ECE3614
Introduction to
Communications Systems
Fall 2007

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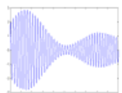
Lecture #13: Introduction to Angle Modulation (FM and PM)



Overview

- **The objective of today's lecture is to introduce angle modulation**
- Angle modulation is a non-linear analog modulation technique that maps the message signal to the angle of a transmitted carrier
- There are two basic forms: Phase Modulation (PM) and Frequency Modulation (FM) with the latter being more common
- Reading
  - 4.1 – 4.3

Analog Angle Modulation

- Phase Modulation (PM) and Frequency Modulation (FM) are termed "angle" modulation because the data changes the angle of the transmitted carrier signal.

$$s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)$$

Time-varying angle or phase

Constant amplitude

- This can be written as

$$s(t) = \frac{A_c}{2}\cos\{\theta_i(t)\}\cos(2\pi f_c t) - \frac{A_c}{2}\sin\{\theta_i(t)\}\sin(2\pi f_c t)$$

- Which results in the following complex baseband representation:

$$g(t) = \frac{A_c}{2}\left(\cos\{\theta(t)\} + j\sin\{\theta(t)\}\right)$$
$$= \frac{A_c}{2}e^{j\theta(t)}$$

FM and PM

- There are two basic forms of angle modulation
  - Frequency Modulation (FM)
  - Phase Modulation (PM)
- FM – the frequency of the carrier is varied linearly with the message signal

$$f(t) = f_c + k_f m(t)$$

- PM – the phase of the carrier is varied linearly with the message signal

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

Frequency Modulation

- FM – the frequency of the carrier is varied linearly with the message signal

$$f(t) = f_c + k_f m(t)$$

  - k_f is frequency sensitivity constant
- Phase is the integral of frequency:

$$\theta(t) = 2\pi \int_0^t f(\tau) d\tau$$

$$= 2\pi \int_0^t \left(f_c + k_f m(\tau) \right) d\tau$$

$$= 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau$$

- Thus, the transmit signal is

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right)$$

Frequency Modulation - cont.

- Alternatively since the instantaneous frequency is

$$f(t) = f_c + k_f m(t)$$

we can write FM as

$$s(t) = A_c \cos\left(2\pi f_c t + \theta(t)\right)$$

where

$$\theta(t) = 2\pi k_f \int_0^t m(\tau) d\tau$$

Phase Modulation

- PM – the phase of the carrier is varied linearly with the message signal

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

  - where k_p is the phase sensitivity constant
- Thus, the transmit signal is

$$s(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

Non-linear Nature of Angle Modulation

- Each of the AM schemes discussed in the previous weeks is linear, however angle modulation is a non-linear modulation scheme
- This can be seen from the definition of linearity
  - If

$$m_1(t) \to s_1(t)$$
$$m_2(t) \to s_2(t)$$

  - Then

$$\{m_1(t) + m_2(t)\} \to s_1(t) + s_2(t)$$

- For DSB-SC we have

$$m_1(t) \to s_1(t) = A_c m_1(t)\cos(2\pi f_c t)$$
$$m_2(t) \to s_2(t) = A_c m_2(t)\cos(2\pi f_c t)$$
$$\{m_1(t) + m_2(t)\} \to A_c\left[m_1(t) + m_2(t)\right]\cos(2\pi f_c t) = s_1(t) + s_2(t)$$

- Thus, DSB-SC is linear

Non-linear modulation – cont.

- For PM we have

$$m_1(t) \to s_1(t) = A_c \cos\left(2\pi f_c t + k_p m_1(t)\right)$$
$$m_2(t) \to s_2(t) = A_c \cos\left(2\pi f_c t + k_p m_2(t)\right)$$

- Now,

$$\{m_1(t) + m_2(t)\} \to A_c \cos\left(2\pi f_c t + k_p\left[m_1(t) + m_2(t)\right]\right) \neq s_1(t) + s_2(t)$$

- Thus, this modulation scheme is *non-linear*
  - The same is true of FM

Transmitted Signals for Angle Modulation

- Phase Modulation: $s_{PM}(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$
- Frequency Modulation: $s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$

  where
  - *m(t)* - message signal
  - A_c - signal amplitude
  - f_c - carrier frequency
  - k_p - phase sensitivity constant (radians/volt)
  - k_f - frequency sensitivity constant (radians/volt-second)

Instantaneous Frequency

- The above form is more natural for PM than for FM
  - It is easier to visualize FM using the *instantaneous frequency*
- Instantaneous frequency for FM:

$$\frac{d}{dt}\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda\right] = 2\pi f_c + 2\pi k_f m(t)\,\frac{\text{rads}}{\text{sec}}$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = \left(f_c + k_f m(t)\right)\text{Hz}$$

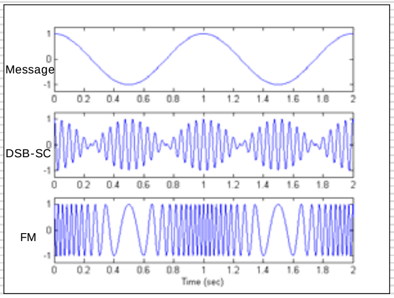
- Instantaneous frequency for PM:

$$\frac{d}{dt}\left[2\pi f_c t + k_p m(t)\right] = 2\pi f_c + k_p \frac{d}{dt} m(t) \Rightarrow f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

Description of Frequency Modulation

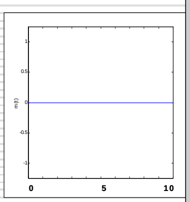
- Peak Frequency Deviation
  - $\Delta f = \max\left\{k_f m(t)\right\} = k_f V_p,$
  - where $V_p = \max|m(t)|$
  - Largest difference between the instantaneous frequency and carrier frequency
- FM Modulation Index
  - Sinusoidal message: $\beta = \frac{\Delta f}{f_m}$, General message: $D = \frac{\Delta f}{W}$,
  - where *W* is the bandwidth of *m(t)*
  - Modulation index controls the BW of the FM signal

FM vs. AM

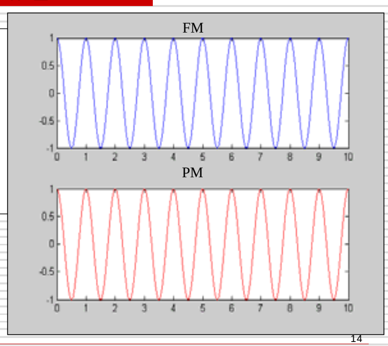


- Example with sinusoidal message signal
- DSB-SC signal has constant frequency but varying amplitude
- FM signal has constant amplitude but varying frequency

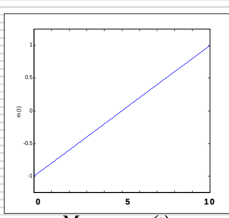
Comparison of PM and FM: Example #1



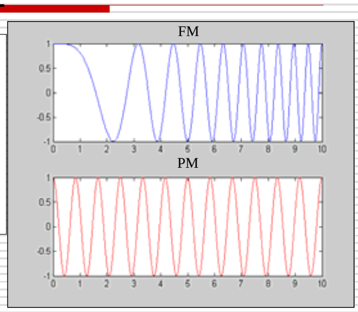
Message *m(t)*



Comparison of PM and FM: Example #2

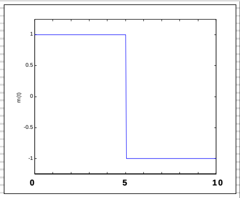


Message *m(t)*

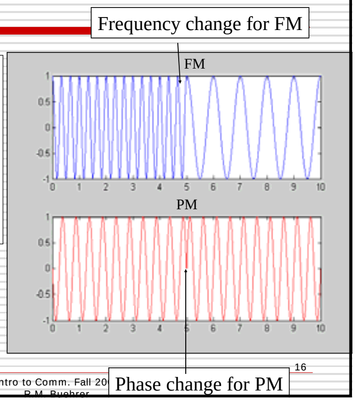


FM signal shows continuous frequency increase
PM signal has single frequency shift

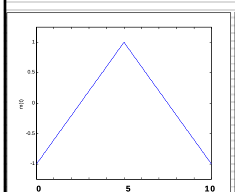
Comparison of PM and FM: Example #3



Message *m(t)*



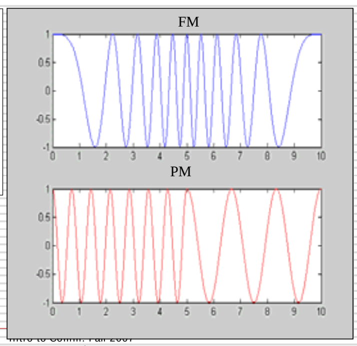
Comparison of PM and FM: Example #4



Message *m(t)*

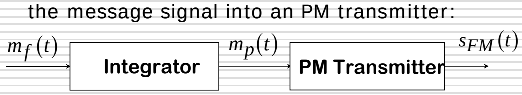
FM signal shows continuous frequency increase followed by decrease

PM signal has positive frequency shift followed by negative frequency shift

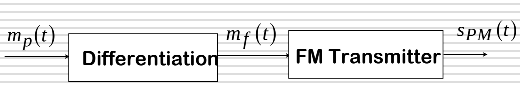


Equivalency of Frequency Modulation and Phase Modulation

- An FM signal can be generated by integrating the message signal into an PM transmitter:



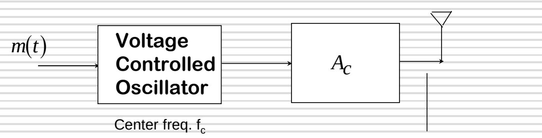
- A PM signal can be generated by differentiating the message signal into an FM transmitter:



Comparison of FM and PM

- They are very similar in signal structure
- Both allow designer to trade bandwidth for power efficiency
- FM has advantages in:
  - Ease of implementation
  - Performance in noise
  - Larger range of BW/performance trade-off
- FM is used almost exclusively

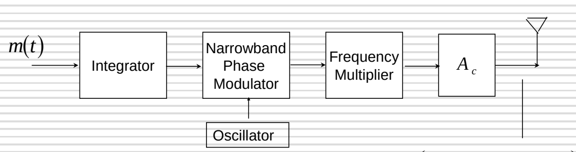
Transmitter for FM



$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$

Direct method

Transmitter for FM

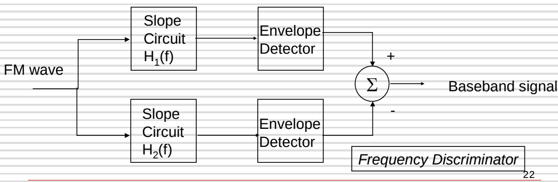


$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\lambda)\,d\lambda\right)$$

Indirect method

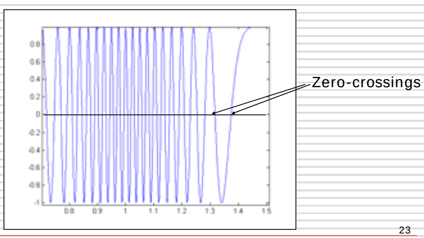
Demodulator For FM

- Many ways of implementing:
  - Discriminator (differentiator) - used in analysis
  - PLL - widely used in practical receivers
  - Zero crossings detector



Zero-Crossings

- Zero-crossings are defined as the instants where the waveform changes polarity (i.e., goes from positive to negative or negative to positive)
- The information content of the message is imbedded in the zero-crossings of the transmitted signal for Angle Modulated waveforms



Example 13.1

- Consider the message signal

$$m(t) = tu(t)$$

- Examine the zero-crossings of PM and FM signals for the following parameters: f_c = 0.25Hz, k_p = $\pi/2$, k_f = 1, A_c = 1;

$$s_{PM}(t) = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

$$s_{FM}(t) = A_c \cos\left(2\pi\left(f_c + k_f m(t)\right)t\right)$$

Example 13.1 – cont.

- We can determine the zero-crossings by finding the values of *t* where s(t) = 0. This is the same as finding the values of *t* where the argument of the cosine wave equals odd multiples of π
- For PM: $s_{PM}(t_n) = A_c \cos\left(2\pi f_c t_n + k_p m(t_n)\right) = 0$
- The zero-crossings are then:

$$2\pi f_c t_n + k_p t_n = \frac{\pi}{2} + n\pi \quad t \geq 0$$

$$2\pi f_c t_n = \frac{\pi}{2} + n\pi \quad t < 0$$

Example 13.1 – cont.

- This results in the following zero-crossing times:

$$t_n = \frac{\frac{\pi}{2} + n\pi}{\left(2\pi f_c + k_p\right)} \quad t \geq 0$$

$$t_n = \frac{\frac{\pi}{2} + n\pi}{2\pi f_c} \quad t < 0$$

- Substituting our values results in:

$$t_n = \frac{\frac{\pi}{2} + n\pi}{\left(\frac{\pi}{2} + \frac{\pi}{2}\right)} \quad t \geq 0 \qquad \rightarrow \qquad t_n = \left(n + \frac{1}{2}\right) \quad t \geq 0 \quad n = 0,1,2\ldots$$

$$t_n = \frac{\frac{\pi}{2} + n\pi}{\frac{\pi}{2}} \quad t < 0 \qquad\qquad t_n = 2n+1 \quad t < 0 \quad n = -1,-2,..$$

Example 13.1 – cont.

- For FM:

$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t \tau u(\tau)\, d\tau\right)$$

$$= A_c \cos\left(2\pi\left(f_c + \frac{k_f}{2} t u(t)\right) t\right)$$

$$= \begin{cases} A_c \cos\left(2\pi\left(f_c + \frac{k_f}{2} t\right) t\right) & t \geq 0 \\ A_c \cos\left(2\pi f_c t\right) & t < 0 \end{cases}$$

$$= \begin{cases} A_c \cos\left(2\pi f_c t + \pi k_f t^2\right) & t \geq 0 \\ A_c \cos\left(2\pi f_c t\right) & t < 0 \end{cases}$$

- Setting the argument equal to odd multiples of π:

$$2\pi f_c t_n + \pi k_f t_n^2 = \frac{\pi}{2} + n\pi \quad t \geq 0 \qquad \rightarrow \qquad 2\pi k_f t_n^2 + \pi f_c t_n - \frac{\pi}{2} - n\pi = 0 \quad t \geq 0$$

$$2\pi f_c t_n = \frac{\pi}{2} + n\pi \quad t < 0 \qquad\qquad t_n = \frac{\frac{\pi}{2} + n\pi}{2\pi f_c} \quad t < 0$$

Example 13.1 – cont.

- Taking the positive root:

$$t_n = \frac{1}{k_f}\left(-f_c + \sqrt{f_c^2 + k_f\left(n + \frac{1}{2}\right)}\right) \quad t \geq 0$$

$$t_n = \frac{\frac{\pi}{2} + n\pi}{2\pi f_c} \quad t < 0$$

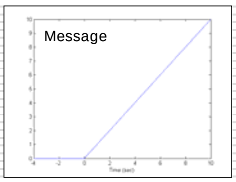
- Substituting our values we have:

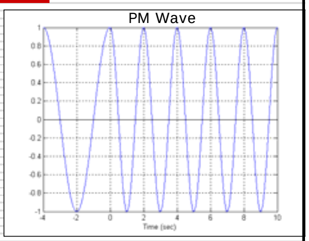
$$t_n = \left(\frac{1}{4}\left(-1 + \sqrt{16n + 9}\right)\right) \quad t \geq 0$$

$$t_n = 2n + 1 \quad t < 0$$

→ Before zero: -1,-3,...
After zero: 0.5, 1, 1.35,

Example 13.1 – cont.

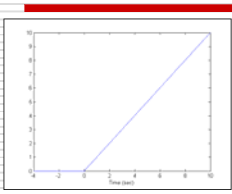


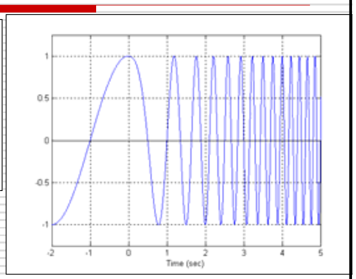


The zero crossings are:
before zero → -1, -3, …
After zero → ½, 3/2, 5/2, …

Equi-spaced zero crossings are consistent with a constant frequency. Since the frequency is derivative of the phase and phase is linear function of time, constant zero-crossings correspond to a linearly increasing message signal.

Example 13.1 – cont.

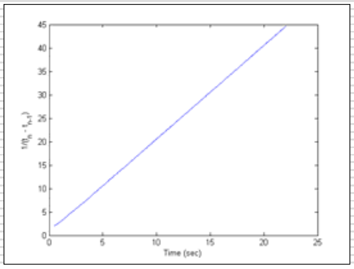




The zero crossings are:
before zero → -1, -3, …
After zero → ½, 1, 1.35, …

Zero crossings are consistent with an increasing frequency (i.e., they are getting closer together.

Example 13.1 –Zero Crossing Detector



- If we plot $\frac{1}{t_n - t_{n-1}}$ versus t_n, we get the plot on the right
- Thus, the zero crossings contain the information signal.

Summary

- Today we have introduced a new analog bandpass modulation technique termed angle modulation
- There are two basic forms of angle modulation
  - Phase Modulation (PM)
  - Frequency Modulation (FM)
- FM is more common and will be discussed at more length in the course

Appendix

- Question: When can we use the expression

$$s_{FM}(t) = A_c \cos\left(2\pi\left(f_c + k_f m(t)\right)t\right)$$

instead of the full expression

$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

- Answer: Rarely!

Appendix – cont.

- When the message signal is a constant, *m(t) = C,* we have

$$\begin{aligned} s_{FM}(t) &= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t C \, d\tau\right) \\ &= A_c \cos\left(2\pi f_c t + 2\pi k_f C t\right) \\ &= A_c \cos\left(2\pi \left(f_c + k_f C\right) t\right) \end{aligned}$$

- Thus, in this case we can safely use the expression

$$s_{FM}(t) = A_c \cos\left(2\pi\left(f_c + k_f m(t)\right)t\right)$$

Appendix – cont.

- When the message signal is a general function of time of the form

$$m(t) = \sum_{n=0}^{K} a_n t^n$$

- We have

$$\begin{aligned} s_{FM}(t) &= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t \left(\sum_{n=0}^{K} a_n \tau^n\right) d\tau\right) \\ &= A_c \cos\left(2\pi f_c t + 2\pi k_f \left(\sum_{n=0}^{K} \frac{a_n}{n+1} \tau^{n+1}\right)\right) \\ &\neq A_c \cos\left(2\pi \left(f_c + 2\pi k_f \left(\sum_{n=0}^{K} a_n t^n\right)\right) t\right) \end{aligned}$$

- In general we require

$$\int_0^t m(\tau)\, d\tau = t m(t)$$