ECE3614
Introduction to Communications Systems
Fall 2007

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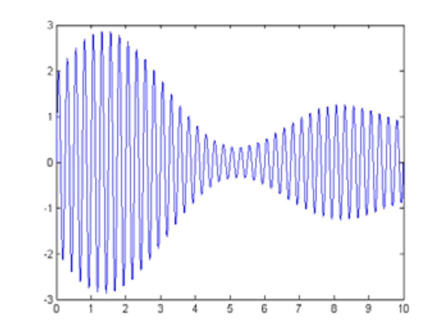
Final Review



Course Objectives

- After successfully completing this course the student should be able to
  - Compute the Fourier transform and the energy/power spectral density of communications signals.
  - Calculate the bandwidth and signal-to-noise ratio of a signal at the output of a linear time-invariant system given the signal and the power spectral density of the noise at the input of the system.
  - Explain the operation of amplitude and angle modulation systems in both the time and frequency domains including plotting the magnitude spectra and computing the power and bandwidth requirements of each type of signal.

Course Objectives (cont.)

- After successfully completing this course the student should be able to
  - Design a basic analog or digital communications system including: (1) the selection of a digital or analog modulation format, (2) the block-diagram design of a transmitter for the system, (3) the block-diagram design of a superheterodyne receiver for the system, (4) the design of a time or frequency division multiplexing scheme, as appropriate, and (5) the choice of an appropriate pulse shape and analog to digital converter (if needed) to meet performance requirements.
  - Evaluate a given analog or digital communications system in terms of the complexity of the required transmitters and receivers and the power and bandwidth requirements of the system.

Final Exam Format

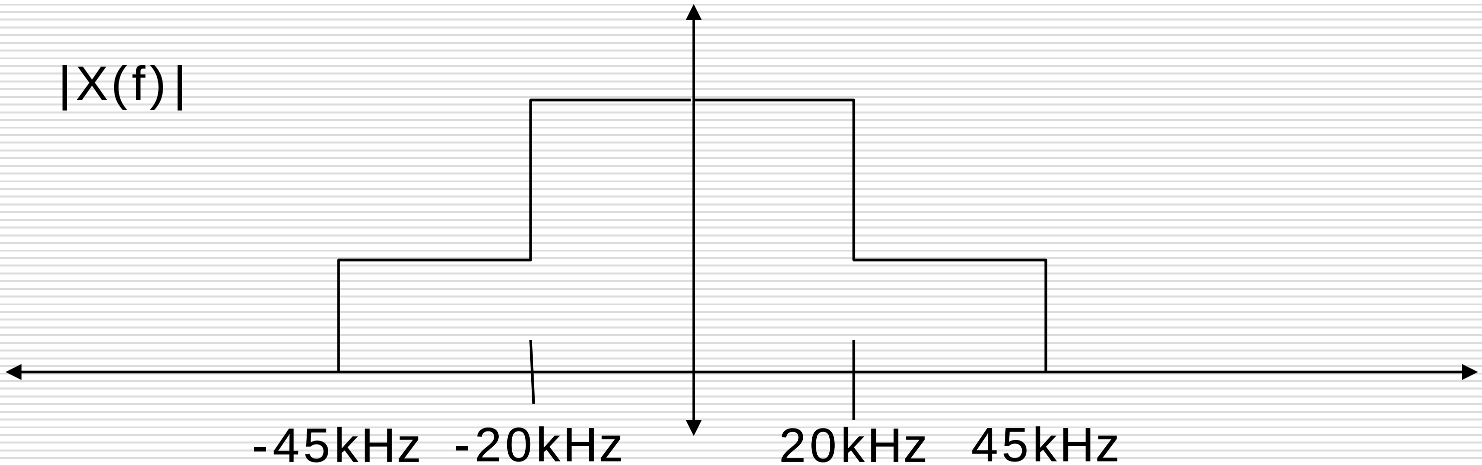
- Six Questions
- Two Hours
- Closed book - Two sheets of notes *PLUS* Fourier Transform and Fourier Series tables on a third sheet
- One problem of multiple choice/short answer and 5 analysis/design problems
  - Similar format to midterms

Likely Exam Topics

- Fourier Transforms – Time/frequency relationships
  - Use of Fourier Series for periodic signals
- System response – Time/frequency relationships between system input and output
- Time/frequency description of AM Systems (Large Carrier AM, DSBSC, SSB, VSB)
- Time/frequency description of FM
- Relationship between PM and FM
- Basics of frequency translation and the Super Het receiver
- SNR calculations for AM systems
- SNR calculations for FM systems
- Basic digital calculations – sampling theorem and quantization

Example Problem

- Consider a signal that is uniformly distributed with the following magnitude spectrum



- Determine the minimum bit rate of a digital bit stream if the required quantization SNR (using a uniform quantizer) must be greater than 64dB.

Example – cont.

- The bit rate for a digital signal is equal to

$$R_b = f_s \times n$$

or including the units for clarity:

$$R_b\left(\text{bits}/\text{sec}\right) = f_s\left(\text{samples}/\text{sec}\right) \times n\left(\text{bits}/\text{sample}\right)$$

- Thus, we need to determine the sampling rate, f_s and the number of bits per sample (i.e., quantization levels)

Example – cont.

- The sampling rate f_s can be determined from the Sampling Theorem

$$
\begin{aligned}
f_s &\geq 2B \\
&\geq 2*45kHz \\
&\geq 90kHz
\end{aligned}
$$

- The number of bits per sample can be determined from the quantization SNR of a uniform quantizer with a uniformly distributed source

Example – Quantizer SNR

$$
\begin{aligned}
SNR &= M^2 \\
&= \left(2^n\right)^2 \\
&= 2^{2n} \\
SNR(dB) &= 10\log_{10}\left(2^{2n}\right) \\
&= 20n\log_{10}(2) \\
&= 6.02n
\end{aligned}
$$

- Applying the SNR requirement:

$$
\begin{aligned}
SNR(dB) &> 64dB \\
6.02n &> 64dB \\
n &> 10.7 \\
n &= 11
\end{aligned}
$$

Example – cont.

- The minimum bit rate is then

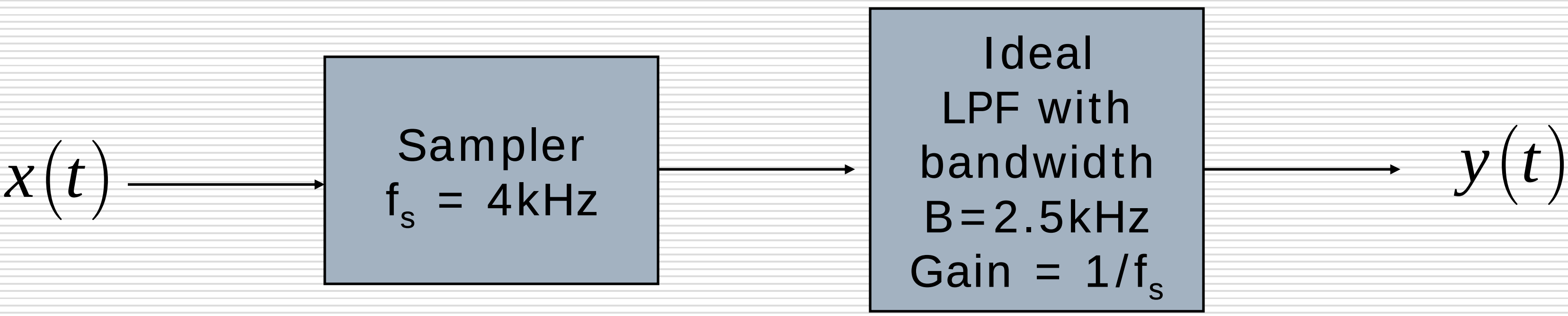
$$\begin{aligned} R_b &= f_s \times n \\ &= 90kHz \times 11bits \\ &= 990kbps \end{aligned}$$

Example 2

- Consider the following signal:

$$x(t) = 5\cos(2000\pi t)$$

- The signal is passed through the following system:



- (a) Determine *y(t)*
- (b) Determine *y(t)* if f_s = 2.5kHz and B = 2kHz

Example 2 – cont.

- This type of problem is most readily solved in the frequency domain.
- The spectrum of the input signal is

$$
\begin{aligned}
X(f) &= F\{x(t)\} \\
&= F\{5\cos(2000\pi t)\} \\
&= \frac{5}{2}\{\delta(f-1000)+\delta(f+1000)\}
\end{aligned}
$$

Example 2 – cont.

- Using the Sampling Theorem the spectrum at the output of the sampler is

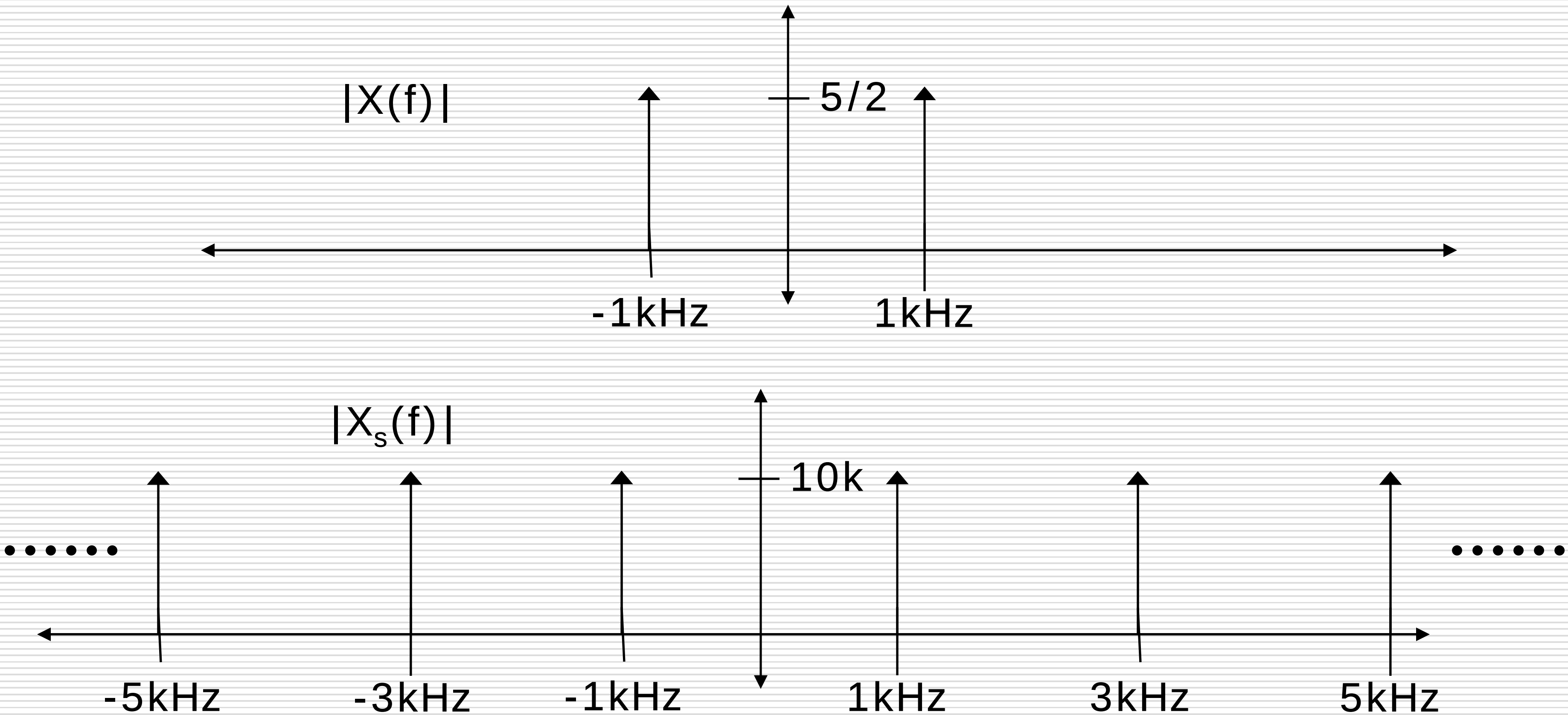
$$X_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$= 4000\sum_{n=-\infty}^{\infty} X(f - 4000n)$$

$$= 10000\sum_{n=-\infty}^{\infty}\left[\delta(f - 1000 - 4000n) + \delta(f + 1000 - 4000n)\right]$$

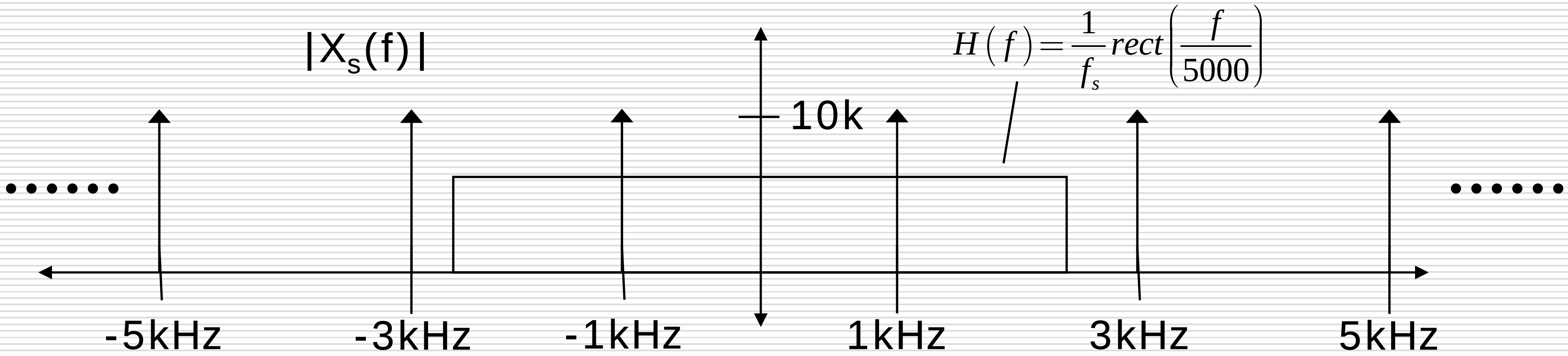
Example 2 – cont.

- Graphically we have

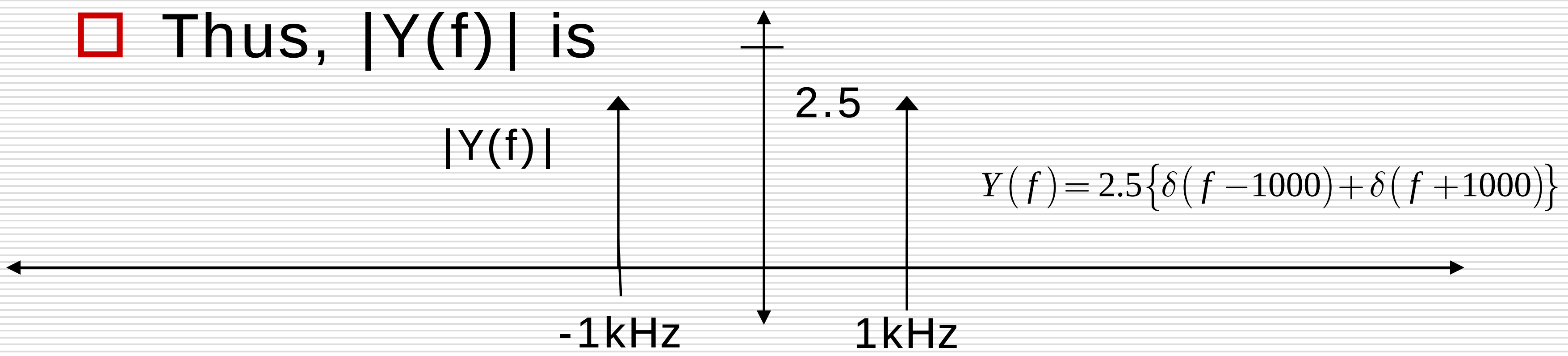


Example 2 – cont.

- Applying the ideal low pass filter:



- Thus, |Y(f)| is



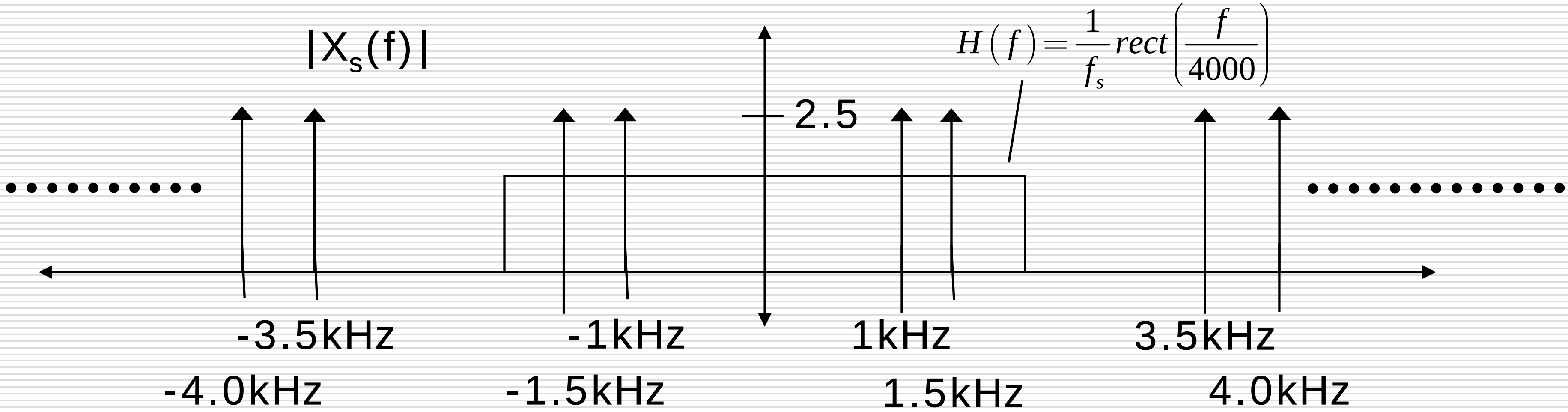
Example 2 – final (part a)

- Thus, in the time domain the output signal is

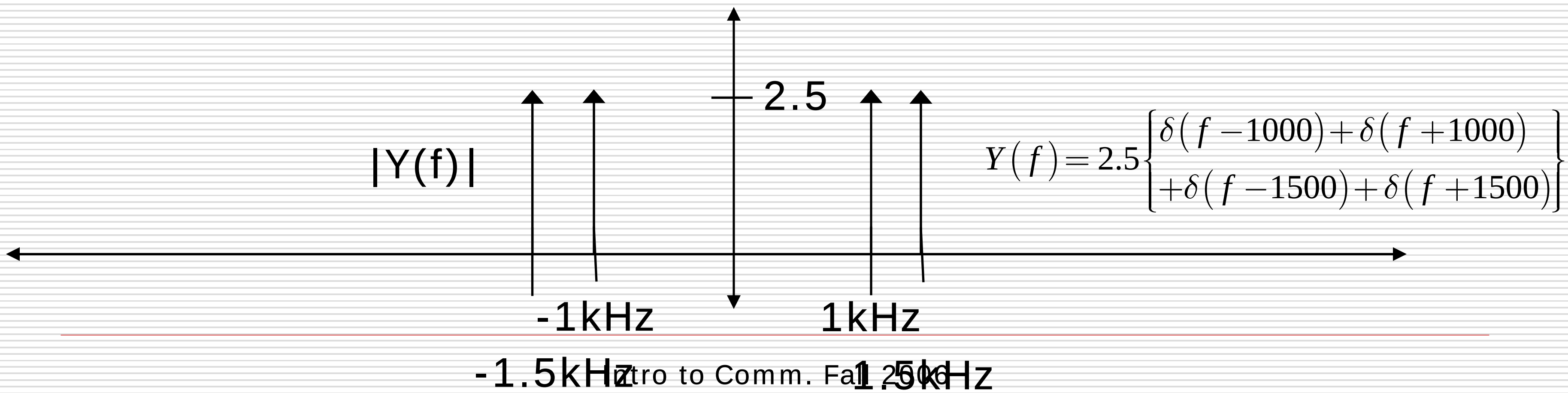
$$y(t) = 5\cos(2000\pi t)$$

Example 2 – part b

- Now, if f_s =2.5kHz and B = 2kHz we have



$$H(f)=\frac{1}{f_s}rect\left(\frac{f}{4000}\right)$$



$$Y(f)=2.5\left\{\begin{array}{l}\delta(f-1000)+\delta(f+1000)\\+\delta(f-1500)+\delta(f+1500)\end{array}\right\}$$

Example 2b - final

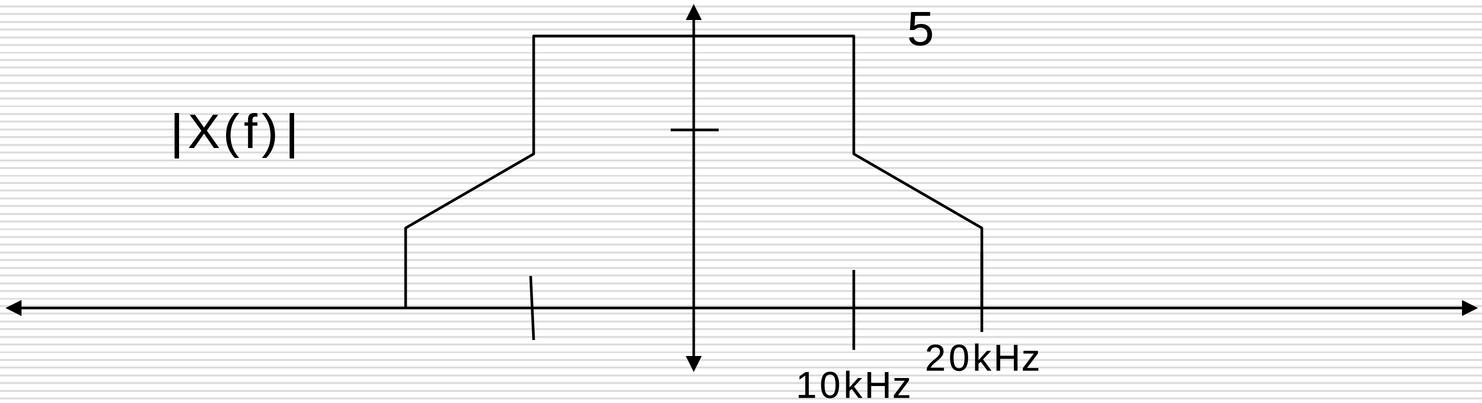
- In the time domain we have

$$y(t) = 5\cos(2000\pi t) + 5\cos(3000\pi t)$$

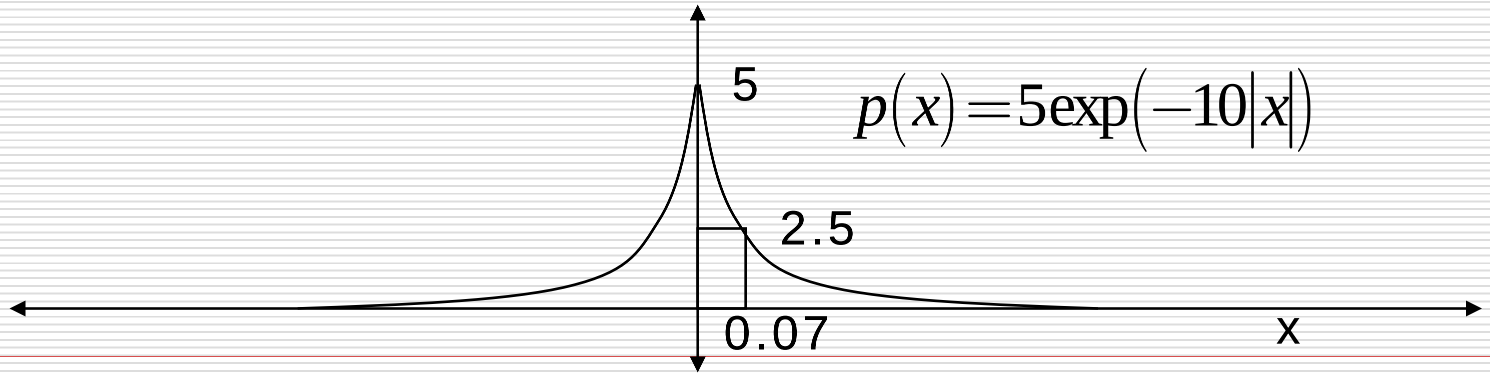
- The filter bandwidth was too large for the sampling rate resulting in a harmonic of the signal to appear in the output

FM Spectra (Wideband)

- Consider a signal which has a spectrum defined as



- And a probability density function defined as



$$p(x) = 5\exp(-10|x|)$$

FM Spectra – cont.

- The message signal with a peak voltage of 1V and bandwidth W=2kHz is used to modulate an FM signal with k_f = 10kHz and a carrier frequency of f_c = 250kHz. Plot the approximate spectrum (either magnitude spectrum or power spectral density) of the FM signal

FM Spectrum Example – cont.

- The approximate spectrum depends on whether the signal is narrowband or wideband. This is determined by the modulation index:

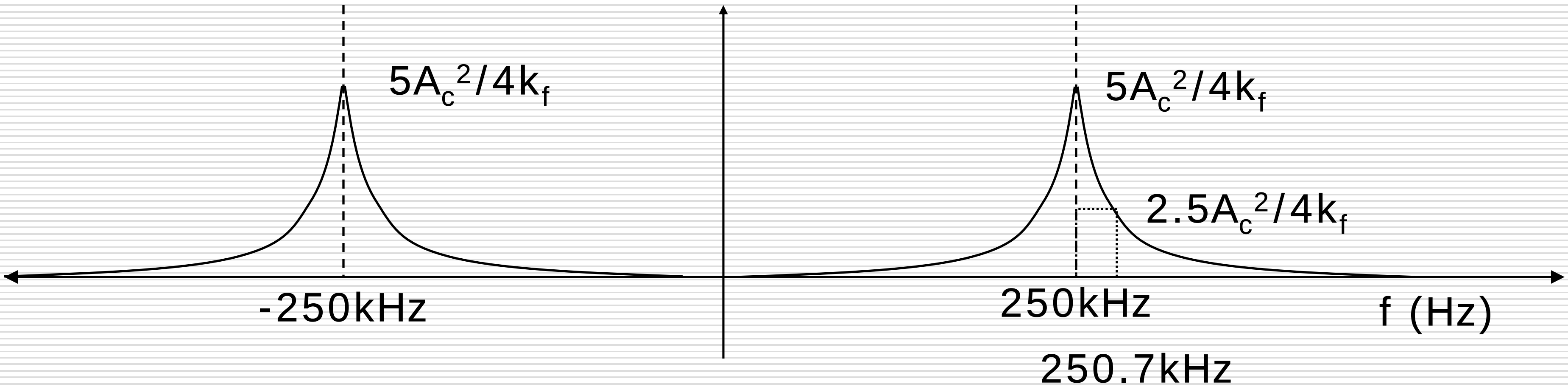
$$D = \frac{\Delta f}{W} = \frac{V_p k_f}{W} = \frac{10000}{2000} = 5$$

- Since this is a wideband FM signal we use the approximation

$$S_{WBFM}(f) = \frac{A_c^2}{4k_f}\left[p_m\left(\frac{1}{k_f}(f - f_c)\right) + p_m\left(\frac{1}{k_f}(-f - f_c)\right)\right]$$

Example - final

$$S_x(f) = \frac{5A_c^2}{4k_f}\exp\left(-\frac{10}{k_f}\left|f - f_c\right|\right) + \frac{5A_c^2}{4k_f}\exp\left(-\frac{10}{k_f}\left|-f - f_c\right|\right)$$



FT of Complex Exponentials

- Using the generalized Fourier Transform of a constant and the frequency shift property we can find the FT of a complex exponential:

$$z(t) = e^{j2\pi f_o t}$$

- This is just a "frequency shift" of a constant, thus

$$Z(f) = \delta(f - f_o)$$

- This results in

$$e^{j2\pi f_o t} \overset{=}{\rightleftharpoons} \delta(f - f_o)$$

$$\sin(2\pi f_o t) \rightleftharpoons \frac{1}{2j}\delta(f - f_o) - \frac{1}{2j}\delta(f + f_o)$$

$$\cos(2\pi f_o t) \rightleftharpoons \frac{1}{2}\delta(f - f_o) + \frac{1}{2}\delta(f + f_o)$$

Fourier Transform of Periodic Signals

- Using this same approach we can develop the Fourier Transform for any periodic signal.
- Specifically, we can define a Fourier Series for any periodic signal that is valid over all time by making $T_F = T_o$.
- Using the linearity property of the Fourier Transform we can write the FT of any periodic signal:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}$$

$$X(f) = F\left\{ \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t} \right\}$$

$$= \sum_{n=-\infty}^{\infty} c_n \delta(f - n f_o)$$

Example 5.2

- Find the Fourier Transform of a 50% duty cycle square wave with period T_o, amplitude 1 and average value 1/2 .
- The signal can be written as

$$x(t) = \sum_{n=-\infty}^{\infty} rect\left(\frac{t - nT_o}{T_o / 2}\right)$$

- Clearly this signal is not integrable and thus does not have a Fourier Transform in the strict sense.
- However, we can write this in terms of its Fourier Series coefficients as

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_F t}$$
$$= \frac{1}{2} \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{k}{2}\right) e^{j2\pi k f_o t}$$

$$c_k = f_o T_w \text{sinc}\left(k f_o T_w\right)$$
$$= \frac{1}{2} \text{sinc}\left(\frac{k}{2}\right)$$

Example 5.2 (cont.)

- Writing the signal in terms of its Fourier Series coefficients (using $T_F = T_o$):

$$x(t) = \frac{1}{2}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\frac{k}{2}\right) e^{j2\pi k f_o t}$$

- Using the linearity property and the FT for a complex exponential:

$$F\left\{e^{j2\pi f_o t}\right\} = \delta\left(f - f_o\right)$$

$$\begin{aligned} X(f) &= F\left\{\frac{1}{2}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\frac{k}{2}\right) e^{j2\pi k f_o t}\right\} \\ &= \frac{1}{2}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\frac{k}{2}\right) F\left\{e^{j2\pi k f_o t}\right\} \\ &= \frac{1}{2}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\frac{k}{2}\right) \delta\left(f - k f_o\right) \end{aligned}$$

Note that since the signal is *periodic* the spectrum (i.e., Fourier Transform) is *discrete*

Example 5.2 (cont.)

- Plotting the spectrum: