3614: Introduction to Communication Systems

Final Exam

December 13, 2006

I pledge that I have neither given nor received any assistance on this exam.

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(signed)

Name (print)

Student Number

1. (20 points) Multiple Choice – Choose the answer which best completes the sentence

1.1 [5 points] The Fourier Transform of a periodic time-domain signal is

(a) periodic in the frequency domain
(b) discrete in the frequency domain
(c) continuous in the frequency domain
(d) None of the above

1.2 [5 points] The transfer function of an ideal low pass filter with bandwidth 20kHz can be written as

(a) $H(f) = rect\left(\frac{f - f_o}{40000}\right) e^{-j2\pi f T_o}$
(b) $H(f) = rect\left(\frac{f}{40000}\right) e^{-j2\pi f T_o}$
(c) $H(f) = rect\left(\frac{f}{20000}\right) e^{-j2\pi f T_o}$
(d) None of the above

1.3 [5 points] If a message bandwidth has a B Hz, the bandwidth of the corresponding vestigial sideband AM signal B_T relates to B as

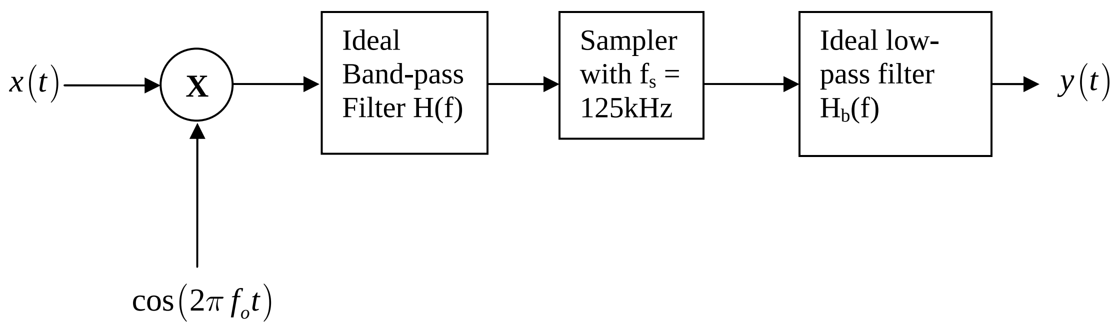
(a) $B_T < B$ Hz
(b) $B_T = 2B$ Hz
(c) $B_T > 2B$ Hz
(d) $B < B_T < 2B$
(e) None of the above

1.4 [5 points] The output SNR of an FM system is

(a) always better than the SNR of a DSBSC system with the same transmit power and message signal
(b) never better than the SNR of DSBSC system with the same transmit power and message signal
(c) never better than the SNR of a large carrier AM system with the same transmit power and message signal
(d) None of the above

2. (25 points) Sampling and Frequency Translation

Consider the following system



where the input signal is

$$x(t) = \cos(2\pi f_c t)$$

with center frequency f_c = 2MHz. The input signal is mixed (multiplied) by a coherent carrier with frequency f_o = 1.95MHz. The resulting signal is passed through in ideal band-pass filter with frequency response

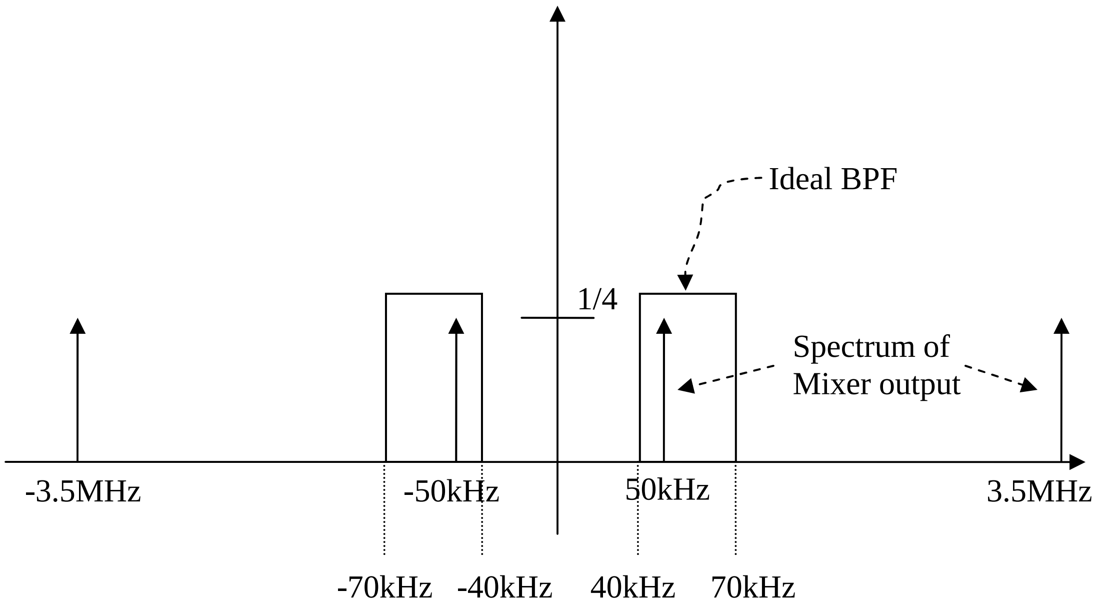
$$H(f) = rect\left(\frac{f - 40000}{30000}\right) + rect\left(\frac{f + 40000}{30000}\right)$$

The output of the ideal BPF is input to an ideal sampling device with sampling frequency f_s = 125kHz. The sampled signal is then passed through an ideal low-pass filter with frequency response

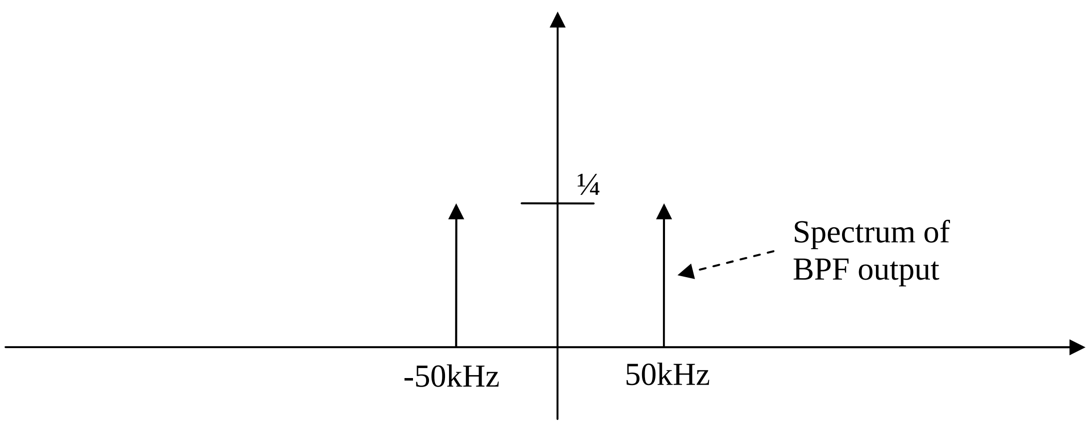
$$H_b(f) = \frac{1}{f_s} rect\left(\frac{f}{160000}\right)$$

(a) [15 points] Plot the magnitude spectrum *Y(f)* at the output. Carefully label all curves and axes.

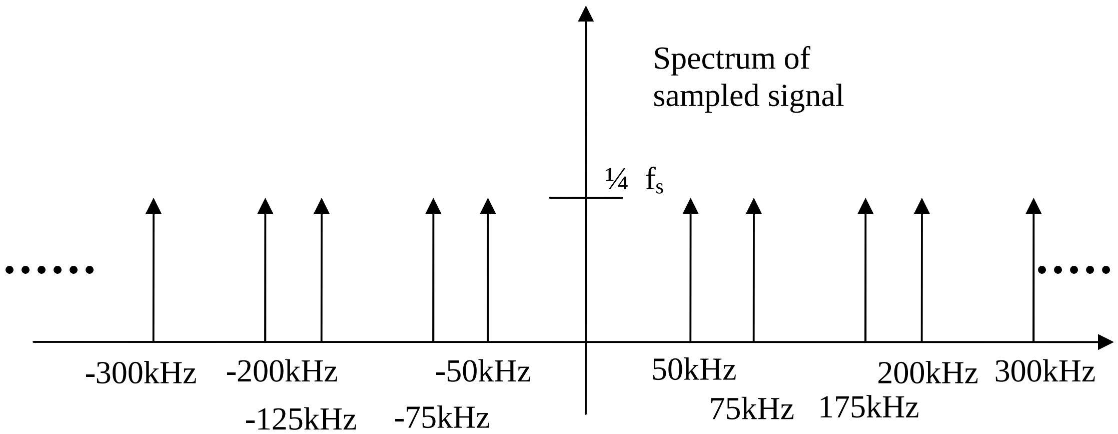
This problem is most easily solved in the frequency domain. The spectrum at the output of the mixer can be drawn as:

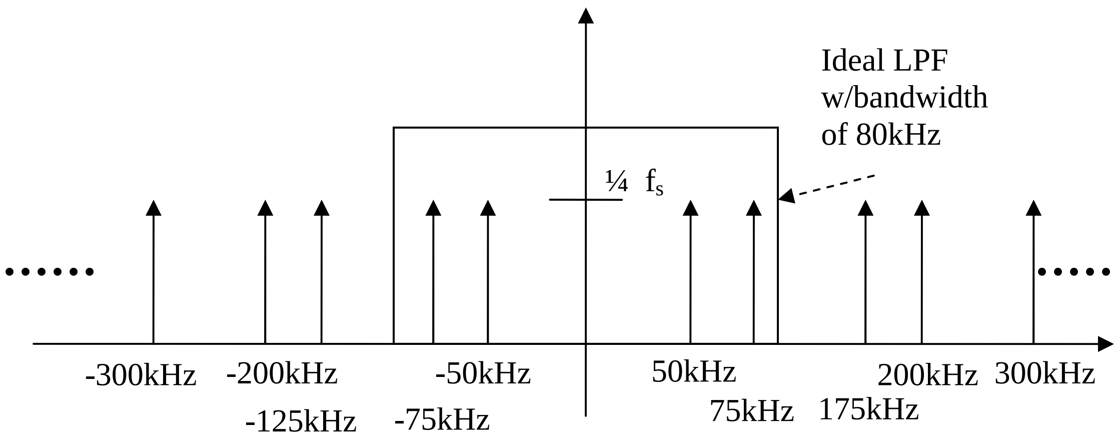


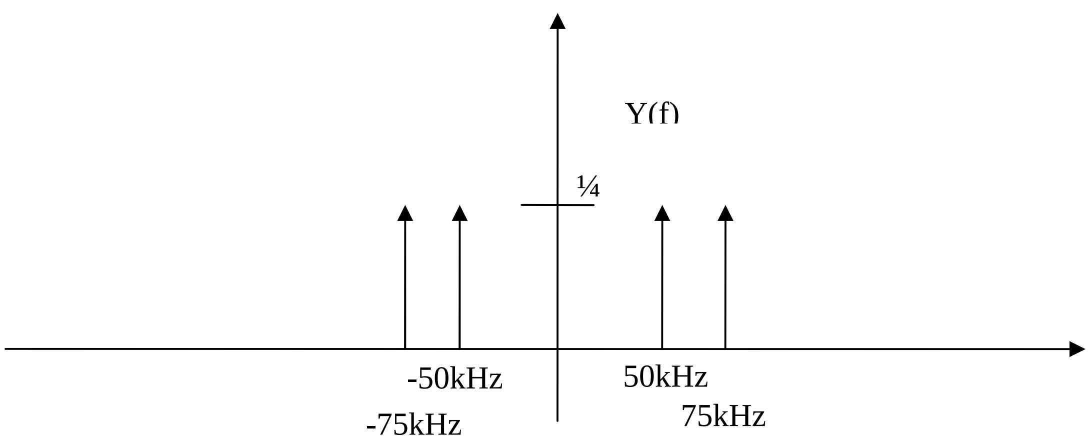
Thus, the spectrum of the signal at the oputput of the ideal bandpass filter is:



The spectrum of the sampled signal is then drawn as:


Spectrum of
sampled signal
¼ fs
-300kHz
-200kHz
-125kHz
-75kHz
-50kHz
50kHz
75kHz
175kHz
200kHz
300kHz


Ideal LPF
w/bandwidth
of 80kHz
¼ fs
-300kHz
-200kHz
-125kHz
-75kHz
-50kHz
50kHz
75kHz
175kHz
200kHz
300kHz


Y(f)
¼
-50kHz
-75kHz
50kHz
75kHz

(b) [10 points] Write an equation of the time-domain output y(t).

From the previous plot, we can write the frequency domain result as:

$$Y(f)=\frac{1}{4}\left\{\delta(f-50000)+\delta(f+50000)+\delta(f-75000)+\delta(f+75000)\right\}$$

Taking the inverse Fourier Transform:

$$Y(f)=\frac{1}{2}\left\{\cos(100000\pi t)+\cos(150000\pi t)\right\}$$

3. (30 points) SNR of AM Systems

If the total received signal power of an amplitude modulated signal is -80dBm, the message bandwidth is 2.5MHz, the message power is 0.1W and the noise power spectral density is N_o = -170dBm/Hz

(a) [10 points] Determine the input (pre-detection) SNR of a DSBSC signal.

The received signal for DSBSC modulation with a message m(t) is

$$s(t) = A_c m(t)\cos(2\pi f_c t)$$

The received signal power can be written as $P = E\left\{s^2(t)\right\} = \frac{A_c^2}{2}P_m$ *where* P_m *is the average message power. Thus,*

$$10\log_{10}\left\{\frac{A_c^2}{2}P_m\right\} = -80dBm$$

The pre-detection SNR for DSBSC is

$$SNR_{pre} = \frac{A_c^2}{2}P_m\frac{1}{2N_oW}$$

or in dB:

$$\begin{aligned} SNR_{pre}(dB) &= 10\log_{10}\left\{\frac{A_c^2}{2}P_m\right\} - 10\log_{10}\{2\} - 10\log_{10}\{W\} - 10\log_{10}\{N_o\} \\ &= -80dBm - 3.01 - 10\log_{10}\left\{2.5*10^6\right\} - (-170dBm/Hz) \\ &= 23dB \end{aligned}$$

(b) [10 points] Determine the output (post-detection) SNR of a DSBSC signal.

There is a 3dB increase in the SNR at the output of the DSBSC detector. Thus,

$$\begin{aligned} SNR_{pre}(dB) &= SNR_{post}(dB) + 3.01 \\ &= 26dB \end{aligned}$$

(c) [10 points] Determine the output (post-detection) SNR if the signal is a large carrier AM signal with amplitude sensitivity k_a = 0.75.

The large-carrier AM signal is written as

$$s(t) = A_c\left(1 + k_a m(t)\right)\cos\left(2\pi f_c t\right)$$

The power is thus $P = E\left\{s^2(t)\right\} = \frac{A_c^2}{2}\left(1 + k_a^2 P_m\right)$ *where* P_m *is the average message power. Thus,*

$$10\log_{10}\left\{\frac{A_c^2}{2}\left(1 + k_a^2 P_m\right)\right\} = -80dBm$$

$$SNR_{pre} = \frac{A_c^2}{2}\left(1 + k_a^2 P_m\right)\frac{1}{2N_oW}$$

$$\begin{aligned} SNR_{pre}(dB) &= 10\log_{10}\left\{\frac{A_c^2}{2}\left(1 + k_a^2 P_m\right)\right\} - 10\log_{10}\{2\} - 10\log_{10}\{W\} - 10\log_{10}\{N_o\} \\ &= -80dBm - 3.01 - 10\log_{10}\left\{2.5*10^6\right\} - \left(-170dBm/Hz\right) \\ &= 23dB \end{aligned}$$

$$\begin{aligned} SNR_{post}(dB) &= SNR_{pre}(dB) + 10\log_{10}\left(\frac{2k_a^2 P_m}{1 + k_a^2 P_m}\right) \\ &= 23 + 10\log_{10}\left(\frac{2*0.75^2*0.1}{1+*0.75^2*0.1}\right) \\ &= 13.3dB \end{aligned}$$

4. (20 points) Consider a message signal

$$m(t) = rect\left(\frac{t - i/10}{0.0005}\right)$$

where a_i is a random value equally probable on {-1, +1}.

(a) [10 points] Plot the magnitude spectrum of a large carrier AM signal if the carrier frequency is f_c = 150kHz and k_a = 0.25.

For large carrier AM:

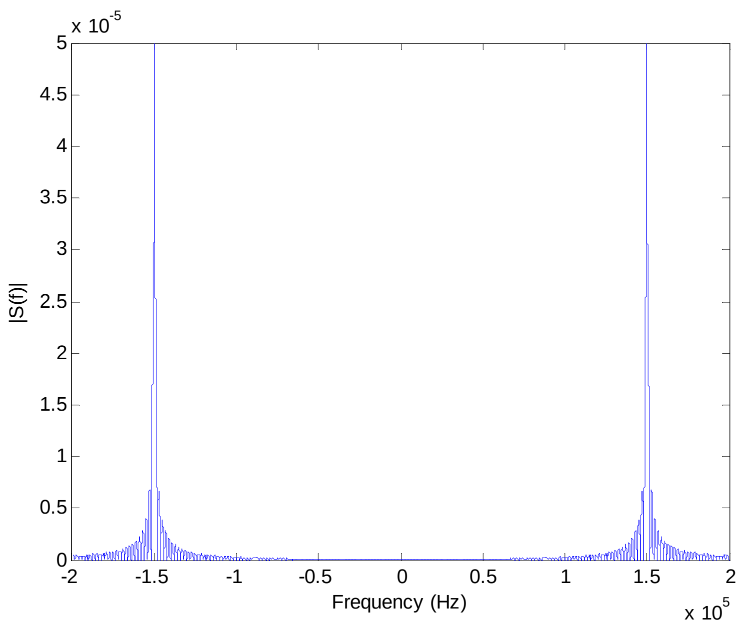
$$S(f) = \frac{A_c}{2}\left[k_a M(f - f_c) + k_a M(f + f_c) + \delta(f - f_c) + \delta(f + f_c)\right]$$

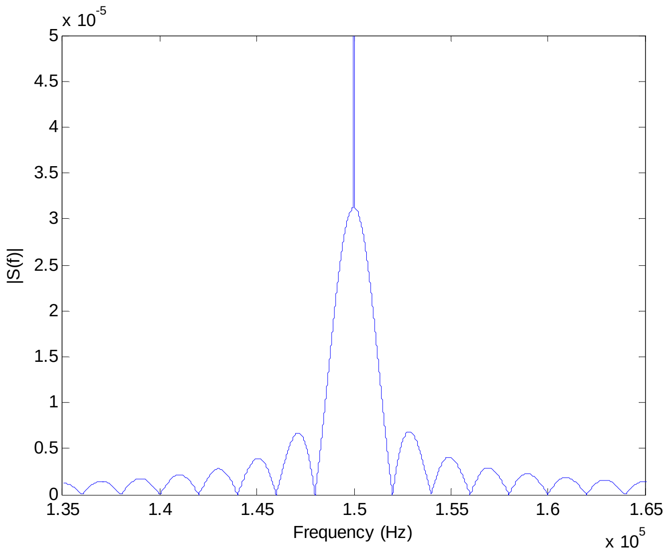
The spectrum of the message signal is

$$M(f) = 0.05\text{sinc}\left(\frac{f}{20}\right)$$

Thus, we have

$$S(f) = \frac{A_c}{2}\left[0.25 * 0.05\text{sinc}\left(\frac{f - f_c}{20}\right) + 0.25 * 0.05\text{sinc}\left(\frac{f + f_c}{20}\right) + \delta(f - f_c) + \delta(f + f_c)\right]$$

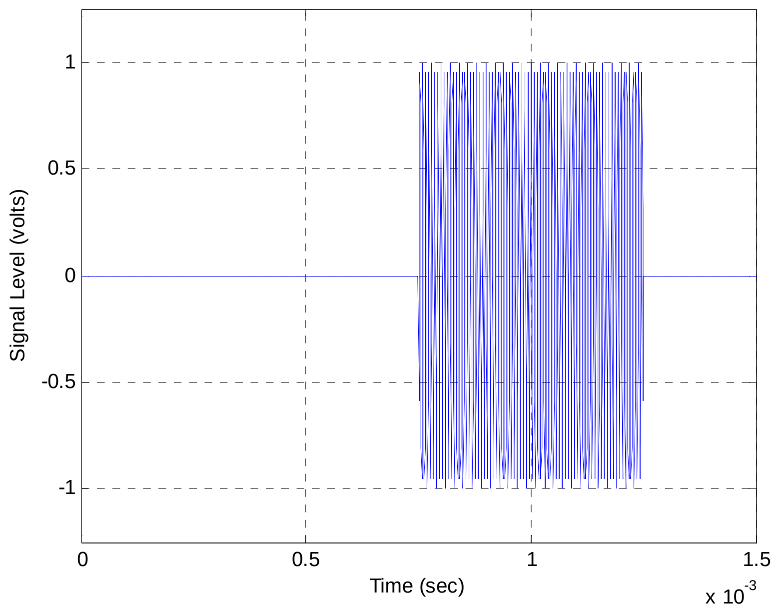



x 10^-5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
|S(f)|
1.35
1.4
1.45
1.5
1.55
1.6
1.65
Frequency (Hz)
x 10^5

(b) [10 points] Plot the time-domain function of a DSBSC signal for time 0 < t < 0.0015. Label all axes and curves.

$$s(t) = A_c m(t)\cos(2\pi f_c t)$$

Assuming $A_c = 1$:



5. [25 points] SNR of FM signals

Consider a received signal with total received power P_r = -90dBm, noise power spectral density N_o = -165dBm/Hz, a message bandwidth of 10kHz and a normalized message power of 0.2W.

(a) [10 points] If the threshold input SNR is 20dB, determine the maximum SNR attainable for an FM system.

The received power for an FM signal is $P = E\left\{s^2(t)\right\} = \frac{A_c^2}{2}$, *thus,* $10\log_{10}\left\{\frac{A_c^2}{2}\right\} = -80dBm$. *The pre-detection SNR is*

$$SNR_{pre} = \frac{A_c^2}{2}\frac{1}{N_o B_T}$$

In dB we have

$$\begin{aligned} SNR_{pre} &= 10\log_{10}\left\{\frac{A_c^2}{2}\right\} - 10\log_{10}\left\{N_o\right\} - 10\log_{10}\left\{B_T\right\} \\ &= -90dBm - \left(-165dBm/Hz\right) - 10\log_{10}\left\{B_T\right\} \end{aligned}$$

Thus, the pre-detection SNR goes down with input bandwidth. Since, the pre-detection SNR must be greater than 20dB, the input bandwidth is the limiting factor.

The output SNR for FM systems (if above the threshold SNR)

$$SNR_{post} = \frac{3A_c^2 D_f^2 \overline{\left(m(t)/V_p\right)^2}}{2N_o W}$$

As D_f *increases the output SNR increases. However,* $B_T = 2\left(D_f + 1\right)W$. *Thus, as* D_f *increases, the input SNR goes down. To maximize the post-detection SNR, we need to maximize* D_f *without allowing the pre-detection SNR to go below 20dB. Thus,*

$$SNR_{pre} = 75dBHz - 10\log_{10}\left\{2\left(D_f + 1\right)W\right\} \geq 20dB$$
$$10\log_{10}\left\{2\left(D_f + 1\right)W\right\} \leq 55dBHz$$
$$D_f \leq 14.8$$
$$SNR_{post}\left(dB\right) = 10\log_{10}\left(3\right) + 10\log_{10}\left(\frac{A_c^2}{2}\right) + 10\log_{10}\left(D_f^2\right) + 10\log_{10}\left(\overline{\left(m(t)/V_p\right)^2}\right) - 10\log_{10}\left(W\right) - 10\log_{10}\left(N_o\right)$$
$$= 5dB - 90dBm + 23.4dB - 7dB - 40 - \left(-165dBm/Hz\right)$$
$$= 56.4dB$$

(b) [5 points] How does the resulting transmit bandwidth compare to a DSBSC AM system?

$$B_T = 2\left(D_f + 1\right)W$$
$$= 316kHz$$

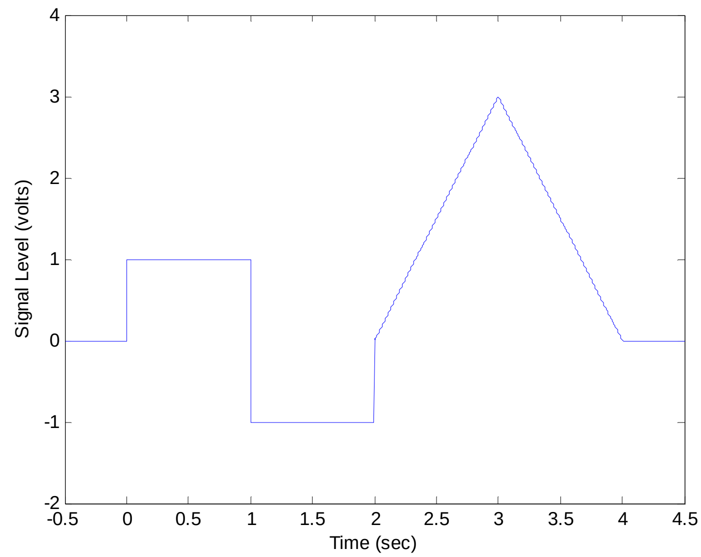
$$B_T = 2W$$
$$= 20kHz$$

(c) [10 points] How does the output SNR compare to an AM system?

$$SNR_{post} = \frac{A_c^2}{2} P_m \frac{1}{N_o B_T}$$
$$SNR_{post}\left(dB\right) = -90dBm + 165 - 10\log_{10}\left(2*10000\right)$$
$$= -90dBm + 165 - 43$$
$$= 32dB$$

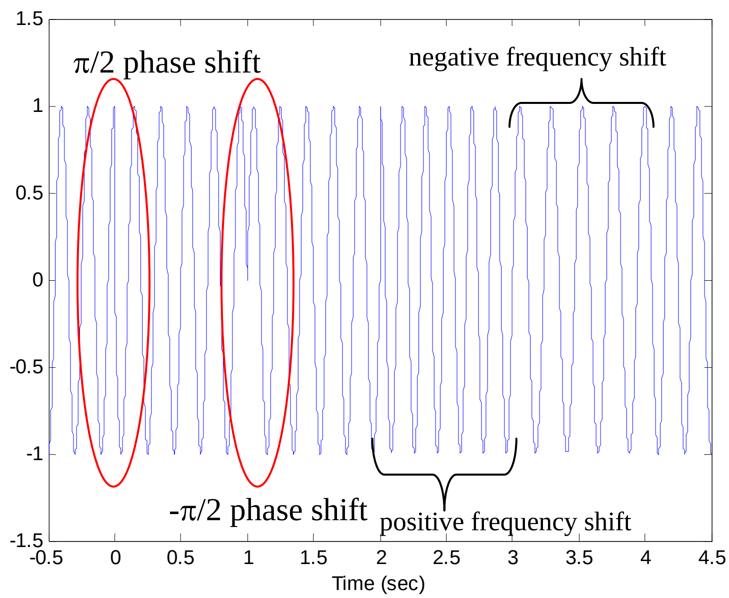
6. (20 points) PM/FM

Consider the message signal



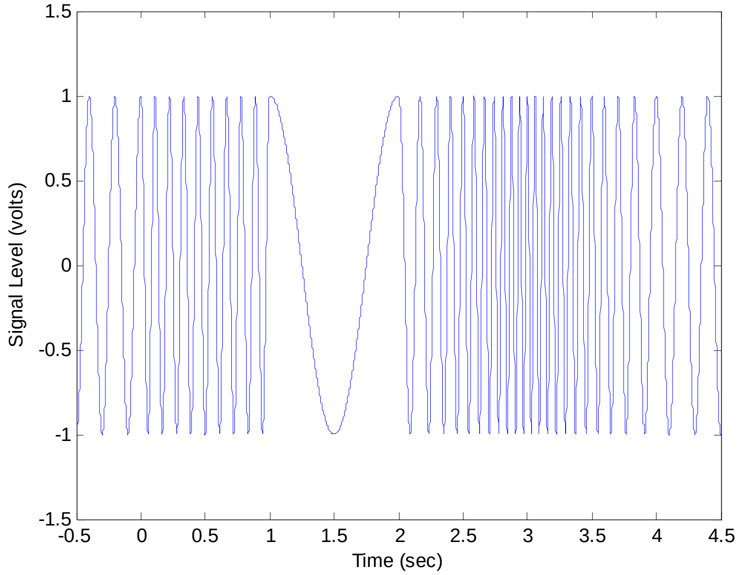
Message Signal for Problem 6

(a) [10 points] Draw the time domain PM signal over the time -0.25 to 4.25 seconds for a carrier frequency of 5Hz and $k_p = \pi/2$ rad/V.



*Over the interval -0.5 < t < 0, the carrier is unmodulated. Over the interval 0 < t < 1 the signal is phase shifted by positive π/2 radians. Over the interval 1 < t < 2 the signal is phase shifted by negative π/2 radians. Over the interval 2 < t < 3 the signal phase is changed linearly 3*π/2 radians/sec which corresponds to a ¾ Hz frequency increase (f = 5.75Hz). Over the interval 3 < t < 4 the phase of the signal changed linearly -3*π/2 radians/sec which corresponds to a -¾ Hz frequency increase (f = 4.25Hz). For the interval t>4, the carrier is unmodulated.*

(b) [10 points] Draw the time domain FM signal over the time -0.25 to 4.25 for a carrier frequency of 5Hz and k_f = 4Hz/V.



Over the interval -0.5 < t < 0, the carrier is unmodulated. Over the interval 0 < t < 1 the signal is frequency shifted by positive 4 Hz (f_c = 9Hz). Over the interval 1 < t < 2 the signal is frequency shifted by negative 4 Hz (f_c = 1Hz).. Over the interval 2 < t < 3 the signal frequency is changing linearly 12Hz/sec so that at the end of the interval fc = 17Hz. Over the interval 3 < t < 4 the signal frequency is changing linearly -12Hz/sec so that at the end of the interval f_c = 5Hz. . For the interval t>4, the carrier is unmodulated.