ECE3614
Introduction to Communications Systems
Fall 2007

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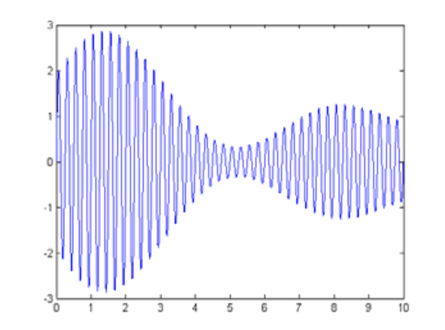
Lecture #16: Generation and Demodulation of FM Signals

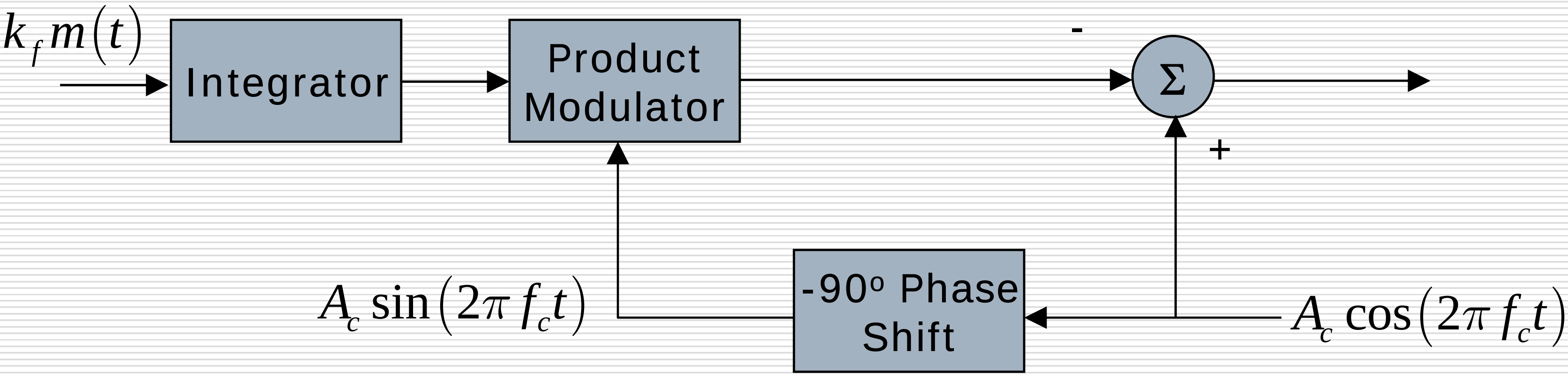


Overview

- Today we continue our discussion of Frequency Modulation (FM) be examining the techniques for generating and demodulating FM signals
- First we will examine transmission techniques
  - Narrowband FM signal generation
  - Wideband FM signal generation
    - Direct
    - Indirect
- Second we will examine three types of demodulation circuits
  - Frequency Discriminator
  - Phase-Locked Loop
  - Zero-crossings detector
- Reading
  - Sections 4.7-4.8

Narrowband FM

- Due to the similarity to between AM and Narrowband FM we can directly create the modulator:

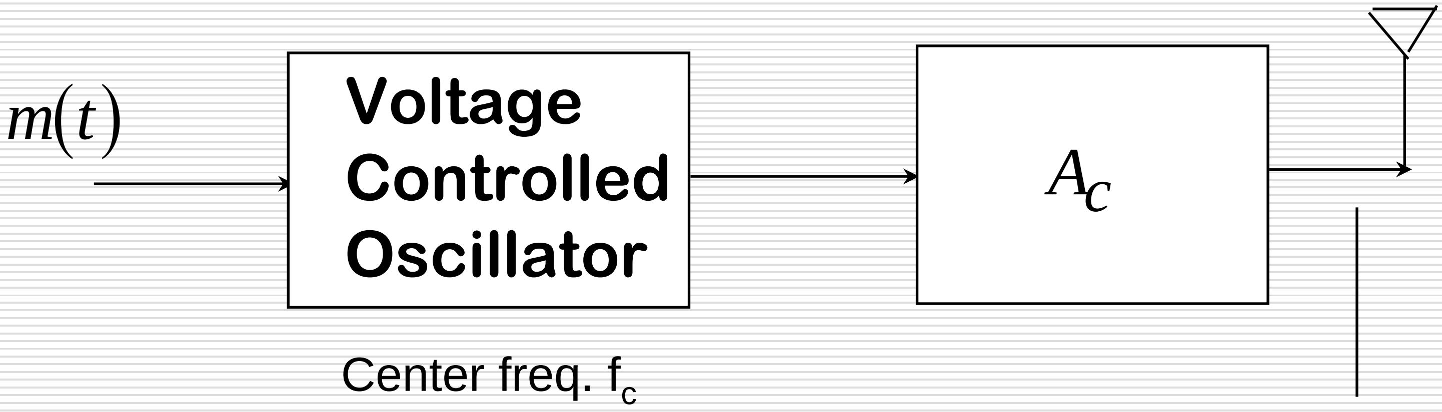


$$s(t) \approx A_c \cos(2\pi f_c t) - \left(2\pi k_f \int_0^t m(\tau)\, d\tau\right) A_c \sin(2\pi f_c t)$$

Generation of Wideband FM Signals

- There are two basic techniques for generating wideband FM waves
  - Direct Method
    - Relies on a voltage-controlled oscillator
  - Indirect Method
    - Relies on the creation of a narrowband FM signal followed by a frequency multiplier

Transmitter for FM – Direct Method

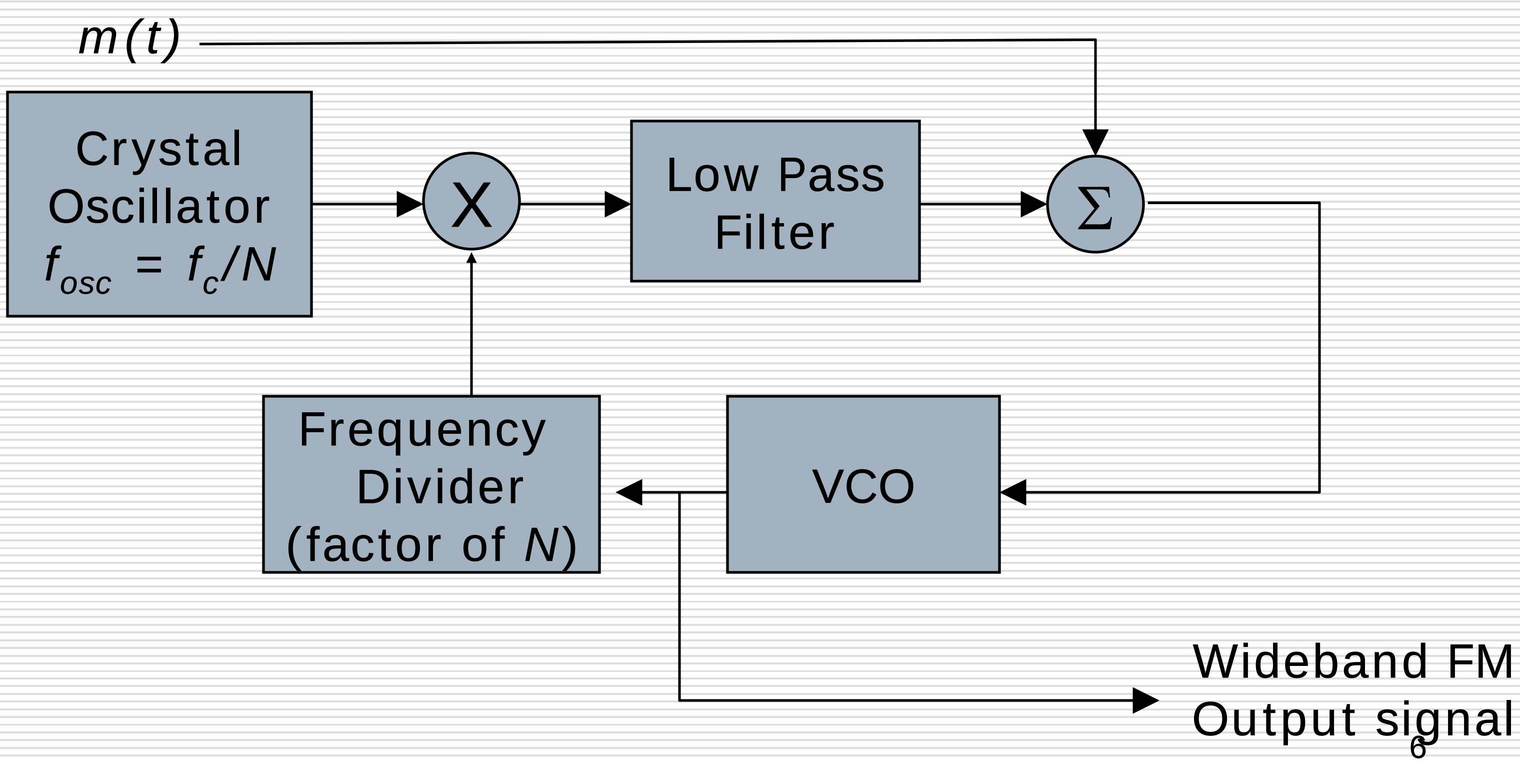


$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\right)$$

The VCO (voltage controlled oscillator) has a wide frequency range and can thus handle wideband FM. One problem is that the center frequency of the oscillator tends to drift. Thus we require a technique to stabilize the oscillator.

Frequency Stabilization

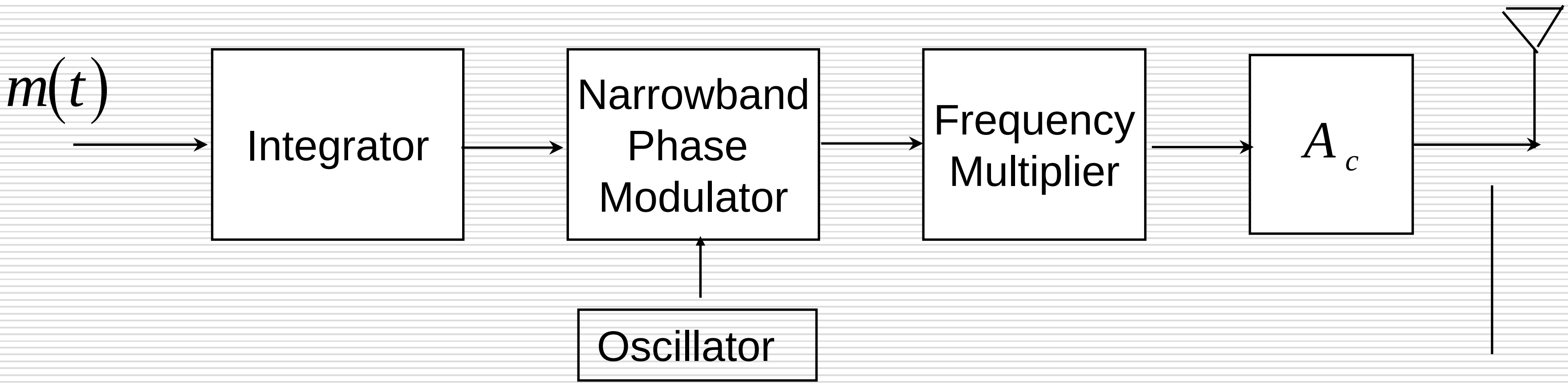
- The VCO can be stabilized by putting it in a frequency lock loop with a crystal oscillator that is very stable
- Frequency divider is needed to create a narrowband FM signal
  - This is needed to create an unmodulated tone for mixing



FM Transmitter – Indirect Method

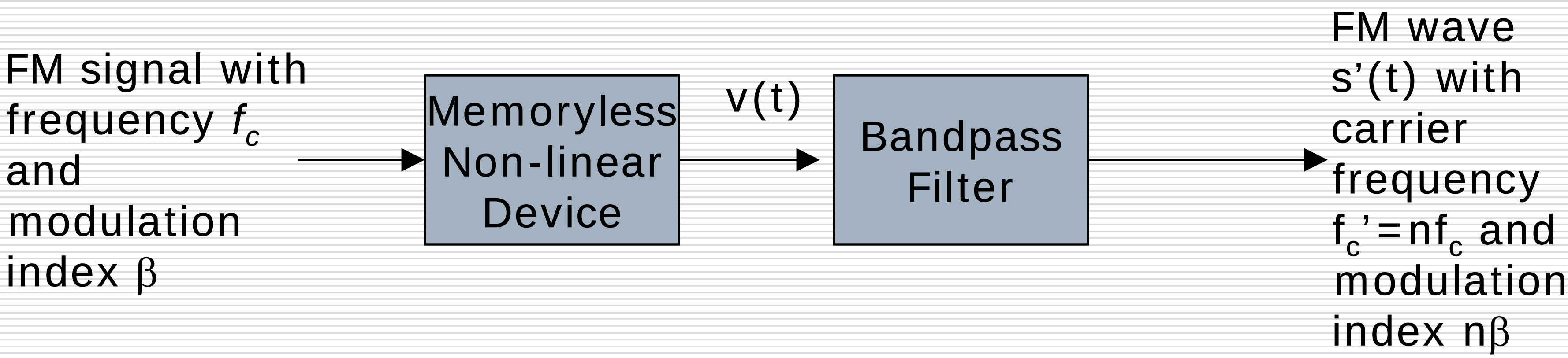
- Another means of getting around the VCO drift problem is to create a narrowband FM signal and increase the frequency modulation using a frequency multiplier
- Narrowband FM signal is created in a manner similar to an AM signal which allows for a stable oscillator to be used
- Frequency multiplier consists of a non-linear device followed by a bandpass filter

Transmitter for FM – Indirect Method



$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\right)$$

Frequency Multiplier



Memoryless non-linear device creates an output signal

$$v(t) = a\, s(t) + a_2 s^2(t) + a_3 s^3(t) + \ldots$$

Frequency Multiplier – cont.

- Consider the non-linear device

$$v(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t) \ldots + a_n s^n(t)$$

when the input signal is $\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$

- Let us consider the terms individually
- First consider the second order term

$$a_2 s^2(t) = a_2 \cos^2\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

$$= \frac{a_2}{2} + \frac{a_2}{2}\cos\left(2\pi(2 f_c)t + (2\pi 2k_f)\int_0^t m(\tau)\,d\tau\right)$$

- Thus, the second order non-linearity produces a term with twice the carrier frequency and twice the frequency modulation

Frequency Multiplier – cont.

- Now consider the 3rd order term:

$$a_3 s^3(t) = a_3 \cos^3\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

$$= a_3\left\{\frac{1}{2} + \frac{1}{2}\cos\left(2\pi(2f_c)t + 2\pi 2k_f \int_0^t m(\tau)\,d\tau\right)\right\}\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

$$= a_3\left\{\frac{1}{2}\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right) + \frac{1}{2}\cos\left(2\pi(2f_c)t + 2\pi 2k_f \int_0^t m(\tau)\,d\tau\right)\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)\right\}$$

$$= \frac{a_3}{2}\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right) + \frac{a_3}{4}\left\{\cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right) + \cos\left(2\pi(3f_c)t + 2\pi 3k_f \int_0^t m(\tau)\,d\tau\right)\right\}$$

- The third order term has components at the original frequency and three times the original frequency.

Frequency Multiplier – cont.

- In general for a non-linear device with up to an *n*th order non-linearity the output signal will be

$$v(t) = c_0 + c_1 \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right) + c_2 \cos\left(2\pi(2f_c)t + 2\pi 2k_f \int_0^t m(\tau)\,d\tau\right) + \dots$$

$$+ c_3 \cos\left(2\pi(3f_c)t + 2\pi 3k_f \int_0^t m(\tau)\,d\tau\right) \dots + c_n \cos\left(2\pi(nf_c)t + 2\pi nk_f \int_0^t m(\tau)\,d\tau\right)$$

- We can obtain the desired frequency term using a bandpass filter provided that $f_c >> 2W + 2\Delta f$

Demodulators for FM Signals

- A demodulator is the inverse of a modulator
- The demodulator recovers the message signal based on the received modulated carrier
- The ideal frequency demodulator creates the output

$$x(t) = K\frac{d}{dt}\left\{2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right\}$$

$$= K2\pi f_c + K2\pi k_f m(t)$$

Practical Demodulators

- There are three practical demodulator circuits
  - Discriminator (differentiator)
    - Attempts to create the derivative through the frequency domain
  - PLL
    - Widely used in practical receivers
  - Zero crossings detector
    - Detects the zero crossings as discussed in a previous lecture

Frequency Discriminator

- Recall the frequency modulated transmit signal *s(t)* is

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

- Further, the derivative of s(t) is

$$\frac{d}{dt}\left\{s(t)\right\} = \frac{d}{dt}\left\{A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)\right\}$$

$$= -\left(2\pi f_c + 2\pi k_f m(t)\right) A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

Frequency Discriminator – cont.

- The signal

$$v(t) = -\left(2\pi f_c + 2\pi k_f m(t)\right) A_c \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right)$$

is simply an amplitude modulated signal, where the amplitude modulation is dependent on the message signal
- Provided that $f_c >> 2\pi k_f m(t)$ an envelope detector can be used to determine the message.
- However, we have yet to show how we can obtain the derivative of the original signal
- This can be accomplished in the frequency domain

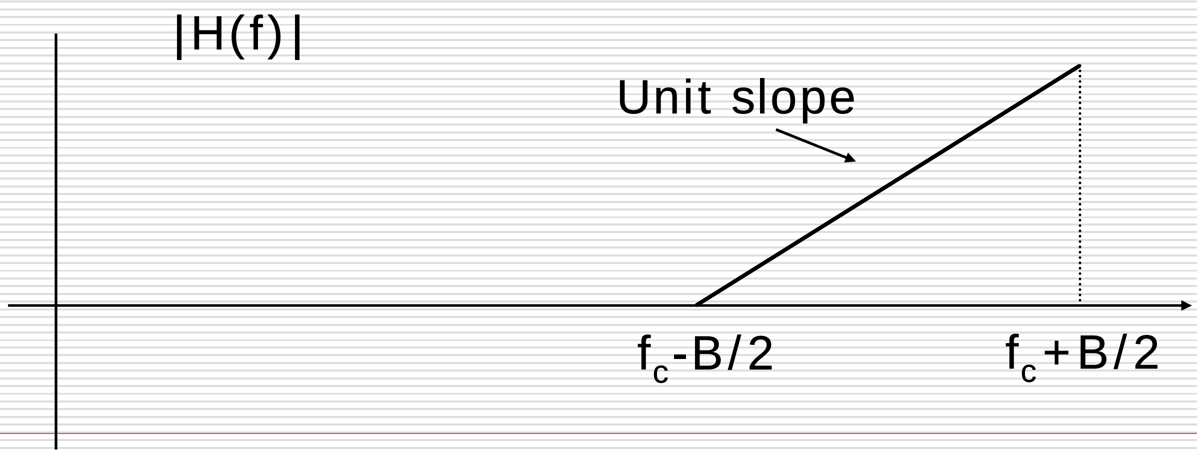
Taking the derivative

- The derivative can be taken in the frequency domain by recalling the relationship between time and frequency:

$$\frac{d}{dt} \rightleftarrows j2\pi f$$

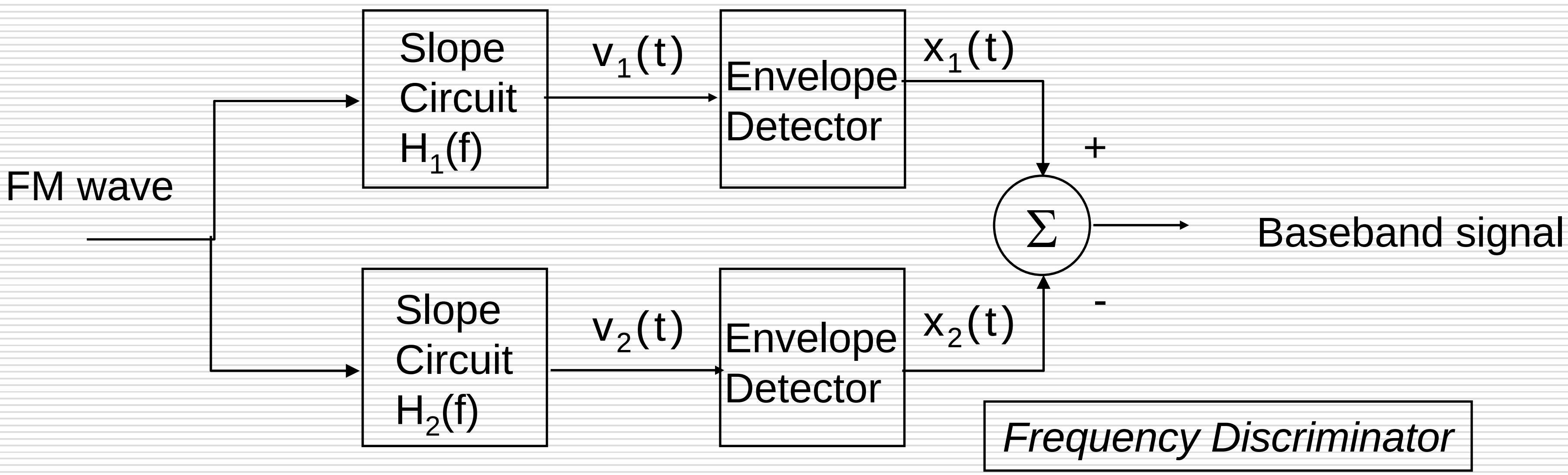
- This can be accomplished by creating a filter with the transfer function:

$$H(f) = \begin{cases} j2\pi\left[f - \left(f_c - B/2\right)\right] & \left(f_c - B/2\right) \le |f| \le \left(f_c + B/2\right) \\ 0 & else \end{cases}$$



Balanced Frequency Discriminator

- A practical implementation of the frequency discriminator is shown below:



- The second filter and envelope detector allow for the removal of the bias due to the carrier

Balanced Discriminator – cont.

- The two filters are

$$H_1(f)=\begin{cases} j2\pi\left[f-\left(f_c-B/2\right)\right] & \left(f_c-B/2\right)\le |f| \le \left(f_c+B/2\right) \\ 0 & else \end{cases}$$

$$H_2(f)=\begin{cases} -j2\pi\left[f-\left(f_c+B/2\right)\right] & \left(f_c-B/2\right)\le |f| \le \left(f_c+B/2\right) \\ 0 & else \end{cases}$$

- At the output of the two filters we have

$$V_1(f)=S(f)H_1(f)$$

$$V_2(f)=S(f)H_2(f)$$

$$v_1(t)=-\left(2\pi f_c+2\pi k_f m(t)\right)A_c \sin\left(2\pi f_c t+2\pi k_f\int_0^t m(\tau)d\tau\right)$$

$$v_2(t)=-\left(2\pi f_c-2\pi k_f m(t)\right)A_c \sin\left(2\pi f_c t+2\pi k_f\int_0^t m(\tau)d\tau\right)$$

Balanced Discriminator – cont.

- At the output of the two envelope detectors

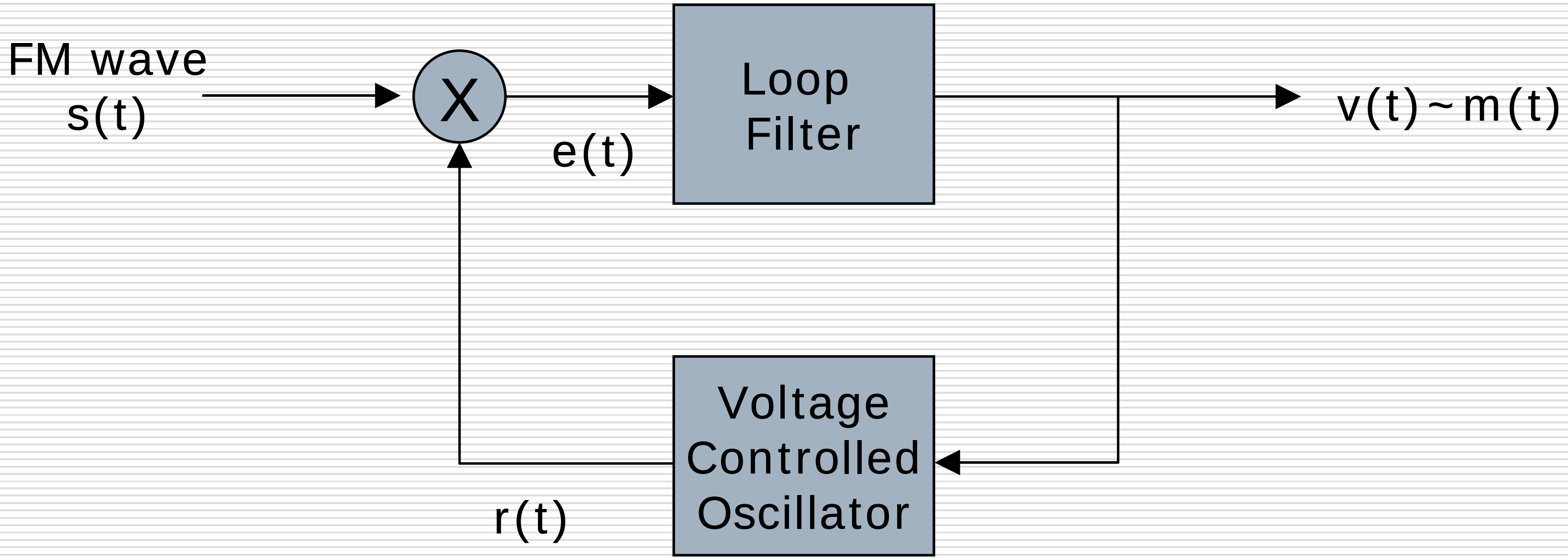
$$x_1(t) = 2\pi f_c + 2\pi k_f m(t)$$
$$x_2(t) = 2\pi f_c - 2\pi k_f m(t)$$

- Finally, at the output of the detector we have

$$\begin{aligned}\widehat{m}(t) &= x_1(t) - x_2(t) \\ &= 2\pi f_c + 2\pi k_f m(t) - 2\pi f_c + 2\pi k_f m(t) \\ &= C\, m(t)\end{aligned}$$

Phase-Locked Loop

- Feedback loop causes VCO to track the phase of the incoming signal
- The output of the LPF tracks the derivative of the phase or the message signal



Phase-Locked Loop

- VCO – Voltage Controlled Oscillator is a circuit which outputs a periodic signal with a frequency that is varied about some free-running frequency f_o according to the input voltage signal (if input voltage is zero, the output frequency is f_o)
- Multiplier – Acts as a Phase Detector - i.e., a circuit which produces an output signal that is proportional to the phase difference between the two input signals
- Low Pass Filter – removes high frequency components and produces the control signal for the VCO

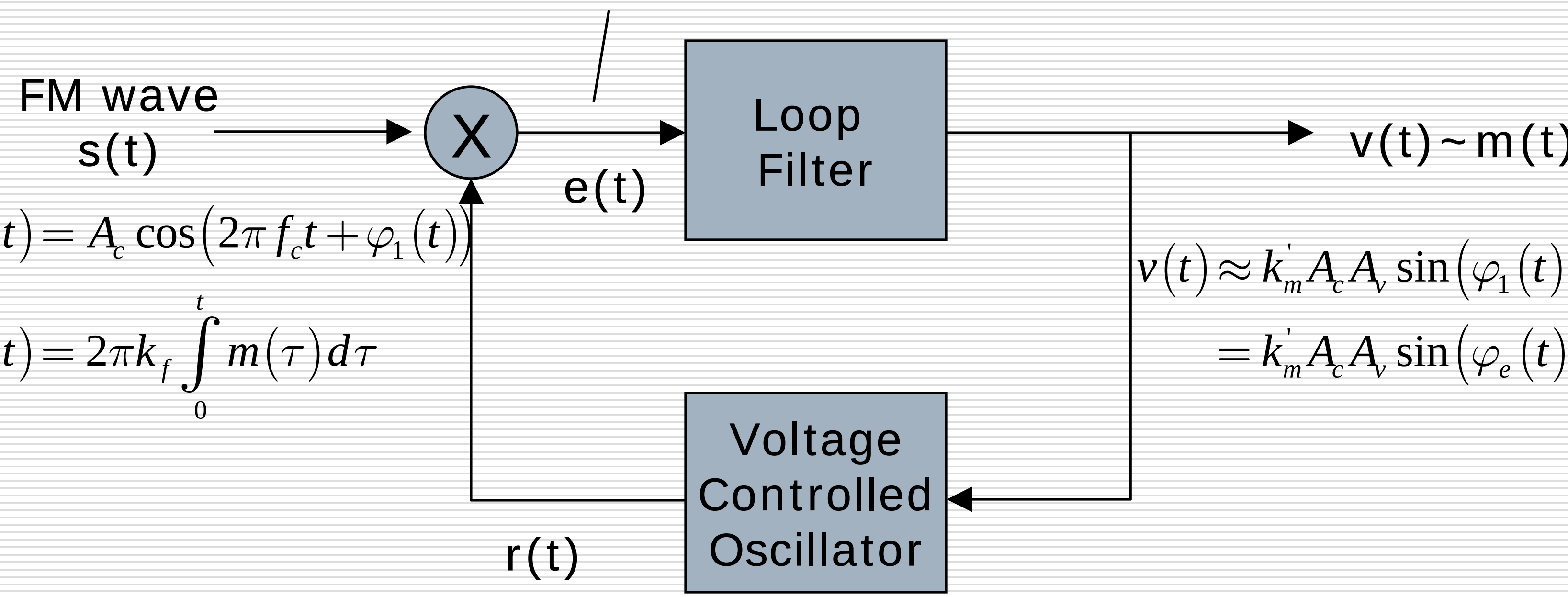
Examining the PLL

- The PLL is a closed-loop feedback system that produces a signal *r(t)* which is in phase lock (i.e., has the same time-varying phase) as the input signal.
- This circuit can be used for creating a phase-coherent reference signal for demodulation or can be used to track the *frequency* of the incoming signal since the VCO input will correspond to the time-varying frequency
- For our analysis let us assume
  - The VCO is calibrated such that when the input voltage *v(t)* is zero the output is a sinusoid at frequency f_c and out of phase by 90^o from the incoming carrier
  - $$s(t) = A_c \cos\left(2\pi f_c t + \varphi_1(t)\right) \quad \text{where} \quad \varphi_1(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau$$
    $$r(t) = A_v \sin\left(2\pi f_c t + \varphi_2(t)\right)$$

Examining the PLL – cont.

$$k_m s(t) r(t) = k_m A_c \sin\left(2\pi f_c t + \varphi_1(t)\right) A_v \cos\left(2\pi f_c t + \varphi_2(t)\right)$$

$$= k_m^{'} A_c A_v \left\{ \sin\left(4\pi f_c t + \varphi_1(t) + \varphi_2(t)\right) + \sin\left(\varphi_1(t) - \varphi_2(t)\right) \right\}$$



$$s(t) = A_c \cos\left(2\pi f_c t + \varphi_1(t)\right)$$

$$\varphi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$v(t) \approx k_m^{'} A_c A_v \sin\left(\varphi_1(t) - \varphi_2(t)\right)$$

$$= k_m^{'} A_c A_v \sin\left(\varphi_e(t)\right)$$

$$r(t) = A_v \sin\left(2\pi f_c t + \varphi_2(t)\right)$$

$$\varphi_2(t) = 2\pi k_v \int_0^t v(\tau)\, d\tau$$

Phase-Locked Loop (cont.)

- Examining $v_1(t)$:

$$v_1(t) = K_m A_i A_o \sin\left(\omega_c t + \theta_i(t)\right)\cos\left(\omega_c t + \theta_o(t)\right)$$
$$= \frac{K_m A_i A_o}{2}\sin\left(\theta_i(t) - \theta_o(t)\right) + \frac{K_m A_i A_o}{2}\sin\left(2\omega_c t + \theta_i(t) + \theta_o(t)\right)$$

- Now, the double frequency term is eliminated by the low pass filter leaving:

$$v_2(t) = \left\{K_d \sin\left(\theta_e(t)\right)\right\} * h(t)$$

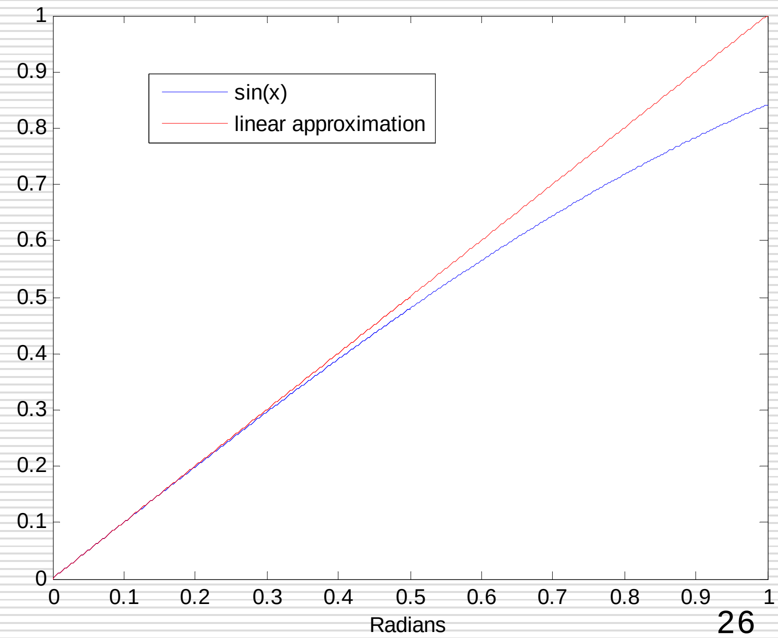
$$K_d = \frac{K_m A_i A_o}{2}$$
$$\theta_e(t) = \theta_i(t) - \theta_e(t)$$

Equivalent Linearized Model

- When the phase-locked loop is *nearly* in lock the phase error $\phi_e(t)$ will be small. In this case we can say that

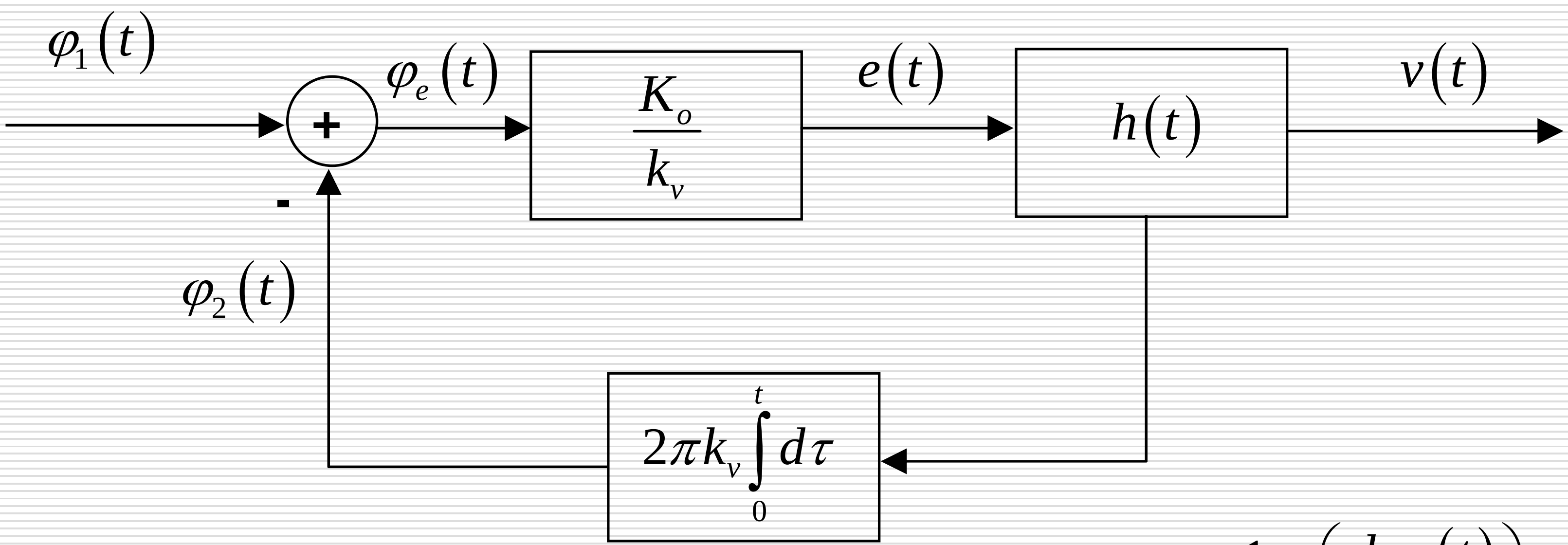
$$k_m^{'} A_c A_v \sin\left(\varphi_e\left(t\right)\right) \approx k_m^{'} A_c A_v \varphi_e\left(t\right)$$

- Using this approximation, we can create a linearized model of the phase of the signals at each point in the loop



Equivalent model

Equivalent Model



$$v(t) = \frac{1}{2\pi k_v}\left(\frac{d\varphi_2(t)}{dt}\right)$$

PLL Output

- It can be shown that provided that K_o is large, the $\phi_2(t)$ will follow $\phi_1(t)$ and thus the output v(t) is

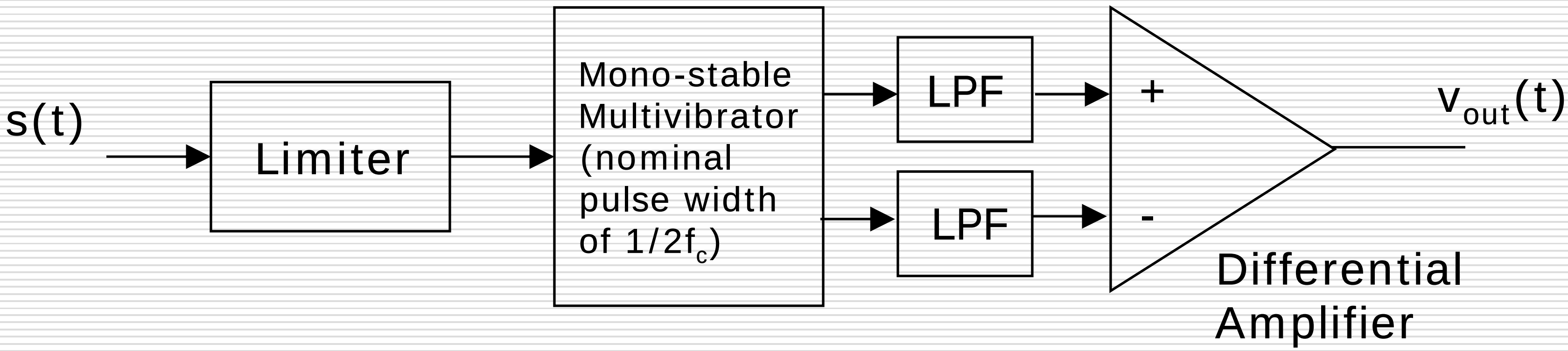
$$v(t) = \frac{1}{2\pi k_v}\left(\frac{d\varphi_1(t)}{dt}\right)$$

$$= \frac{1}{2\pi k_v}\frac{d}{dt}\left(2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

$$= \frac{k_f}{k_v} m(t)$$

Zero-crossing Detector

- The output of the ideal FM detector is directly proportional to the instantaneous frequency of the input signal
- The instantaneous frequency may obtained by counting zero crossings of the input waveform



Summary

- In this lecture we have examined two basic techniques for generating FM waves
  - Direct method using a VCO
  - Indirect method using product modulator and a frequency multiplier
- We have also briefly examined three techniques for demodulating FM waves
  - Frequency discriminator
  - Phase-Locked Loop
  - Zero-crossing detector
- Next class we will look at some practical examples of FM including broadcast radio and TV