4) for a signal with peak value 1.

_2__ Large Carrier AM with $k_a = 0.5$
_1__ Single Sideband AM
_4__ FM with $k_f = 5000$
_3__ FM with $k_f = 2000$

1.2 [5 points] The Fourier Transform of the time domain signal $x(t) = \sum_{n=-\infty}^{\infty} p(t - nT)$ given that $p(t) \rightleftharpoons P(f)$ is

(a) $X(f) = \sum_{n=-\infty}^{\infty} P(f - nT)$
(b) $X(f) = \sum_{n=-\infty}^{\infty} P(nf)\delta\left(f - \frac{n}{T}\right)$
(c) $X(f) = \frac{1}{T} P(f) \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$
(d) None of the above

1.3 [5 points] The basic trade-off involved in Vestigial Sideband modulation is

(a) bandwidth vs. implementation complexity
(b) bandwidth vs. power efficiency
(c) power efficiency vs. implementation complexity
(d) transmit bandwidth vs. message bandwidth
(e) None of the above

1.4 [5 points] Which of the following is an example of an AM receiver?

(a) Envelope detector
(b) Product detector
(c) Both of the above
(d) None of the above

2. (40 points) Amplitude Modulation

Consider a message signal with the following spectrum

$$M(f) = \frac{1}{1 + j2\pi f}$$

(a) [10 points] Sketch the time domain signal of a large carrier AM modulated signal if k_a = 0.5 and f_c = 10Hz. Clearly label all axes.

The transmit signal for a Large Carrier AM signal is written as

$$x(t) = A_c\left[1 + k_a m(t)\right]\cos(2\pi f_c t)$$

The message signal in this case is

$$\begin{aligned} m(t) &= F^{-1}\left\{\frac{1}{1 + j2\pi f}\right\} \\ &= e^{-t}u(t) \end{aligned}$$

ka is given as 0.5 and fc = 10Hz. Since A_c is not specified, we will assume that it is one. Thus, we have

$$x(t) = \left[1 + 0.5e^{-t}u(t)\right]\cos(20\pi t)$$

The plot is given below:

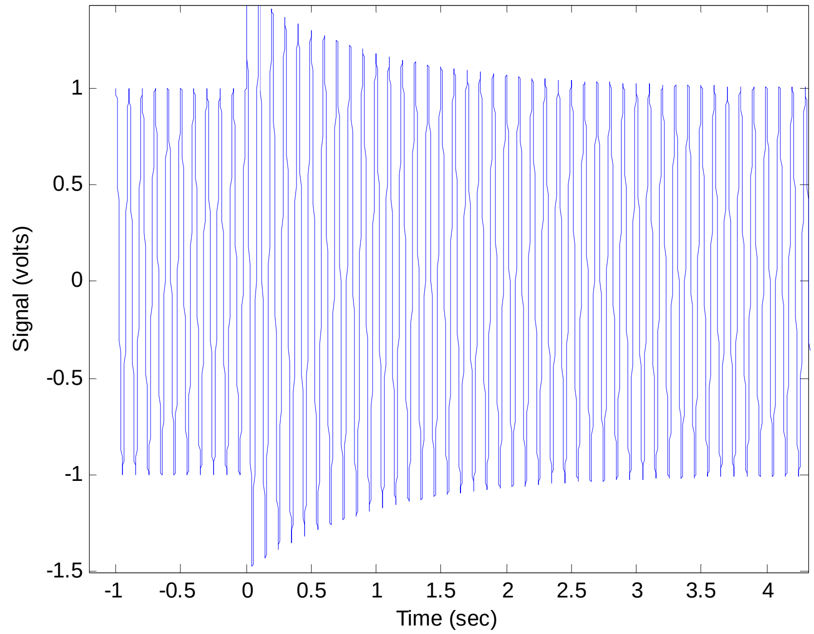

Figure 1: Time Domain Plot for Problem 2a

(b) [10 points] Sketch the frequency domain of the transmit signal described in part (a).

The frequency domain of an AM signal can be written as

$$
\begin{aligned}
S(f) &= \frac{k_a}{2} M(f-f_c) + \frac{k_a}{2} M(f+f_c) + \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c) \\
&= \frac{k_a}{2\left(1+j2\pi(f-f_c)\right)} + \frac{k_a}{2\left(1+j2\pi(f+f_c)\right)} + \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)
\end{aligned}
$$

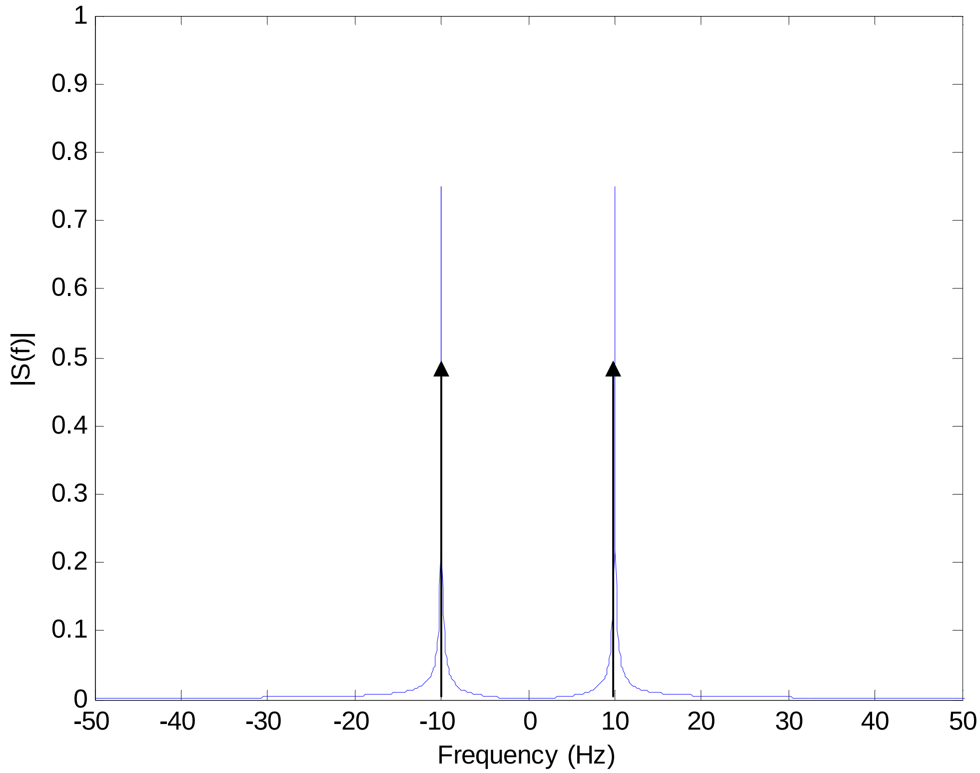

Figure 2: Frequency Domain Plot for Problem 2b

(c) [10 points] Sketch the time domain of a DSB-SC signal when $f_c = 10$Hz. Clearly label all axes and points.

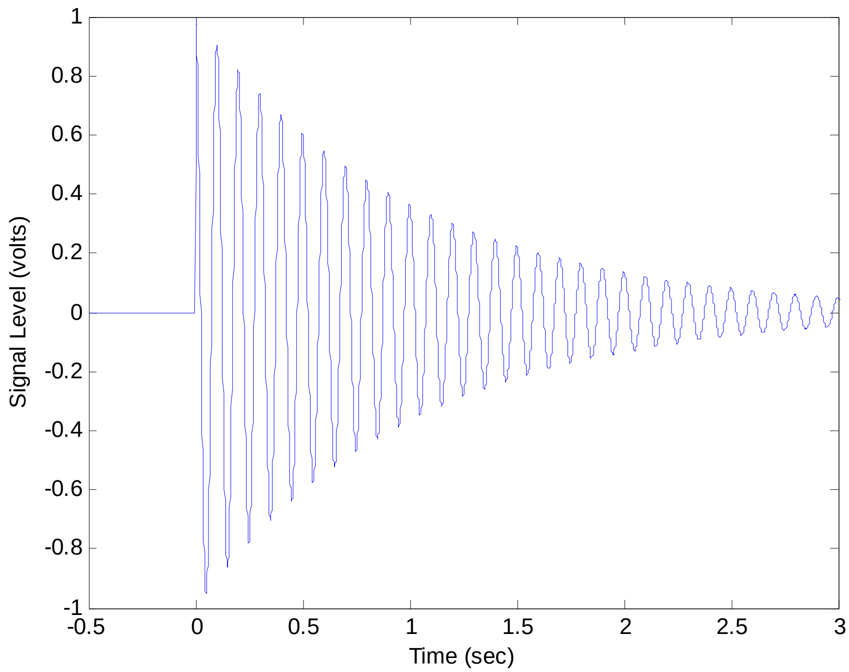

Figure 3: Time Domain Plot for Problem 2c

(d) [10 points] Sketch the frequency domain of a SSB signal (upper sideband) when $f_c = 10$Hz.

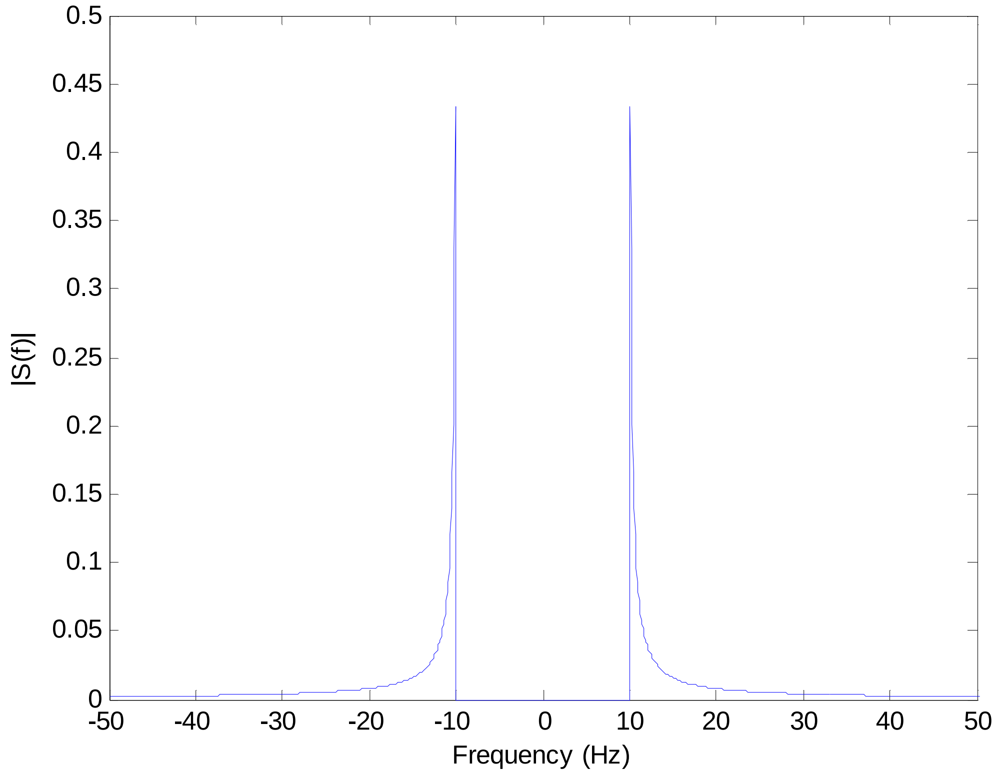

Figure 4: Frequency Domain Plot for Problem 2d

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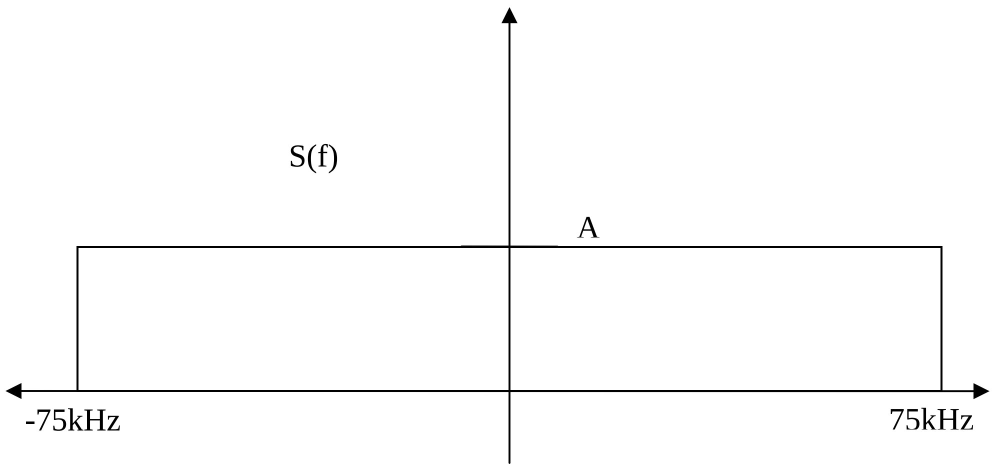
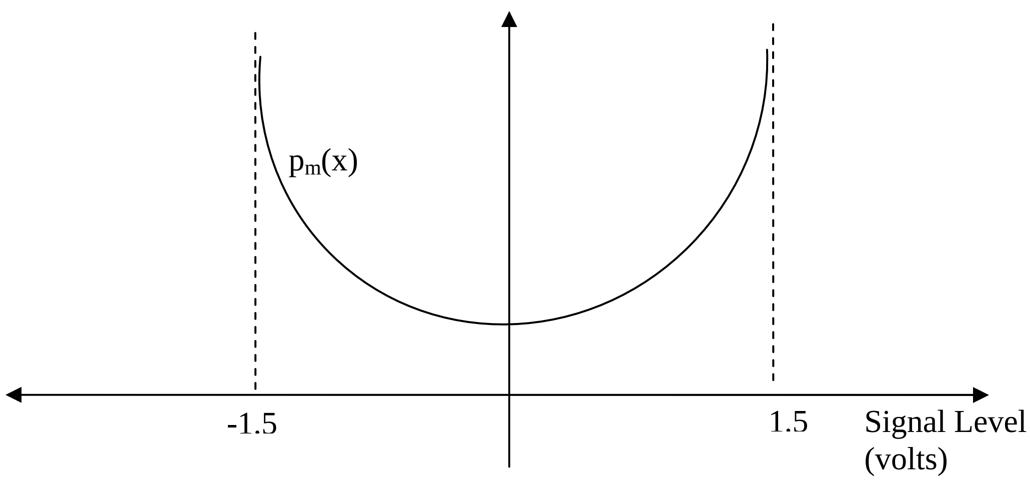
3. (30 points) Frequency Modulation

Consider a message signal with spectrum



and probability density function



(a) [10 points] Plot the spectrum of an FM signal if f_c = 1MHz and k_f = 200.

In order to approximate the spectrum of an FM signal we need to determine whether the signal is narrowband or wideband.

$$\begin{aligned} D &= \frac{\Delta f}{W} \\ &= \frac{V_p k_f}{W} \\ &= \frac{1.5 * 200}{75000} \\ &= 0.004 \end{aligned}$$

Thus, this is narrowband FM. For narrowband we can approximate the spectrum using the AM approximation:

$$\begin{aligned} S(f) &= \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ &\quad + \frac{A_c k_f}{2}\left[\frac{1}{|f - f_c|} M(f - f_c) + \frac{1}{|f + f_c|} M(f + f_c)\right] \\ &= \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ &\quad + 100\left[\frac{1}{|f - f_c|} rect\left(\frac{f - f_c}{150000}\right) + \frac{1}{|f - f_c|} rect\left(\frac{f + f_c}{150000}\right)\right] \end{aligned}$$

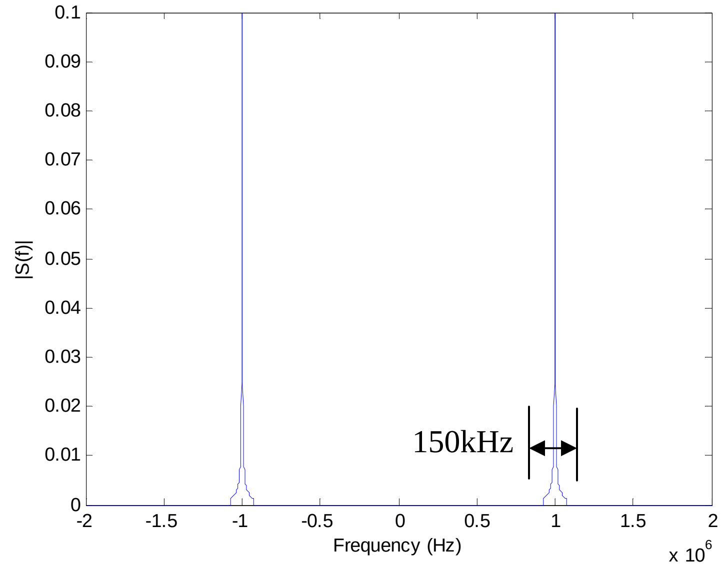


Figure 5: Frequency Domain Plot for Problem 3a

(b) [10 points] Plot the Power spectral density of an FM signal if f_c = 1MHz and k_f = 200000.

$$\begin{aligned} D &= \frac{\Delta f}{W} \\ &= \frac{V_p k_f}{W} \\ &= \frac{1.5 * 200000}{75000} \\ &= 4 \end{aligned}$$

Since this is a wideband FM signal, we can use the pdf approximation:

$$S_{WBFM}(f) = \frac{A_c^2}{4k_f}\left[p_m\left(\frac{1}{k_f}(f - f_c)\right) + p_m\left(\frac{1}{k_f}(-f - f_c)\right)\right]$$

The pdf approxaches asymptotes at 1.5V and -1.5V. There are two images of the pdf centered at +/- f_c. For the image in the positive frequency range, the asymptotes are at

$$\frac{1}{k_f}(f - f_c) = 1.5$$

$$\begin{aligned} f &= 1.5k_f + f_c \\ &= 1.3e6 \end{aligned}$$

and

$$\frac{1}{k_f}\left(f - f_c\right) = -1.5$$
$$f = -1.5k_f + f_c$$
$$= 0.7e6$$

This is also true for the negative frequency range. Thus, the approximate PSD for the FM signal is:

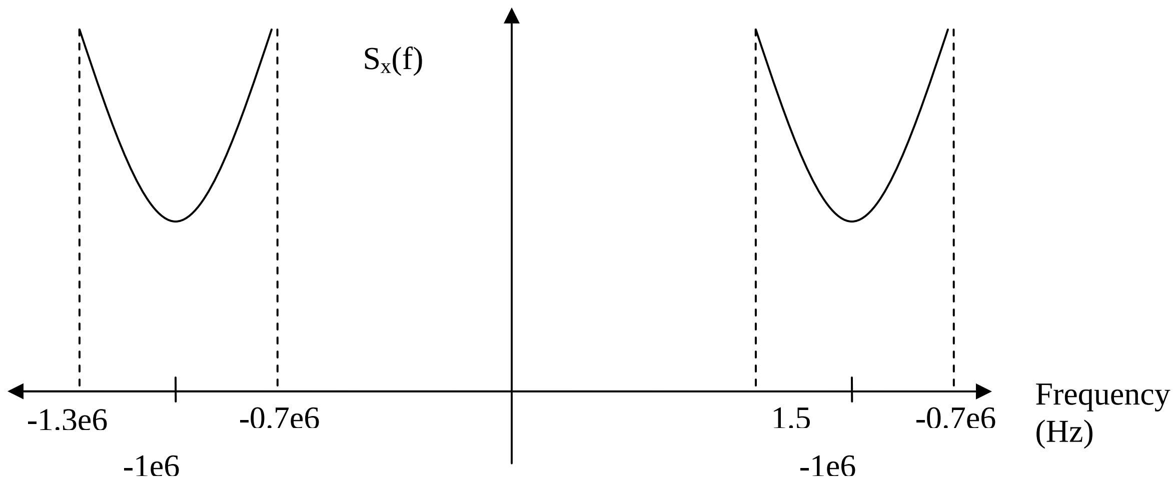


Figure 6: Frequency Domain Plot for Problem 3b

(c) [5 points] How does the bandwidth of the signal plotted in part (a) compare to Carson's Rule?

The bandwidth in Figure 150kHz. Carson's rule states that the bandwidth is

$$B = 2\Delta f + 2W$$
$$= 2*200 + 2*75kHz$$
$$= 150.4kHz$$

Thus, the two measures are very close.

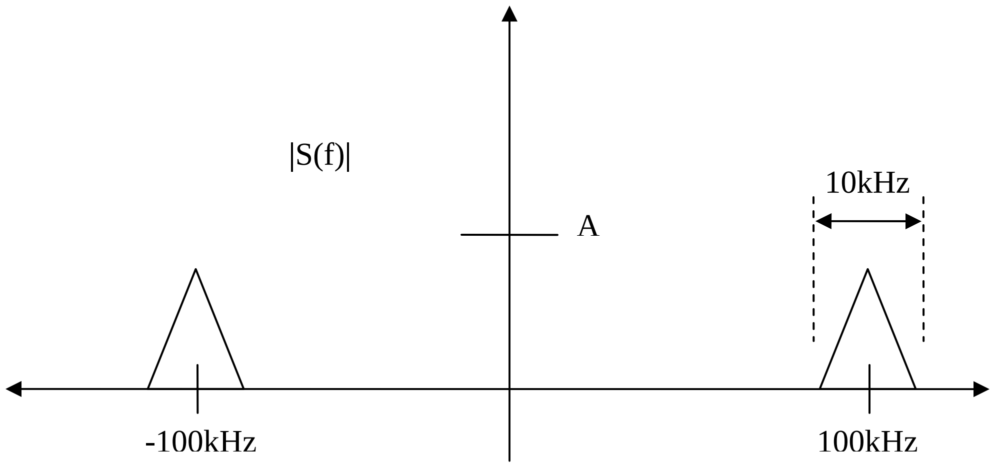
(d) [5 points] How does the bandwidth of the signal plotted in part (b) compare to Carson's Rule?

From Figure 6 we can see that the bandwidth is 600kHz. From Carson's rule we have

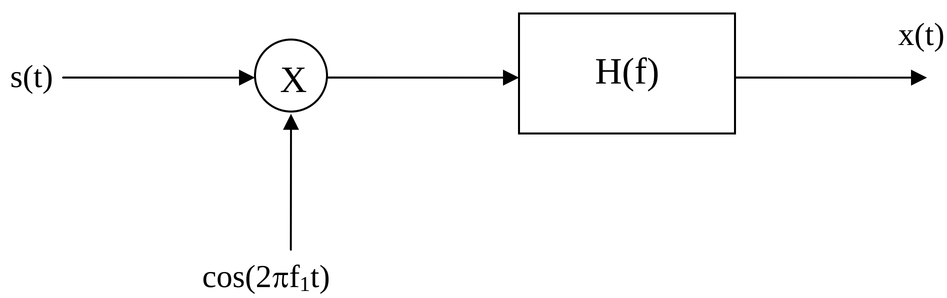
$$\begin{aligned} B &= 2\Delta f + 2W \\ &= 2*300kHz + 2*75kHz \\ &= 750kHz \end{aligned}$$

Carson's rule is also close to the approximation in this case, but not nearly as close as in the previous part. This is because while the signal is wideband, it is not very wideband.

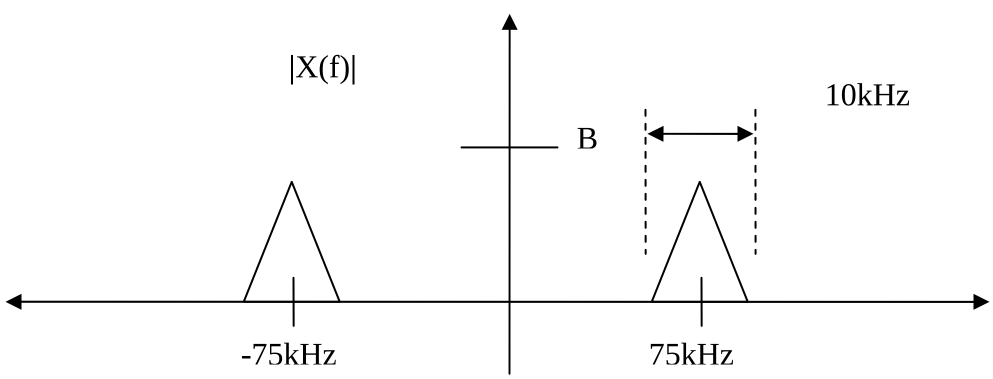
4. (10 points) Consider the following received signal.



This received signal is passed through the following down-conversion system:



where $H(f) = rect\left(\frac{f - f_o}{10000}\right) + rect\left(\frac{f + f_o}{10000}\right)$. If the output signal is



Determine proper values of f_1 *and* f_o.

If we write the received signal as

$$S(f) = tri((f - 100kHz)/5000) + tri((f + 100kHz)/5000)$$

The signal coming out of the mixer is

$$X(f) = S(f) * \left(\frac{1}{2}\delta(f - f_1) + \frac{1}{2}\delta(f - f_1)\right)$$

$$= \left\{tri((f - 100kHz)/5000) + tri((f + 100kHz - f_1)/5000)\right\} * \left(\frac{1}{2}\delta(f - f_1) + \frac{1}{2}\delta(f - f_1)\right)$$

$$= \frac{1}{2}tri((f - 100kHz - f_1)/5000) + \frac{1}{2}tri((f + 100kHz - f_1)/5000) +$$

$$\frac{1}{2}tri((f - 100kHz + f_1)/5000) + \frac{1}{2}tri((f + 100kHz + f_1)/5000)$$

Now, the output of the filter we desire the signal to be centered at 75kHz. Thus, we clearly desire

$$H(f) = rect\left(\frac{f - 75kHz}{10000}\right) + rect\left(\frac{f + 75kHz}{10000}\right)$$

Thus, we need

$$X(f)H(f) = B\,tri((f - 75kHz)/5000) + B\,tri((f + 75kHz)/5000)$$

Thus, we clearly require that f_1 *= 25kHz.*