Introduction to Communication Systems
Homework 8
SOLUTIONS

1. Consider a message signal with a magnitude spectrum given in Figure 1. Sketch the spectrum of a Large Carrier AM system with k_a =0.5, a peak message value of 1 and a normalized average message power P_m = -12dBW. Determine the post-detection SNR if the pre-detection SNR is 10dB.

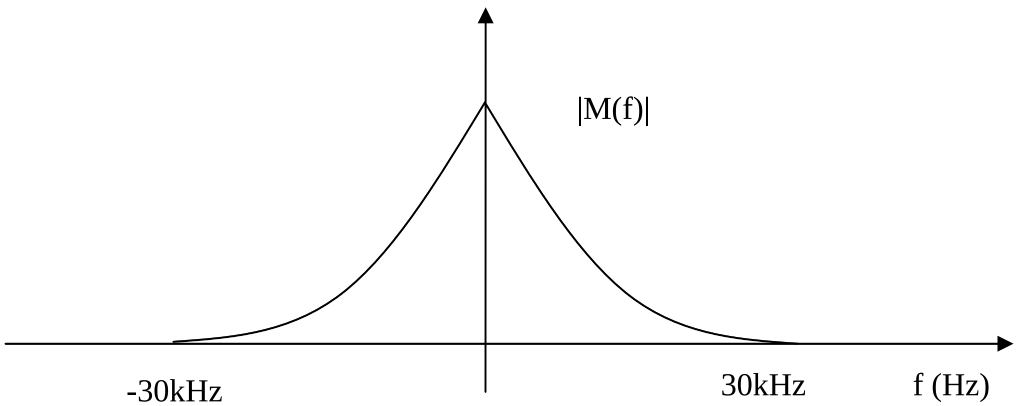


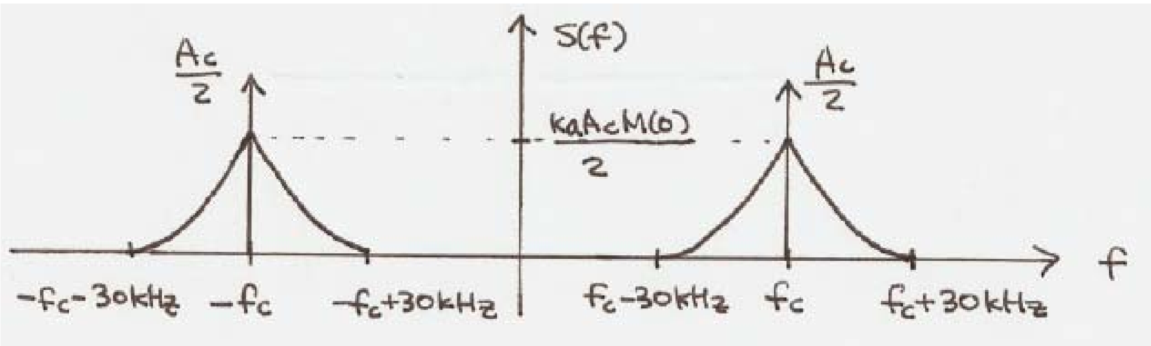
Figure 1: Magnitude Spectrum of the Message Signal

SOLUTION:

$$k_a = 0.5, A_m = 1$$
$$P_m = -12dBW = 10^{-1.2} = 0.0631W$$

a) Sketch the spectrum of a Large Carrier AM.

$$s(t) = A_c\left(1 + k_a m(t)\right)\cos\left(2\pi f_c t\right)$$
$$S(f) = \frac{A_c}{2}\left[\delta\left(f - f_c\right) + \delta\left(f + f_c\right)\right] = \frac{k_a A_c}{2}\left[M\left(f - f_c\right) + M\left(f + f_c\right)\right]$$



b) Determine the post-detection SNR

$$SNR_{pre}^{AM} = 10dB$$
$$SNR_{pre}^{AM} = \frac{A_c^2\left(1+k_a^2 P\right)}{2N_o B_T} = 10dB = 10$$
$$B_T = 2W$$
$$W = 30kHz$$
$$\frac{A_c\left(1+0.5^2 \cdot 0.0631\right)}{2N_o \cdot 2 \cdot 30000} = 10 \rightarrow N_o = 8.465\times10^{-7} A_c^2$$
$$SNR_{post}^{AM} = \frac{A_c^2 k_a^2 P}{2N_o W} = \frac{A_c^2 k_a^2 P}{N_o B_T}$$

Using the SNR AM pre equation,

$$= \frac{20N_o B_T - A_c^2}{N_o B_T} = 20 - \frac{A_c^2}{N_o B_T}$$
$$= 20 - \frac{1}{8.465\times10^{-7}\cdot 60000}$$
$$= 0.311 = -5.07dB$$

2. Consider the message signal $m(t) = e^{-2t}\cos(2\pi t)u(t)$. Sketch the time-domain plot of a Large Carrier AM signal with f_c = 200Hz, A_c = 2 and k_a = 0.25.

SOLUTION:

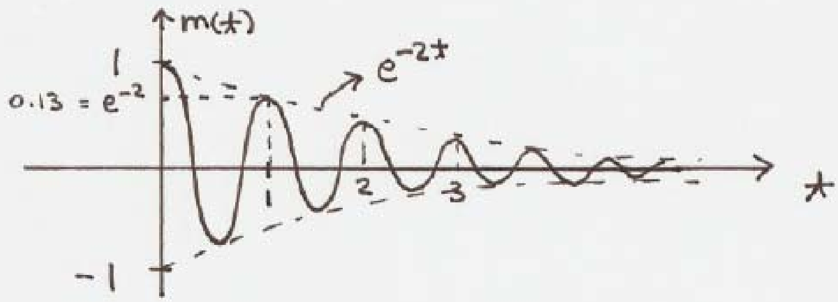
$$m(t) = e^{-2t}\cos(2\pi t)u(t)$$

Time domain plot of LC-AM

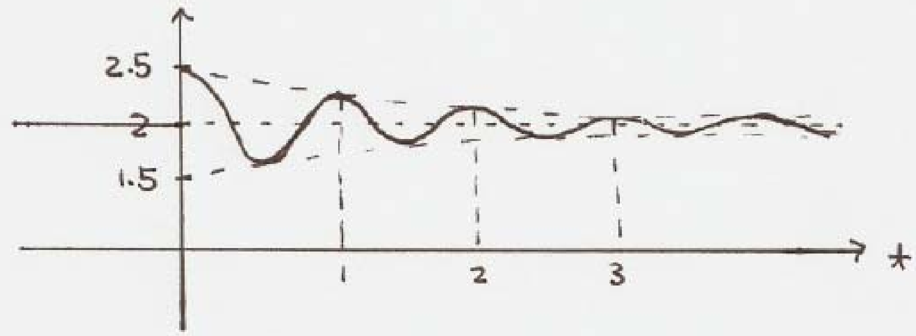
$$s(t) = A_c\left(1+k_a m(t)\right)\cos\left(2\pi f_c t\right)$$
$$= 2\left(1+0.25m(t)\right)\cos\left(400\pi t\right)$$

Start from the function inside

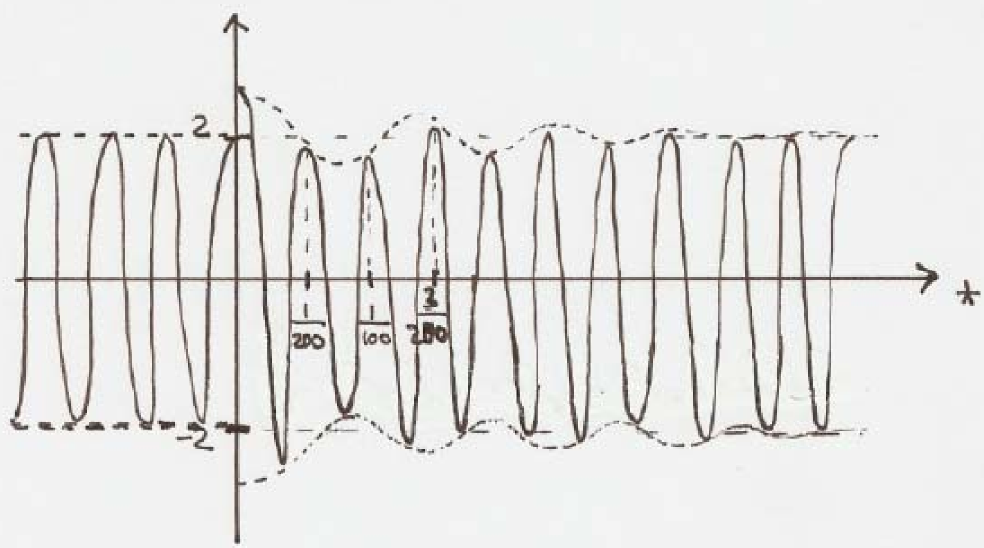
① $m(t)$



② $2(1+0.25\,m(t))$



③ $2(1+0.25\,m(t))\cos(400\pi t)$



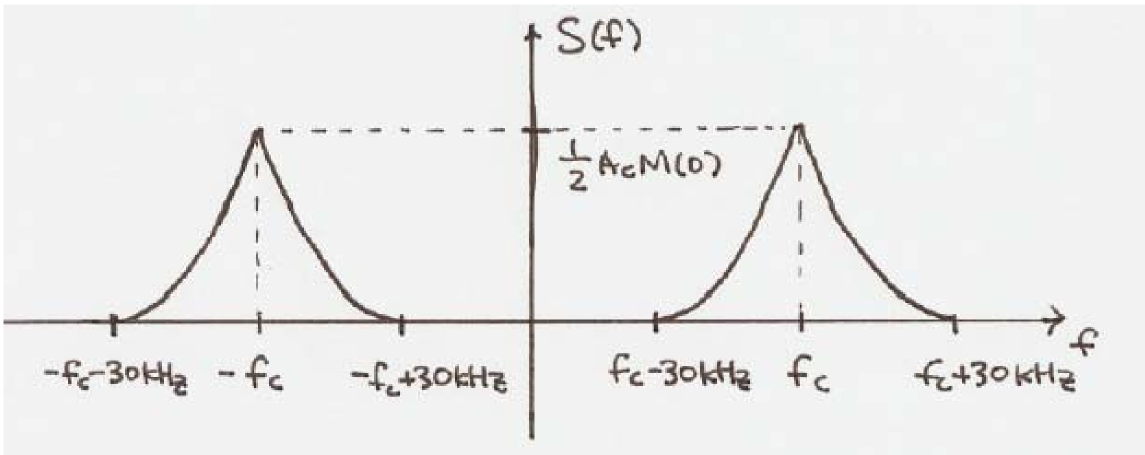
3. Repeat problem #1 using DSB-SC.

SOLUTION:

a)

$$s(t) = A_c \cos\left(2\pi f_c t\right) m(t)$$

$$S(f) = \frac{1}{2} A_c \left[M\left(f - f_c\right) + M\left(f + f_c\right) \right]$$



b)

$$SNR_{pre}^{DSB} = \frac{A_c^2 P}{2N_o B_T} = 10dB = 10$$

$$SNR_{post}^{DSB} = \frac{A_c^2 P}{2N_o W} = \frac{A_c^2 P}{N_o B_T} = 20 = 13dB$$

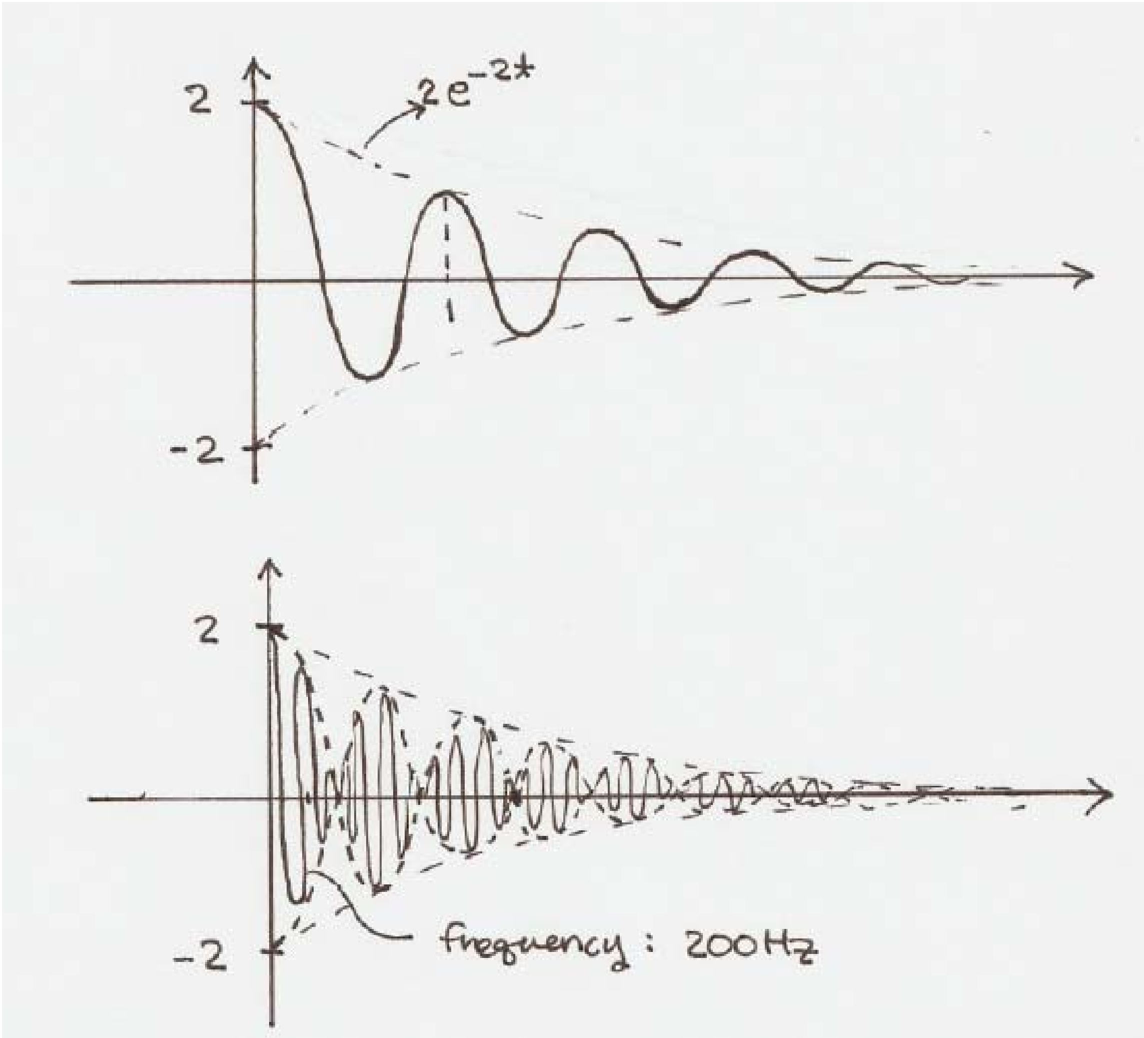
$$W = \frac{B_T}{2}$$

4. Repeat problem #2 using DSB-SC.

SOLUTION:

$$s(t) = A_c \cos\left(2\pi f_c t\right) e^{-2t} \cos\left(2\pi t\right) u(t)$$

$$= 2 \cdot e^{-2t} \cos\left(2\pi t\right) u(t) \cdot \cos\left(400\pi t\right)$$



5. A SSB signal is received with power -100dBm. The front end bandwidth is B_T = 125kHz (assume that the receiver filter is just large enough to pass the desired signal). If the signal noise temperature is 875K, determine the pre-detection SNR. What is the post-detection SNR? Could you do better with DSB-SC ?

SOLUTION:

$$P_{SSB} = -100dBm = 10^{-10}\,mW = 10^{-13}W$$
$$B_T = 125kHz$$
$$T_n = 875K$$

$W = B_T$ for the SSB signal

$$\rightarrow P_{n,SSB} = kT_nB = 1.38\times10^{-23}\cdot 875\cdot 125\times10^{3} = 1.51\times10^{-15}W$$

Received SNR,

$$SNR_{pre}^{SSB} = \frac{P_{SSB}}{P_n} = \frac{10^{-13}}{1.51\times10^{-15}} = 66.2 = 18.2dB$$
$$SNR_{post}^{SSB} = SNR_{pre}^{SSB} = 18.2dB$$

Could you do better with DSB-SC? With DSB-SC,

$$B_T = 2W$$
$$P_{n,DSB} = P_{n,SSB}\cdot 2 = 3.02\times10^{-15}W$$
$$SNR_{pre}^{DSB} = \frac{10^{-13}}{3.02\times10^{-15}} = 33.1 = 15.2dB$$
$$SNR_{post}^{DSB} = 2\cdot SNR_{pre}^{DSB} = 66.2 = 18.2dB$$

Therefore, for the pre-detection SNR, SSB can do better than the DSB, but for the post detection SNR, both DSB and SSB show the same performance.

6. Consider the received signal $r(t) = A_c \cos(2\pi f_c t) + n(t)$ where *n(t)* is AWGN with power spectral density $N_o/2$. If the received signal is applied to an RC filter with noise-equivalent bandwidth $B_N = \frac{1}{4RC}$, assuming that the sinusoid is unaffected by the filter, derive an expression for the output SNR in terms of *R*, *C*, N_o and A_c.

SOLUTION:

$$r(t) = A_c \cos(2\pi f_c t) + n(t)$$

$$S_n(f) = \frac{N_o}{2}$$

RC filter with $B_N = \frac{1}{4RC}$

$$E\left[n^2(t)\right] = B_N(2)\cdot\frac{N_o}{2} = \frac{N_o}{4RC} \rightarrow \text{Noise Power}$$

The signal power is $\frac{A_c^2}{2}$ for a sinusoid, so the signal to noise ratio is given by

$$SNR = \frac{\frac{A_c^2}{2}}{\frac{N_o}{4RC}} = \frac{2A_c^2 RC}{N_o}$$

7. Derive an expression for the post detection SNR of a DSBSC signal if the message is $m(t) = A_m \cos(2\pi f_m t)$. Your answer should be in terms of N_o, f_m and A_m, and A_c.

SOLUTION:

$$m(t) = A_m \cos(2\pi f_m t)$$

$$SNR_{post}^{DSB} = \frac{A_c^2 P}{2N_o W}$$

$$P = E\left[m^2(t)\right] = \frac{A_m^2}{2}$$

$$W = \frac{B_T}{2} = \frac{2f_m}{2} = f_m$$

$$\therefore SNR_{post}^{DSB} = \frac{A_c^2\left(\frac{A_m^2}{2}\right)}{2N_o f_m} = \frac{A_c^2 A_m^2}{4N_o f_m}$$

8. Assume that a message signal has a power spectral density of

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W} & |f| \le W \\ 0 & else \end{cases}$$

Determine the post-detection SNR of a DSBSC signal in terms of *a, W,* A_c *and* N_o.

SOLUTION:

$$SNR_{post}^{DSB} = \frac{A_c^2 P}{2N_o W}$$

For the given message spectrum, the power is

$$\begin{aligned} P_m &= \int_{-\infty}^{\infty} S_M(f)df \\ &= \int_{-W}^{W} \frac{a|f|}{W} df \\ &= \frac{a}{W} \cdot 2\int_0^W f df \\ &= \frac{2a}{W} \frac{f^2}{2}\bigg|_0^W = aW \end{aligned}$$

$$SNR_{post}^{DSB} = \frac{A_c^2 \cdot aW}{2N_o W} = \frac{A_c a}{2N_o}$$

9. Repeat problem #8 for Large Carrier AM with $k_a = 0.25$.

SOLUTION:

$$\begin{aligned} SNR_{post}^{AM} &= \frac{A_c^2 k_a^2 P}{2N_o W} \\ &= \frac{A_c^2 a}{2N_o} \cdot k_a^2 \\ &= 0.0625 \frac{A_c^2 a}{2N_o} = \frac{A_c^2 a}{32N_o} \end{aligned}$$

10. A 10kW transmitter amplitude modulates (large carrier AM) a message signal $m(t) = \sin(2000\pi t)$ using 50% modulation. Propagation losses between the transmitter and receiver are 90dB. The receiver has a front end with spectral density N_o = -113dBW/Hz. and includes a bandpass filter with bandwidth B_T = 2W = 10kHz. What is the post-detection SNR assuming that the receiver uses a product detector?

SOLUTION:

If the output of a 10kW transmitter is attenuated by 90dB through propagation, then the received signal level R is

$$R = 10^4 \times 10^{-90/10} = 10^{-5} W$$

For an amplitude modulated signal, this received power corresponds to

$$R = \frac{A_c^2}{2}\left(1 + k_a^2 P\right)$$

From eq. 9.30

$$\begin{aligned} SNR_{post}^{AM} &= \frac{A_c^2 k_a^2 P}{2 N_o W} \\ &= \frac{r}{1 + k_a^2 P} \cdot \frac{k_a^2 P}{N_o W} \end{aligned}$$

50% Modulations means that

$$\begin{aligned} &\mu = 0.5 \\ &k_a = \mu A_m = \mu \cdot 1 = 0.5 \\ &P = P_m = E\left\{m^2(t)\right\} = \frac{1}{2} \\ &N_o = 10^{-11.3} W \end{aligned}$$

$$\therefore SNR_{post}^{AM} = \frac{10^{-5} \cdot (0.5)^2 \cdot (0.5)}{\left(1 + (0.5)^2 \cdot 0.5\right)\left(10^{-11.3}\right)\left(5000\right)} = 44.4 = 16.5 dB$$