

Problem 4.12

(a) From Table A3.1 in Appendix 3, we find (by interpolation) that $J_0(\beta)$ is zero for the following values of modulation index:

$$\beta = 2.44,$$

$$\beta = 5.52,$$

$$\beta = 8.65,$$

$$\beta = 11.8,$$

and so on.

(b) The modulation index is defined by

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

Therefore, the frequency sensitivity factor is

$$k_f = \frac{\beta f_m}{A_m} \quad (1)$$

We are given $f_m = 1$ kHz and $A_m = 2$ volts. Hence, with $J_0(\beta) = 0$ for the first time when $\beta = 2.44$, the use of Eq. (1) yields

$$\begin{aligned} k_f &= \frac{2.44 \times 10^3}{2} \\ &= 1.22 \times 10^3 \text{ hertz/volt} \end{aligned}$$

Next, we note that $J_0(\beta) = 0$ for the second time when $\beta = 5.52$. Hence, the corresponding value of A_m for which the carrier component is reduced to zero is

$$\begin{aligned} A_m &= \frac{\beta f_m}{k_f} \\ &= \frac{5.52 \times 10^3}{1.22 \times 10^3} \\ &= 4.52 \text{ volts} \end{aligned}$$

Problem 4.13

- (a) The frequency deviation is

$$\Delta f = k_f A_m = 25 \times 10^3 \times 20 = 5 \times 10^5 \text{ Hz}$$

The corresponding value of the modulation index is

$$\beta = \frac{\Delta f}{f_m} = \frac{5 \times 10^5}{10^5} = 5$$

Using Carson's rule, the transmission bandwidth of the FM wave is therefore

$$B_T = 2f_m(1 + \beta) = 2 \times 100(1 + 5) = 1200 \text{ kHz} = 1.2 \text{ MHz}$$

- (b) Using the universal curve of Fig. 4.9, we find that for $\beta = 5$:

$$\frac{B_T}{\Delta f} = 3$$

Therefore, the transmission bandwidth is

$$B_T = 3 \times 500 = 1500 \text{ kHz} = 1.5 \text{ MHz}$$

which is greater than the value calculated by Carson's rule.

- (c) If the amplitude of the modulating wave is doubled, we find that

$$\Delta f = 1 \text{ MHz} \text{ and } \beta = 10$$

Thus, using Carson's rule we now obtain the transmission bandwidth

$$B_T = 2 \times 100(1 + 10) = 2200 \text{ kHz} = 2.2 \text{ MHz}$$

On the other hand, using the universal curve of Fig. 4.9, we get

$$\frac{B_T}{\Delta f} = 2.75$$

and $B_T = 2.75 \text{ MHz}$.

- (d) If f_m is doubled, $\beta = 2.5$. Then, using Carson's rule, $B_T = 1.4 \text{ MHz}$. Using the universal curve,

$$(B_T/\Delta f) = 4, \text{ and}$$

$$B_T = 4\Delta f = 2 \text{ MHz}$$