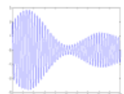
ECE3614
Introduction to
Communications Systems
Fall 2007

Instructor: Dr. R. Michael Buehrer

Lecture #11: Single Sideband and Vestigial Sideband Amplitude Modulation



Overview

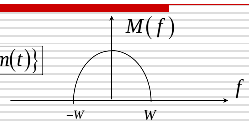
- The original AM signal that we discussed is wasteful of both bandwidth and power
  - DSB-SC improves power efficiency by eliminating the unmodulated carrier
  - We now wish to improve the bandwidth efficiency
  - We can reduce the bandwidth requirements by introducing two new forms of AM
    - Single Sideband (SSB) AM
    - Vestigial Sideband (VSB) AM
- Reading
  - 3.6 – 3.7

Objectives

- The objectives of this lecture are to
  - Demonstrate that the bandwidth efficiency of LC-AM and DSB-SC AM can be improved through two techniques SSB and VSB
  - Describe how SSB and VSB signals are created and demodulated

Spectrum of DSB-SC AM

$M(f) = F\{m(t)\}$

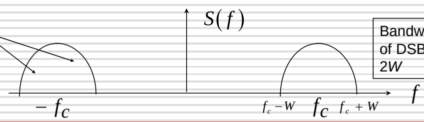


Bandwidth of message : *W*

Message

$$S(f) = \frac{A_c}{2}\{M(f - f_c) + M(f + f_c)\}$$

Since both "sidebands" are present we call this *double* sideband

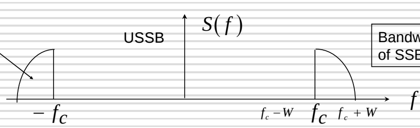


Bandwidth of DSBSC: 2*W*

Single Sideband (SSB)

- Single Sideband (SSB) AM is an AM modulation scheme where only one of the two sidebands are transmitted
- If the lower sideband is transmitted, we call this Lower SSB or LSSB
- If the upper sideband is transmitted, we call this Upper SSB or USSB

Since only one of the "sidebands" are present we call this *single* sideband



Bandwidth of SSB: *W*

Example 11.1

- To develop SSB, let us first consider a sinusoidal message signal:

$$m(t) = A_m \cos(2\pi f_m t)$$

- The transmit signal for a DSB-SC AM wave is

$$\begin{aligned} s(t) &= A_c m(t)\cos(2\pi f_c t) \\ &= A_c A_m \cos(2\pi f_m t)\cos(2\pi f_c t) \\ &= \frac{A_c A_m}{2}\{\cos(2\pi(f_c - f_m)t) + \cos(2\pi(f_c + f_m)t)\} \end{aligned}$$

- Thus, the transmit signal has two "side frequencies",
  - One at $f_c - f_m$ and one at $f_c + f_m$.

Example 11.1 – cont.

$$s(t) = \frac{A_c A_m}{2}\left\{\cos\left(2\pi\left(f_c - f_m\right)t\right) + \cos\left(2\pi\left(f_c + f_m\right)t\right)\right\}$$

- Now suppose that we suppress (somehow) the lower "side-frequency":

$$s_{USF}(t) = \frac{A_c A_m}{2}\cos\left(2\pi\left(f_c + f_m\right)t\right)$$

- This can be written as

$$s_{USF}(t) = \frac{A_c A_m}{2}\cos\left(2\pi f_c t\right)\cos\left(2\pi f_m t\right) - \frac{A_c A_m}{2}\sin\left(2\pi f_c t\right)\sin\left(2\pi f_m t\right)$$

- On the other hand, if we wanted to suppress the higher "side-frequency" we obtain

$$s_{LSF}(t) = \frac{A_c A_m}{2}\cos\left(2\pi f_c t\right)\cos\left(2\pi f_m t\right) + \frac{A_c A_m}{2}\sin\left(2\pi f_c t\right)\sin\left(2\pi f_m t\right)$$

Example 11.1 – cont.

- Thus, we can say that in general, for a single side-frequency we have

$$s_{SSF}(t) = \frac{A_c A_m}{2}\cos\left(2\pi f_c t\right)\cos\left(2\pi f_m t\right) \mp \frac{A_c A_m}{2}\sin\left(2\pi f_c t\right)\sin\left(2\pi f_m t\right)$$

Where the negative sign results in the upper side-frequency and the plus sign results in the lower side-frequency

Example 11.2

- Now, consider a periodic signal that can be written in terms of cosines (i.e., all b_n terms in the Fourier Series are zero)

$$m(t) = \sum_n a_n \cos\left(2\pi f_n t\right)$$

- To obtain only a single side-band (i.e., all of the side-frequencies) we can create the signal

$$s_{SSB}(t) = \frac{A_c A_m}{2}\cos\left(2\pi f_c t\right)\sum_n a_n \cos\left(2\pi f_n t\right)$$
$$\mp \frac{A_c A_m}{2}\sin\left(2\pi f_c t\right)\sum_n a_n \sin\left(2\pi f_n t\right)$$

Example 11.2 – cont.

- If we define a new signal

$$\hat{m}(t) = \sum_n a_n \sin(2\pi f_n t)$$

we can write

$$s_{SSB}(t) = \frac{A_c A_m}{2}\cos(2\pi f_c t)\, m(t) \mp \frac{A_c A_m}{2}\sin(2\pi f_c t)\,\hat{m}(t)$$

- Further, we should note that $\hat{m}(t)$ is simply *m(t) with each cosine term shifted by -90º.*

Comments

- Let us now make two comments:
  - Just as periodic signals can be represented by a Fourier Series, an aperiodic signal (under most conditions) can be represented by a Fourier Transform
  - The -90º phase shifter is simply the *Hilbert Transform*

$$\hat{M}(f) = H(f)M(f)$$

$$H(f) = -j\,\text{sgn}(f) = \begin{cases} -j & f > 0 \\ j & f < 0 \end{cases}$$

Single Sideband AM

- It can be shown that the result in Example 11.2 can be generalized for any message signal (periodic or aperiodic):

$$s_{SSB}(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

where the negative sign produces the upper sideband and the positive sign produces the lower sideband

- In other words

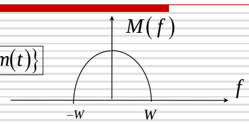
$$S_{USB}(f) = \begin{cases} \frac{A_c}{2} M(f - f_c) & |f| \geq f_c \\ 0 & 0 < |f| < f_c \end{cases}$$

PROOF: See Appendix

$$S_{LSB}(f) = \begin{cases} 0 & |f| > f_c \\ \frac{A_c}{2} M(f - f_c) & 0 < |f| \leq f_c \end{cases}$$

Spectrum of USSB AM

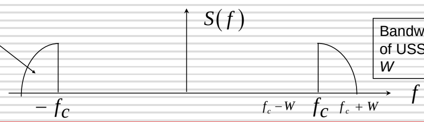
$M(f)=F\{m(t)\}$



Bandwidth of message : *W*

Message

$$S_{USSB}(f)=\begin{cases}\frac{A_c}{2}M(f-f_c) & f>f_c\\ 0 & -f_c>f>f_c\\ \frac{A_c}{2}M(f+f_c) & f<-f_c\end{cases}$$

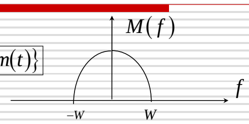


Since only one of the "sidebands" is present we call this *single* sideband

Bandwidth of USSB: *W*

Spectrum of LSSB AM

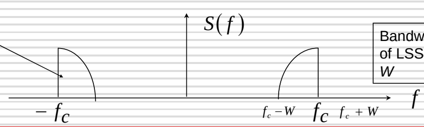
$M(f)=F\{m(t)\}$



Bandwidth of message : *W*

Message

$$S_{LSSB}(f)=\begin{cases}0 & f>f_c\\ \frac{A_c}{2}M(f-f_c)+\frac{A_c}{2}M(f+f_c) & -f_c>f>f_c\\ 0 & f<-f_c\end{cases}$$



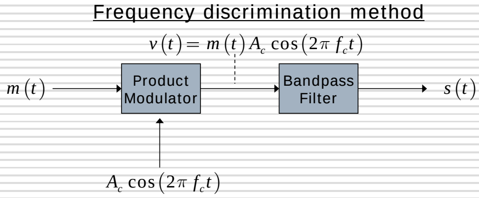
Since only one of the "sidebands" is present we call this *single* sideband

Bandwidth of LSSB: *W*

Modulators for SSB

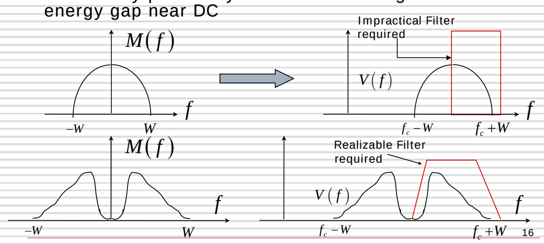
- Frequency discrimination method
- Direct implementation of previous frequency domain equations (phase discriminator)

<u>Frequency discrimination method</u>



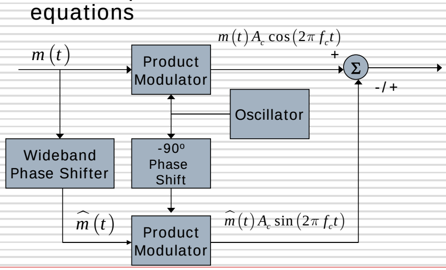
Frequency Discriminator - Issues

- Clearly the most challenging component of the frequency discriminator is the bandpass filter
- In order to transmit one of the two sidebands requires an impractically steep filter
- This is only practically realizable if the signal has an energy gap near DC



Modulators for SSB

- Phase discrimination method
- Direct implementation of the time domain equations

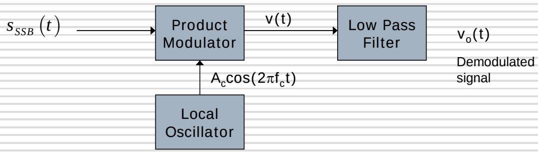


Phase Discriminator

- The most challenging aspect of the phase discrimination method is the wideband phase shifter
  - It must phase shift all components of the signal by 90º
- The technique essentially creates a phase shifted version of the message signal to cause self-interference which cancels either the upper or lower sideband
  - Demonstrates the usefulness of phase

Coherent Detection of SSB

- A coherent product detector which was used for DSBSC can also be used for SSB



Coherent Detection – cont.

$$s_{SSB}(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

- At the output of the product demodulator

$$v(t) = \left\{\frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\right\} A_c\cos(2\pi f_c t)$$

$$= \left\{\frac{A_c^2}{2} m(t)\cos^2(2\pi f_c t) \mp \frac{A_c^2}{2}\hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)\right\}$$

$$= \frac{A_c^2}{4} m(t)\{1+\cos(4\pi f_c t)\} \mp \frac{A_c^2}{4}\hat{m}(t)\sin(4\pi f_c t)$$

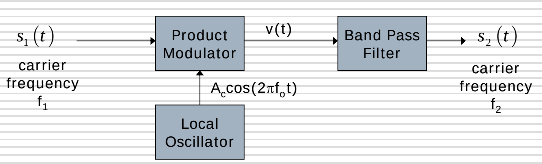
- At the output of the low pass filter

$$v_o(t) = LPF\{v(t)\}$$

$$= \frac{A_c^2}{4} m(t)$$

Frequency Translation

- The basic operation in SSB modulation and demodulation is frequency translation
  - This is also called frequency changing, mixing or heterodyning

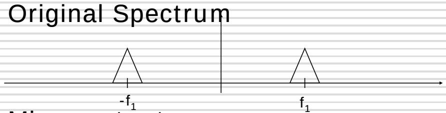


Frequency Translation – cont.

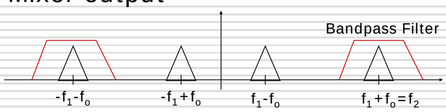
- There are two basic forms of frequency translation
  - Up conversion
  - Down conversion
- Up conversion
  - The translated frequency is higher than the incoming frequency
  - The oscillator frequency $f_o = f_2 - f_1$
- Down conversion
  - The translated frequency is lower than the incoming frequency
  - The oscillator frequency $f_o = f_1 - f_2$

Example – Up conversion

- Original Spectrum



- Mixer output



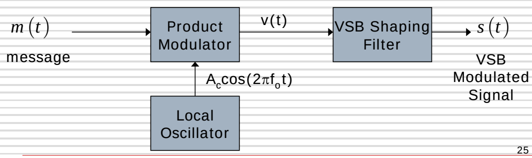
- $f_o = f_2 - f_1$

Vestigial Sideband AM

- SSB transmission has serious challenges, particularly if the message signal has energy near DC
  - Filter challenges associated with frequency discrimination technique
  - Wideband phase shifter associated with the phase discrimination technique
- DSB-SC requires double the bandwidth of SSB but is easier to implement
- A compromise provides a bandwidth between SSB and DSB-SC but is more implementable
  - Termed Vestigial Sideband

Vestigial Sideband AM – cont.

- VSB is different from SSB in that
  - Instead of completely eliminating one of the two sidebands a portion or vestige of the sideband is transmitted
  - Instead of transmitting the entire portion of the other sideband most of the other sideband is transmitted
- Total bandwidth = $W+f_v = W(1+\alpha)$ where $0<\alpha<1$



VSB Shaping Filter

- The key to VSB amplitude modulation is the shaping filter
- Similar to the frequency discrimination method of producing SSB, this filter is a bandpass filter which filters out either the upper or lower sideband
- In contrast to the SSB case, this filter does not need to completely eliminate the unwanted band and does not perfectly retain the desired band
  - This is the key to making the filter practically realizable
- However, the key to the filter design is that *the transmitted vestige must compensate for the distortion caused to the desired sideband.* This is maintained by the following filter requirement:

$$H(f+f_c)+H(f-f_c)=1 \quad -W \le f \le W$$

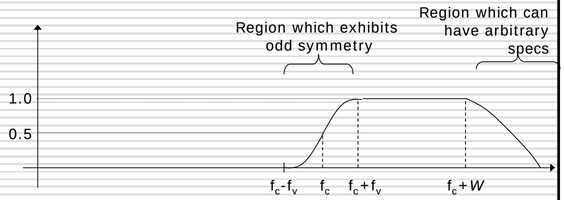
VSB Shaping Filter – cont.

$$H(f+f_c)+H(f-f_c)=1 \quad -W \le f \le W$$

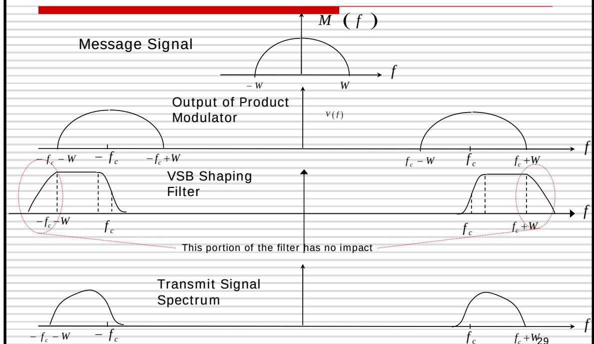
- $H(f+f_c)$ is positive-frequency portion of the bandpass transfer function shifted to the left by f_c.
- $H(f-f_c)$ is negative-frequency portion of the bandpass transfer function shifted to the right by f_c.
- The transfer function of the VSB shaping filter exhibits odd symmetry about the carrier frequency f_c *in the frequency region of the unwanted sideband.*
- The requirement above must hold *only for the interval given.* The filter can have arbitrary specs outside the frequency band of the message.

VSB Shaping filter – cont.

- Positive frequency portion of the VSB shaping filter



Example



Example 11.3 - VSB

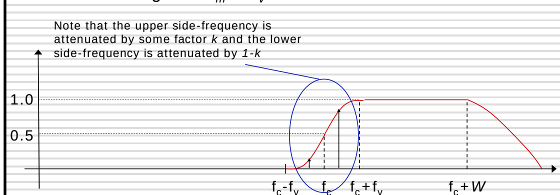
- Consider a sinusoidal message signal

$$m(t) = A_m \cos(2\pi f_m t)$$

and the carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$

We have the following positive portion of the spectrum assuming that $f_m < f_v$



Example 11.3 – cont.

- The transmitted spectrum is then

$$S(f)=\frac{A_cA_m}{4}k\left\{\delta\left(f-(f_c+f_m)\right)+\delta\left(f+(f_c+f_m)\right)\right\}+$$
$$\frac{A_cA_m}{4}(1-k)\left\{\delta\left(f-(f_c-f_m)\right)+\delta\left(f+(f_c-f_m)\right)\right\}$$

- Taking the inverse Fourier Transform we have

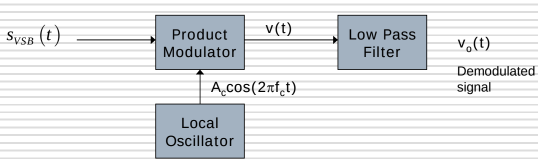
$$s(t)=\frac{A_cA_m}{4}k\left\{e^{j2\pi(f_c+f_m)t}+e^{-j2\pi(f_c+f_m)t}\right\}+$$
$$\frac{A_cA_m}{4}(1-k)\left\{e^{j2\pi(f_c-f_m)t}+e^{-j2\pi(f_c-f_m)t}\right\}$$
$$=\frac{A_cA_m}{2}k\cos\left(2\pi\left(f_c+f_m\right)t\right)+\frac{A_cA_m}{2}(1-k)\cos\left(2\pi\left(f_c-f_m\right)t\right)$$
$$=\frac{A_cA_m}{2}\cos\left(2\pi f_mt\right)\cos\left(2\pi f_ct\right)+\frac{A_cA_m}{2}(1-2k)\sin\left(2\pi f_mt\right)\sin\left(2\pi f_ct\right)$$

Example 11.3 – cont.

- Note that if
  - k =0.5 we have DSB-SC AM
  - k =0 we have LSSB AM
  - k =*1* we have USSB AM
  - $0 < k < 0.5$ we have VSB where the attenuated upper sideband defines the lower sideband vestige
  - $0.5 < k < 1$ we have VSB where the attenuated lower sideband defines the upper sideband vestige

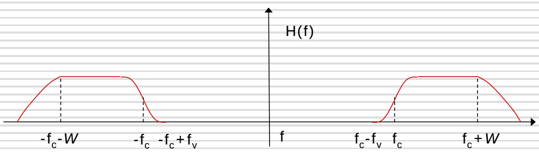
Coherent Detection of VSB

- A coherent product detector which was used for DSBSC can also be used for VSB



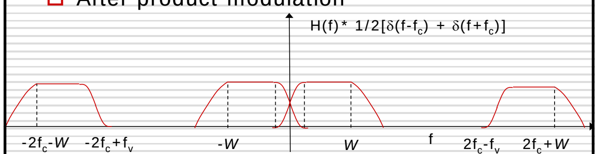
Coherent Detection of VSB – cont.

- Let us examine coherent detection of VSB by examining the band-shaping filter throughout the process
  - Assume perfect coherence ($\phi = 0$)
- Original Band-shaping filter spectrum:

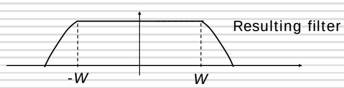


Coherent Detection of VSB – cont.

- After product modulation



- After eliminating high frequency images



Example 11.4 - Demodulation

- Returning to our sinusoidal message example, the transmit signal is

$$s(t) = \frac{A_c A_m}{2}\cos(2\pi f_m t)\cos(2\pi f_c t) + \frac{A_c A_m}{2}(1-2k)\sin(2\pi f_m t)\sin(2\pi f_c t)$$

- After the product detector we have

$$\begin{aligned} v(t) &= s(t)\cos(2\pi f_c t) \\ &= \frac{A_c A_m}{2}\cos(2\pi f_m t)\cos(2\pi f_c t)\cos(2\pi f_c t) \\ &\quad + \frac{A_c A_m}{2}(1-2k)\sin(2\pi f_m t)\sin(2\pi f_c t)\cos(2\pi f_c t) \\ &= \frac{A_c A_m}{4}\cos(2\pi f_m t)\{1+\cos(4\pi f_c t)\} + \frac{A_c A_m}{4}(1-2k)\sin(2\pi f_m t)\sin(4\pi f_c t) \end{aligned}$$

Example 11.4 –cont.

- After low-pass filtering we have

$$v_o(t) = LPF\{v(t)\}$$
$$= LPF\left\{\frac{A_c A_m}{4}\cos(2\pi f_m t)\{1+\cos(4\pi f_c t)\} + \frac{A_c A_m}{4}(1-2k)\sin(2\pi f_m t)\sin(4\pi f_c t)\right\}$$
$$= \frac{A_c A_m}{4}\cos(2\pi f_m t)$$

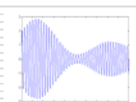
- Which is simply a scaled version of the message signal

Summary

- In this lecture we have examined two techniques to reduce the bandwidth of Amplitude Modulation
  - Single Sideband (SSB)
  - Vestigial Sideband (VSB)
- SSB AM achieves the minimum bandwidth of *W* (equal to the message bandwidth)
  - The cost is difficult implementation (i.e., high complexity)
- VSB trades reduced complexity for increased bandwidth of $(1+\alpha)W$ where $0<\alpha<1$

Appendix 1

Proof of SSB Spectrum



Proof

- Taking the Fourier Transform

$$\begin{aligned} S_{SSB}(f) &= F\{s_{SSB}(t)\} \\ &= F\left\{\frac{A_c}{2}m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\widehat{m}(t)\sin(2\pi f_c t)\right\} \\ &= \frac{A_c}{4}\{M(f-f_c)+M(f+f_c)\} \\ &\mp \frac{A_c}{4j}\{\widehat{M}(f-f_c)-\widehat{M}(f+f_c)\} \\ &= \frac{A_c}{4}\{M(f-f_c)(1\pm jH(f-f_c))+M(f+f_c)(1\mp jH(f+f_c))\} \end{aligned}$$

Proof – cont.

$$\begin{aligned} S_{SSB}(f) &= \frac{A_c}{4}M(f-f_c)(1\pm jH(f-f_c)) \\ &+ \frac{A_c}{4}M(f+f_c)(1\mp jH(f+f_c)) \end{aligned}$$

- To evaluate the upper sideband we choose the signs on the top

$$S_{SSB}(f)=\frac{A_c}{4}M(f-f_c)(1+jH(f-f_c))+\frac{A_c}{4}M(f+f_c)(1-jH(f+f_c))$$

- To evaluate the lower sideband we choose the signs on the bottom

$$S_{SSB}(f)=\frac{A_c}{4}M(f-f_c)(1-jH(f-f_c))+\frac{A_c}{4}M(f+f_c)(1+jH(f+f_c))$$

Upper Sideband

$$S_{SSB}(f)=\frac{A_c}{4}M(f-f_c)(1+jH(f-f_c))+\frac{A_c}{4}M(f+f_c)(1-jH(f+f_c))$$

$$= \begin{cases} \frac{A_c}{4}M(f-f_c)(1+j(-j))+\frac{A_c}{4}M(f+f_c)(1-j(-j)) & f>f_c \\ \frac{A_c}{2}M(f-f_c)(1+j(j))+\frac{A_c}{2}M(f+f_c)(1-j(-j)) & -f_c>f>f_c \\ \frac{A_c}{2}M(f-f_c)(1+j(j))+\frac{A_c}{2}M(f+f_c)(1-j(j)) & f<-f_c \end{cases}$$

$$= \begin{cases} \frac{A_c}{2}M(f-f_c) & f>f_c \\ 0 & -f_c>f>f_c \\ \frac{A_c}{2}M(f+f_c) & f<-f_c \end{cases}$$

Lower Sideband

$$S_{SSB}(f) = \frac{A_c}{4} M(f - f_c)\left(1 - jH(f - f_c)\right) + \frac{A_c}{4} M(f + f_c)\left(1 + jH(f + f_c)\right)$$

$$= \begin{cases} \frac{A_c}{4} M(f - f_c)(1 - j(-j)) + \frac{A_c}{4} M(f + f_c)(1 + j(-j)) & f > f_c \\ \frac{A_c}{4} M(f - f_c)(1 - j(j)) + \frac{A_c}{4} M(f + f_c)(1 + j(-j)) & -f_c > f > f_c \\ \frac{A_c}{4} M(f - f_c)(1 - j(j)) + \frac{A_c}{4} M(f + f_c)(1 + j(j)) & f < -f_c \end{cases}$$

$$= \begin{cases} 0 & f > f_c \\ \frac{A_c}{2} M(f - f_c) + \frac{A_c}{2} M(f + f_c) & -f_c > f > f_c \\ 0 & f < -f_c \end{cases}$$