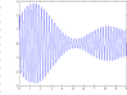


ECE3614
Introduction to
Communications Systems
Fall 2007

Instructor: Dr. R. Michael Buehrer
Lecture #16: Generation and
Demodulation of FM Signals

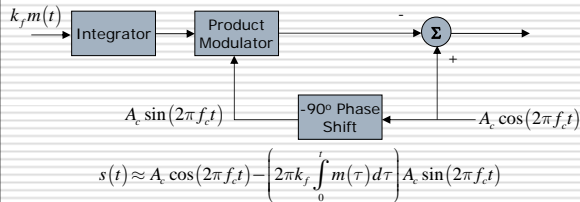


Overview

- Today we continue our discussion of Frequency Modulation (FM) by examining the techniques for generating and demodulating FM signals
- First we will examine transmission techniques
 - Narrowband FM signal generation
 - Wideband FM signal generation
 - Direct
 - Indirect
- Second we will examine three types of demodulation circuits
 - Frequency Discriminator
 - Phase-Locked Loop
 - Zero-crossings detector
- Reading
 - Sections 4.7-4.8

Narrowband FM

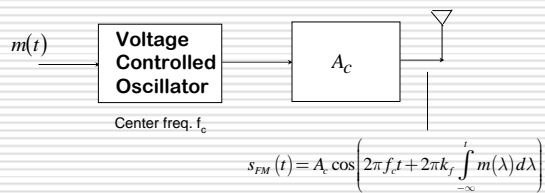
- Due to the similarity to between AM and Narrowband FM we can directly create the modulator:



Generation of Wideband FM Signals

- There are two basic techniques for generating wideband FM waves
 - Direct Method
 - Relies on a voltage-controlled oscillator
 - Indirect Method
 - Relies on the creation of a narrowband FM signal followed by a frequency multiplier

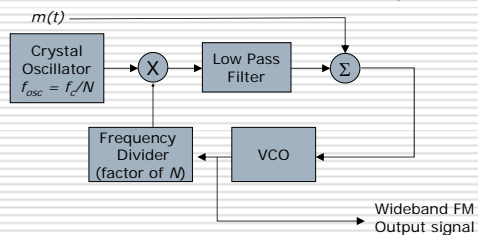
Transmitter for FM – Direct Method



The VCO (voltage controlled oscillator) has a wide frequency range and can thus handle wideband FM. One problem is that the center frequency of the oscillator tends to drift. Thus we require a technique to stabilize the oscillator.

Frequency Stabilization

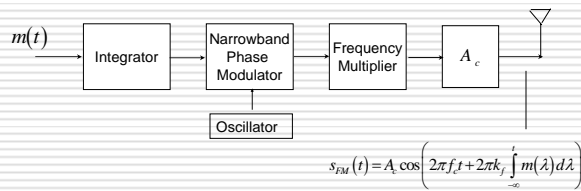
- The VCO can be stabilized by putting it in a frequency lock loop with a crystal oscillator that is very stable
- Frequency divider is needed to create a narrowband FM signal
 - This is needed to create an unmodulated tone for mixing



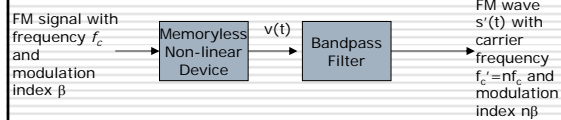
FM Transmitter – Indirect Method

- Another means of getting around the VCO drift problem is to create a narrowband FM signal and increase the frequency modulation using a frequency multiplier
- Narrowband FM signal is created in a manner similar to an AM signal which allows for a stable oscillator to be used
- Frequency multiplier consists of a non-linear device followed by a bandpass filter

Transmitter for FM – Indirect Method



Frequency Multiplier



Memoryless non-linear device creates an output signal

$$v(t) = a s(t) + a_2 s^2(t) + a_3 s^3(t) + \dots$$

Frequency Multiplier – cont.

- Consider the non-linear device

$$v(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t) \dots + a_n s^n(t)$$

when the input signal is $\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$

- Let us consider the terms individually
- First consider the second order term

$$\begin{aligned} a_2 s^2(t) &= a_2 \cos^2\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) \\ &= \frac{a_2}{2} + \frac{a_2}{2} \cos\left(2\pi(2f_c)t + (2\pi 2k_f) \int_0^t m(\tau) d\tau\right) \end{aligned}$$

- Thus, the second order non-linearity produces a term with twice the carrier frequency and twice the frequency modulation

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Frequency Multiplier – cont.

- Now consider the 3rd order term:

$$\begin{aligned} a_3 s^3(t) &= a_3 \cos^3\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) \\ &= a_3 \left[\frac{1}{2} + \frac{1}{2} \cos\left(2\pi(2f_c)t + 2\pi 2k_f \int_0^t m(\tau) d\tau\right) \right] \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) \\ &= a_3 \left[\frac{1}{2} \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) + \frac{1}{2} \cos\left(2\pi(2f_c)t + 2\pi 2k_f \int_0^t m(\tau) d\tau\right) \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) \right] \\ &= \frac{a_3}{2} \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) + \frac{a_3}{4} \left[\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) + \cos\left(2\pi(3f_c)t + 2\pi 3k_f \int_0^t m(\tau) d\tau\right) \right] \end{aligned}$$

- The third order term has components at the original frequency and three times the original frequency.

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Frequency Multiplier – cont.

- In general for a non-linear device with up to an n th order non-linearity the output signal will be

$$\begin{aligned} v(t) &= c_0 + c_1 \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right) + c_2 \cos\left(2\pi(2f_c)t + 2\pi 2k_f \int_0^t m(\tau) d\tau\right) + \dots \\ &+ c_3 \cos\left(2\pi(3f_c)t + 2\pi 3k_f \int_0^t m(\tau) d\tau\right) \dots + c_n \cos\left(2\pi(nf_c)t + 2\pi nk_f \int_0^t m(\tau) d\tau\right) \end{aligned}$$

- We can obtain the desired frequency term using a bandpass filter provided that $f_c \gg 2W + 2\Delta f$

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Demodulators for FM Signals

- A demodulator is the inverse of a modulator
- The demodulator recovers the message signal based on the received modulated carrier
- The ideal frequency demodulator creates the output

$$x(t) = K \frac{d}{dt} \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$
$$= K 2\pi f_c + K 2\pi k_f m(t)$$

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Practical Demodulators

- There are three practical demodulator circuits
 - Discriminator (differentiator)
 - Attempts to create the derivative through the frequency domain
 - PLL
 - Widely used in practical receivers
 - Zero crossings detector
 - Detects the zero crossings as discussed in a previous lecture

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Frequency Discriminator

- Recall the frequency modulated transmit signal $s(t)$ is

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

- Further, the derivative of $s(t)$ is

$$\frac{d}{dt} \{s(t)\} = \frac{d}{dt} \left\{ A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right] \right\}$$
$$= -(2\pi f_c + 2\pi k_f m(t)) A_c \sin \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

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Frequency Discriminator – cont.

- The signal

$$v(t) = -(2\pi f_c + 2\pi k_f m(t)) A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$$

is simply an amplitude modulated signal, where the amplitude modulation is dependent on the message signal

- Provided that $f_c \gg 2\pi k_f m(t)$ an envelope detector can be used to determine the message.
- However, we have yet to show how we can obtain the derivative of the original signal
- This can be accomplished in the frequency domain

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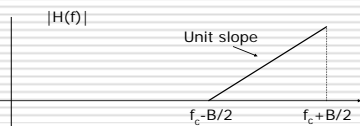
Taking the derivative

- The derivative can be taken in the frequency domain by recalling the relationship between time and frequency:

$$\frac{d}{dt} \Rightarrow j2\pi f$$

- This can be accomplished by creating a filter with the transfer function:

$$H(f) = \begin{cases} j2\pi[f - (f_c - B/2)] & (f_c - B/2) \leq |f| \leq (f_c + B/2) \\ 0 & \text{else} \end{cases}$$

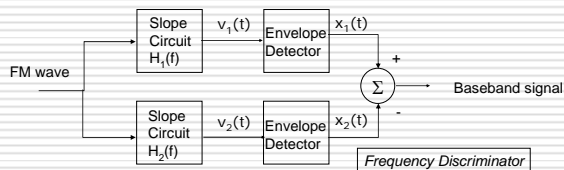


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Balanced Frequency Discriminator

- A practical implementation of the frequency discriminator is shown below:



- The second filter and envelope detector allow for the removal of the bias due to the carrier

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Balanced Discriminator – cont.

- The two filters are

$$H_1(f) = \begin{cases} j2\pi[f - (f_c - B/2)] & (f_c - B/2) \leq |f| \leq (f_c + B/2) \\ 0 & \text{else} \end{cases}$$

$$H_2(f) = \begin{cases} -j2\pi[f - (f_c + B/2)] & (f_c - B/2) \leq |f| \leq (f_c + B/2) \\ 0 & \text{else} \end{cases}$$

- At the output of the two filters we have

$$V_1(f) = S(f)H_1(f)$$

$$V_2(f) = S(f)H_2(f)$$

$$v_1(t) = -(2\pi f_c + 2\pi k_f m(t)) A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$$

$$v_2(t) = -(2\pi f_c - 2\pi k_f m(t)) A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right)$$

Balanced Discriminator – cont.

- At the output of the two envelope detectors

$$x_1(t) = 2\pi f_c + 2\pi k_f m(t)$$

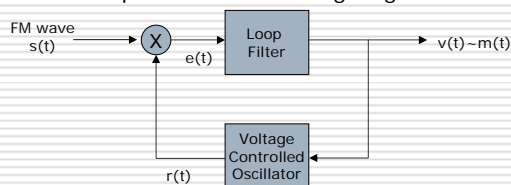
$$x_2(t) = 2\pi f_c - 2\pi k_f m(t)$$

- Finally, at the output of the detector we have

$$\begin{aligned} \hat{m}(t) &= x_1(t) - x_2(t) \\ &= 2\pi f_c + 2\pi k_f m(t) - 2\pi f_c + 2\pi k_f m(t) \\ &= C m(t) \end{aligned}$$

Phase-Locked Loop

- Feedback loop causes VCO to track the phase of the incoming signal
- The output of the LPF tracks the derivative of the phase or the message signal



Phase-Locked Loop

- VCO – Voltage Controlled Oscillator is a circuit which outputs a periodic signal with a frequency that is varied about some free-running frequency f_c according to the input voltage signal (if input voltage is zero, the output frequency is f_c)
- Multiplier – Acts as a Phase Detector - i.e., a circuit which produces an output signal that is proportional to the phase difference between the two input signals
- Low Pass Filter – removes high frequency components and produces the control signal for the VCO

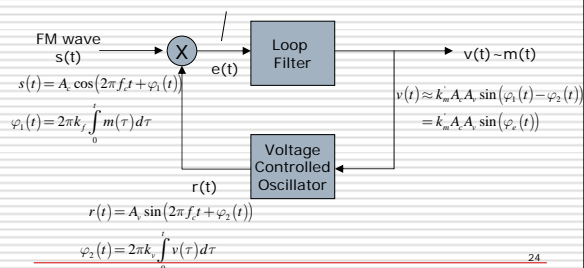
Examining the PLL

- The PLL is a closed-loop feedback system that produces a signal $r(t)$ which is in phase lock (i.e., has the same time-varying phase) as the input signal.
- This circuit can be used for creating a phase-coherent reference signal for demodulation or can be used to track the *frequency* of the incoming signal since the VCO input will correspond to the time-varying frequency
- For our analysis let us assume
 - The VCO is calibrated such that when the input voltage $v(t)$ is zero the output is a sinusoid at frequency f_c and out of phase by 90° from the incoming carrier
 - $s(t) = A_c \cos(2\pi f_c t + \varphi_1(t))$ where $\varphi_1(t) = 2\pi k_f \int_0^t m(\tau) d\tau$
 - $r(t) = A_c \sin(2\pi f_c t + \varphi_2(t))$

Examining the PLL – cont.

$$k_w s(t)r(t) = k_w A_c \sin(2\pi f_c t + \varphi_1(t)) A_c \cos(2\pi f_c t + \varphi_2(t))$$

$$= k_w A_c A_c \{ \sin(4\pi f_c t + \varphi_1(t) + \varphi_2(t)) + \sin(\varphi_1(t) - \varphi_2(t)) \}$$



Phase-Locked Loop (cont.)

Examining $v_1(t)$:

$$v_1(t) = K_m A_c A_s \sin(\omega_c t + \theta_c(t)) \cos(\omega_c t + \theta_s(t))$$

$$= \frac{K_m A_c A_s}{2} \sin(\theta_c(t) - \theta_s(t)) + \frac{K_m A_c A_s}{2} \sin(2\omega_c t + \theta_c(t) + \theta_s(t))$$

Now, the double frequency term is eliminated by the low pass filter leaving:

$$v_2(t) = \{K_d \sin(\theta_e(t))\} * h(t)$$

$$K_d = \frac{K_m A_c A_s}{2}$$

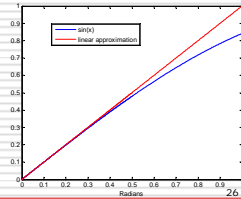
$$\theta_e(t) = \theta_c(t) - \theta_s(t)$$

Equivalent Linearized Model

When the phase-locked loop is *nearly* in lock the phase error $\phi_e(t)$ will be small. In this case we can say that

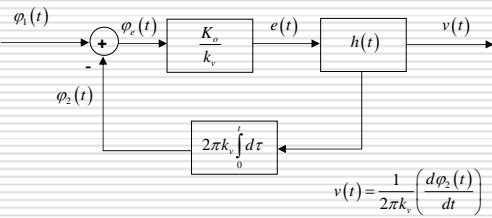
$$k_m A_c A_s \sin(\phi_e(t)) \approx k_m A_c A_s \phi_e(t)$$

Using this approximation, we can create a linearized model of the phase of the signals at each point in the loop



Equivalent model

Equivalent Model



PLL Output

- It can be shown that provided that K_o is large, the $\phi_2(t)$ will follow $\phi_1(t)$ and thus the output $v(t)$ is

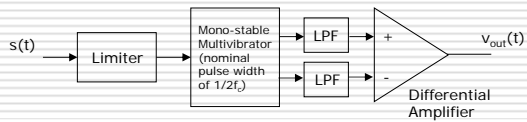
$$\begin{aligned}v(t) &= \frac{1}{2\pi k_v} \left(\frac{d\phi_1(t)}{dt} \right) \\ &= \frac{1}{2\pi k_v} \frac{d}{dt} \left(2\pi k_f \int_0^t m(\tau) d\tau \right) \\ &= \frac{k_f}{k_v} m(t)\end{aligned}$$

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Zero-crossing Detector

- The output of the ideal FM detector is directly proportional to the instantaneous frequency of the input signal
- The instantaneous frequency may be obtained by counting zero crossings of the input waveform



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Summary

- In this lecture we have examined two basic techniques for generating FM waves
 - Direct method using a VCO
 - Indirect method using product modulator and a frequency multiplier
- We have also briefly examined three techniques for demodulating FM waves
 - Frequency discriminator
 - Phase-Locked Loop
 - Zero-crossing detector
- Next class we will look at some practical examples of FM including broadcast radio and TV

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