Introduction to Communication Systems

Homework 9

SOLUTIONS

1. Consider an FM system that has a pre-detection SNR of 14dB and achieves a post-detection SNR of 30dB. If the message has a normalized average power of 1W and a bandwidth of 50kHz. Using Carson's rule (and assuming that the pre-detection SNR exceeds the necessary threshold) estimate the transmission bandwidth of the FM signal.

SOLUTION:

$$SNR_{pre}^{FM} = 14dB$$

$$SNR_{post}^{FM} = 30dB$$

Normalized average power of the message is then,

$$1W = \overline{\left({}^{m}\!/\!{}_{V_p} \right)^2}$$

$$W = 50kHz$$

$$\frac{SNR_{pre}^{FM}}{SNR_{post}^{FM}} = 6D_f^2\left(D_f + 1\right)\overline{\left({}^{m}\!/\!{}_{V_p} \right)^2}$$

$$\frac{10^3}{10^{1.4}} = 6D_f^2\left(D_f + 1\right)\bullet 1$$

$$D_f = 1.598$$

Using Carson's Rule,

$$\begin{aligned} B_T &= 2\left(D_f + 1\right)W \\ &= 2\left(1.598 + 1\right)50kHz \\ &= 259.8kHz \end{aligned}$$

2. Repeat problem #1 assuming that pre-emphasis is used with f_{3dB} = 10kHz.

SOLUTION:

With pre-emphasis,

$$\frac{SNR_{pre}^{FM}}{SNR_{post}^{FM}} = 2D_f^2\left(D_f + 1\right)\left(\frac{W}{f_{3dB}}\right)^2\overline{\left({}^{m}\!/\!{}_{V_p} \right)^2}$$

$$\frac{10^3}{10^{1.4}} = 2D_f^2\left(D_f + 1\right)\left(\frac{50kHz}{10kHz}\right)^2\bullet 1$$

$$D_f = 0.687$$

$$B_T = 2\left(D_f + 1\right)W = 168.7kHz$$

3. Consider a message signal $m(t) = \cos(400\pi t)$ that is transmitted via FM. If the final filter has an ideal characteristic from 100Hz to 300Hz, determine the post-detection SNR. (Assume that k_f = 1kHz/volt and that the pre-detection SNR is 500.)

SOLUTION:

$$m(t) = \cos(400\pi t) \rightarrow FM$$

$$k_f = 1kHz/volt$$

$$SNR_{pre}^{FM} = 500$$

$$SNR_{post}^{FM} = ?$$

$$\Rightarrow SNR_{pre}^{FM} = \frac{A_c^2}{2N_oB_T} = 500$$

$$B_T = 2\left(\Delta f + f_m\right) = 2\left(k_f A_m + f_m\right)$$

$$= 2\left(1000\frac{Hz}{volt}\cdot 1volt + 200Hz\right) = 2400Hz$$

$$\therefore \frac{A_c^2}{N_o} = 1000 \cdot B_T = 2.4\times 10^6 Hz$$

The post detection filter is not ideal with unity gain from 0 to W and zero for higher frequencies. Consequently, we must re-evaluate the post-detection noise using Eq. (9.58)

Average post-detection noise power is equal to

$$= \frac{N_o}{A_c^2}\int_{100}^{300} f^2 df$$

$$= \frac{N_o}{A_c^2}\frac{1}{3}\left(300^3 - 100^3\right) = \frac{N_o}{3A_c^2}\left(2.6\times 10^7\right)$$

$$\therefore SNR_{post}^{FM} = \frac{k_f^2\overline{m^2(t)}\cdot 3A_c^2}{N_o\left(2.6\times 10^7\right)}$$

$$= \frac{A_c^2}{N_o}\cdot\frac{k_f^2\overline{m^2(t)}\cdot 3}{2.6\times 10^7}$$

$$= 2.4\times 10^6 \cdot \frac{(1000)^2(0.5)(3)}{2.6\times 10^7}$$

$$= 138461.5 = 51.4dB$$

4. Suppose that the spectrum of a modulating signal occupies the frequency band $f_1 \le |f| \le f_2$. To accommodate this signal, the receiver of an FM system (without pre-emphasis) uses an ideal band-pass filter connected to the output of the FM detector. The filter passes frequencies in the interval $f_1 \le |f| \le f_2$. Deter mine the post-detection SNR and compare it to the baseband reference SNR.

SOLUTION:

This problem is very similar to problem 3.
Since the post detection filter is no longer an ideal brickwall filter, we must revert to Eq. (9.58) to computer the post-detection noise power. (also in the slide #13 of lecture 20)

$$
\begin{aligned}
N &= \int_{-f_2}^{-f_1} P_n(f)df + \int_{f_1}^{f_2} P_n(f)df \\
&= \frac{N_o}{A_c^2}\left[\int_{-f_2}^{-f_1} f^2 df + \int_{f_1}^{f_2} f^2 df\right] \\
&= \frac{2N_o}{3A_c^2}\left(f_2^3 - f_1^3\right) \\
\therefore SNR_{post}^{FM} &= \frac{3A_c^2 k_f^2 \overline{m^2(t)}}{2N_o\left(f_2^3 - f_1^3\right)}
\end{aligned}
$$

$$SNR_{ref} = \frac{A_c^2}{2N_o\left(f_2 - f_1\right)}$$

The corresponding figure of merit is

$$
\begin{aligned}
&= \frac{SNR_{post}^{FM}}{SNR_{ref}} \\
&= \frac{3A_c^2 k_f^2 \overline{m^2(t)}}{2N_o\left(f_2^3 - f_1^3\right)} \cdot \frac{2N_o\left(f_2 - f_1\right)}{A_c^2} \\
&= \frac{3k_f^2 \overline{m^2(t)}}{f_2^2 + f_2 f_1 + f_1^2}
\end{aligned}
$$

5. The signal $\text{sinc}(100t)$ is sampled at a rate of 300Hz. Plot the spectrum of the sampled signal.

SOLUTION:

$$g(t) = \text{sinc}(100t)$$

Sampled at a rate of 300 Hz.

$$f_s = 300Hz \rightarrow T_s = \frac{1}{300}s$$

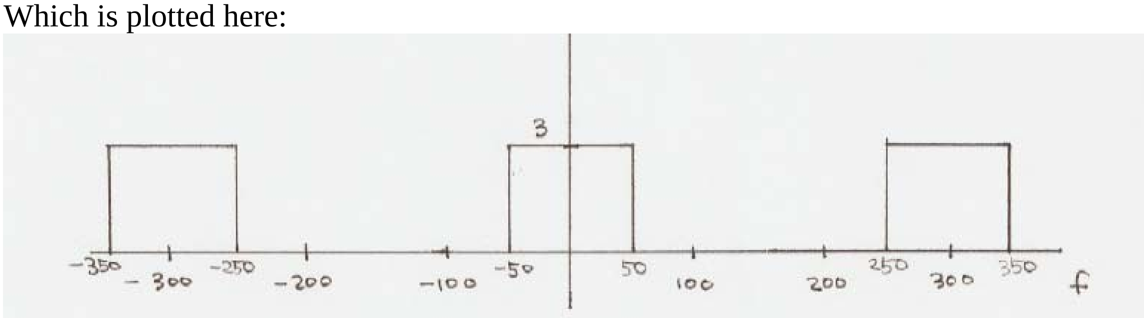
The sampled signal can be represented by

$$\sum_{n=-\infty}^{\infty} g(nT_s)\delta(t-nT_s)$$
$$= \sum_{n=-\infty}^{\infty} \sin c\left(\frac{n}{300}\right)\delta\left(t-\frac{n}{300}\right)$$

The spectrum of this sampled signal is

$$G(f) = \frac{1}{100} rect\left(\frac{f}{100}\right)$$
$$G_s(f) = f_s \sum_{m=-\infty}^{\infty} G\left(f - mf_s\right)$$
$$= 300 \sum_{m=-\infty}^{\infty} \frac{1}{100} rect\left(\frac{f-300m}{100}\right)$$
$$= 3 \sum_{m=-\infty}^{\infty} rect\left(\frac{f-300m}{100}\right)$$

Which is plotted here:



6. Suppose that an analog signal is found to have a bandwidth of 35 kHz and that samples of the signal may be modeled as having a uniform distribution. Find the minimum bit rate at which it would be possible to transmit a digital version of the music signal while maintaining an average SNR of at least 65dB.

SOLUTION:

$$f_s \geq 2B = 35000 \times 2 = 70000 \frac{samples}{\sec}$$
$$\left(\frac{S}{N}\right)_{avg} = M^2$$
$$10\log M^2 \geq 65dB = 1778.28$$

Since *M* is in the form of 2^n when *n* is an integer, the smallest *M* is $2048 = 2^{11}$.

Thus,

$$n \geq 11 \frac{bits}{sample}$$

Therefore, the minimum bit rate is

$$\begin{aligned}
&70000 \frac{samples}{\sec} \cdot \frac{11 bits}{sample} \\
&= 770000 \frac{bits}{\sec} \\
&= 770 kbps
\end{aligned}$$