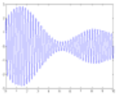
ECE3614
Introduction to Communications Systems
Fall 2007

Instructor: Dr. R. Michael Buehrer

Lecture #17: Frequency Modulation – System Examples and Example Problems

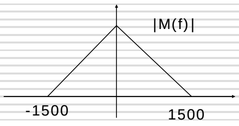


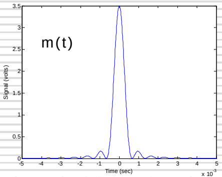
Overview

- ☐ The Objective of today's lecture is to wrap up our discussion of the characteristics of FM signals by discussing a few examples
- ☐ In particular we will examine broadcast FM
- ☐ Reading
  - ■ 4.9

Example 17.1

- ☐ A message signal has the following characteristics





- ☐ If an FM system has a bandwidth of 10kHz, determine the value of k_f that allows for full bandwidth utilization.

Example 17.1 – cont.

- The bandwidth is related to k_f through

$$BW = 2\Delta f + 2W$$

- From the previous slide we know that BW = 10kHz and W = 1500Hz
- Thus:

$$\begin{aligned} 2\Delta f &= BW - 2W \\ &= 10kHz - 3kHz = 7kHz \\ \Delta f &= 3.5kHz \end{aligned}$$

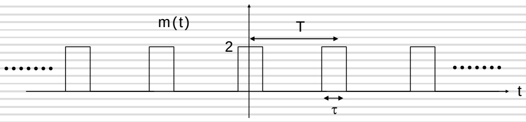
Example 17.1 – cont.

- Further, since $\Delta f = k_f V_p$ and the peak voltage is found to be 3.5V

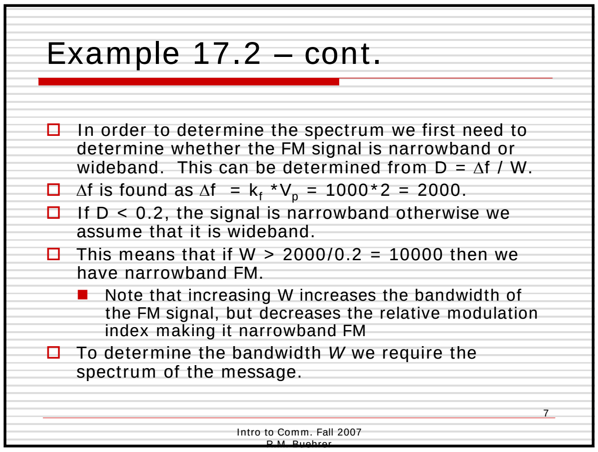
$$\begin{aligned} k_f &= \frac{\Delta f}{V_p} \\ &= \frac{3.5kHz}{3.5} \\ &= 1000 \end{aligned}$$

Example 17.2

- Consider the following message signal:

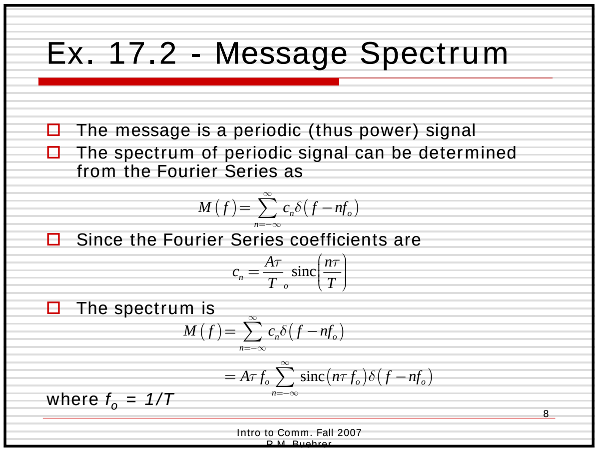


- If the message signal above is frequency modulated with k_f = 1000 and f_c = 100kHz, plot an approximate spectrum. The signal period T = *1ms* and τ = 50μs.



Example 17.2 – cont.

- ☐ In order to determine the spectrum we first need to determine whether the FM signal is narrowband or wideband. This can be determined from D = Δf / W.
- ☐ Δf is found as Δf = $k_f * V_p$ = 1000*2 = 2000.
- ☐ If D < 0.2, the signal is narrowband otherwise we assume that it is wideband.
- ☐ This means that if W > 2000/0.2 = 10000 then we have narrowband FM.
  - ■ Note that increasing W increases the bandwidth of the FM signal, but decreases the relative modulation index making it narrowband FM
- ☐ To determine the bandwidth *W* we require the spectrum of the message.



Ex. 17.2 - Message Spectrum

- ☐ The message is a periodic (thus power) signal
- ☐ The spectrum of periodic signal can be determined from the Fourier Series as

$$M(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_o)$$

- ☐ Since the Fourier Series coefficients are

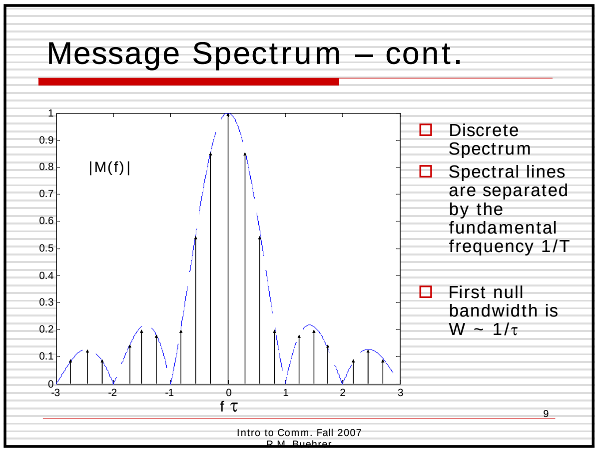
$$c_n = \frac{A\tau}{T}{}_{o}\,\text{sinc}\left(\frac{n\tau}{T}\right)$$

- ☐ The spectrum is

$$M(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_o)$$

$$= A\tau f_o \sum_{n=-\infty}^{\infty} \text{sinc}(n\tau f_o)\delta(f - nf_o)$$

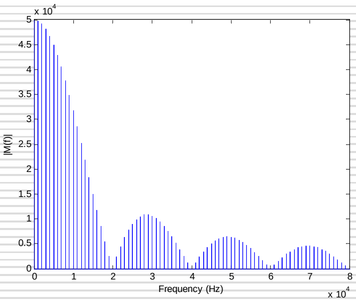
where f_o = *1/T*



Message Spectrum – cont.

- ☐ Discrete Spectrum
- ☐ Spectral lines are separated by the fundamental frequency 1/T
- ☐ First null bandwidth is W ~ 1/τ

Message Spectrum – cont.



- Spectral lines at 1/T = 1kHz
- First null at f = T/τ * 1/T

Example 17.2 – cont.

- With T = 1ms and τ = 50μs, we find that the “first-null” bandwidth is approximated by
  - W ~ f = T/τ∗1/T = 1/τ = 20kHz
- For narrowband FM we require
  W > 10000

Thus, we have a narrowband FM signal.

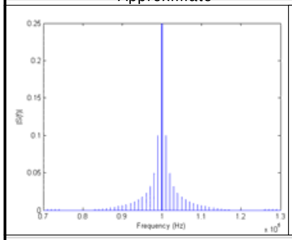
(Note that D = Δf/W = 2000/2000 = 0.1)

Ex. 17.2 - Narrowband Spectrum

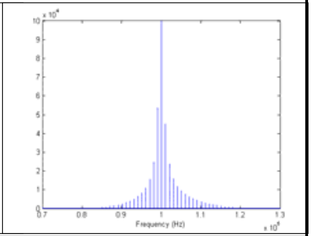
- The narrowband spectrum can be approximated using the AM approximation:

$$
\begin{aligned}
S_{NBFM}(f) &= \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] \\
&\quad + \frac{A_c k_f}{2}\left[\frac{M(f-f_c)}{(f-f_c)}+\frac{M(f+f_c)}{(f+f_c)}\right] \\
&= \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]+\frac{A_c A k_f \tau f_o}{2(f-f_c)}\sum_{n=-\infty}^{\infty}\operatorname{sinc}(n\tau f_o)\delta(f-nf_o-f_c)\ldots \\
&\quad + \frac{A_c A k_f \tau f_o}{2(f+f_c)}\sum_{n=-\infty}^{\infty}\operatorname{sinc}(n\tau f_o)\delta(f-nf_o+f_c)
\end{aligned}
$$

Approximated vs. Actual Spectrum



Approximate



Actual (using Fourier Transform of signal)

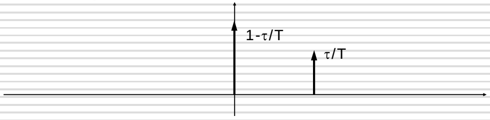
Ex. 17.2 - Wideband Spectrum

- Let's change the message such that T = 100ms and τ = 50ms. In this case
  - W ~ f = T/τ*1/T = 1/τ = 200Hz
  - Since W << 10000, we have wideband FM
  - Specifically, D = Δf / W = 2000/200 = 10
  - We can use the wideband approximation

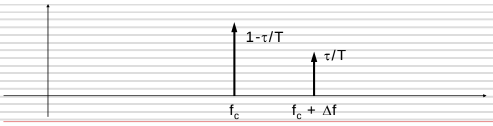
$$S_{WBFM}(f) = \frac{A_c^2}{4k_f}\left[p_m\left(\frac{1}{k_f}(f-f_c)\right) + p_m\left(\frac{1}{k_f}(-f-f_c)\right)\right]$$

Wideband Spectrum – cont.

- The pdf of the message signal is

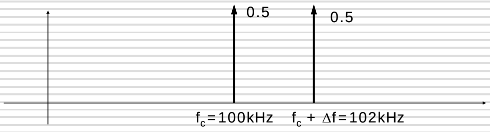


- Thus, the approximate spectrum is



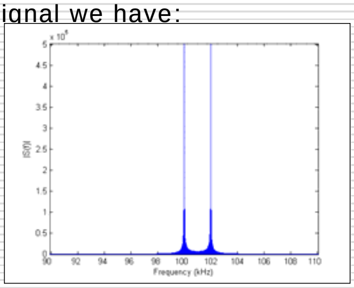
Ex. 17.2 - Approximate Spectrum

- τ/T = 50ms/100ms = 0.5
- 1 – τ/T = 0.5



Ex. 17.2 - Actual Spectrum

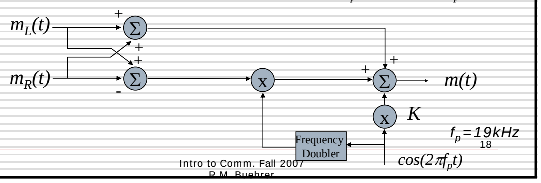
- Taking the Fourier Transform of the FM signal we have:



Broadcast FM

- Frequency range:
  - 88.1MHz - 91.9 MHz Non-commercial stations
  - 92.1MHz - 107.9MHz Commercial stations
  - Bandwidth - 200kHz
  - Message signal is Frequency Division Mulitplexed for stereo sound. Composite message signal contains right and left channels and pilot for coherent demodulation of stereo info.

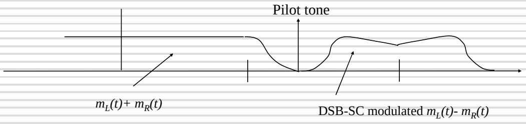
$$m(t) = [m_L(t) + m_R(t)] + [m_L(t) - m_R(t)]\cos(4\pi f_p t) + K\cos(2\pi f_p t)$$



f_p=19kHz

Broadcast FM

- Spectrum of composite message signal
- Monophonic receivers only detect the bottom portion of the signal using standard FM receiver. Stereo receivers demodulate both portions and recreate left and right channels.



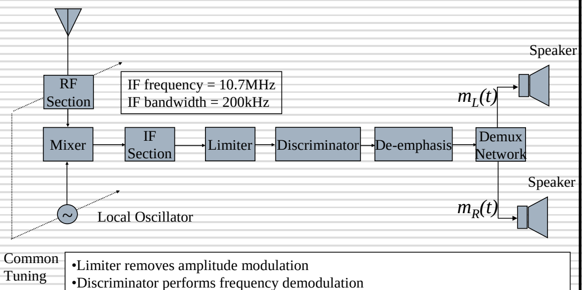
$$m(t)=[m_L(t)+m_R(t)]+[m_L(t)-m_R(t)]\cos(4\pi f_p t)+K\cos(2\pi f_p t)$$

f_p=19kHz

Broadcast FM Specifications

- ☐ The entire composite message signal is applied to an FM transmitter with
  - Δf = 75kHz
    - ☐ k_f = 75000 if V_p = 1
  - *W* = 15kHz (music)
  - f_c = 88.1 – 107.9 in 200kHz increments
    - ☐ B ~200kHz
- ☐ According to Carson’s Rule
  - B = 2Δf + 2W = 180kHz
- ☐ Modulation Index: D = Δf / W = 5
  - Wideband FM

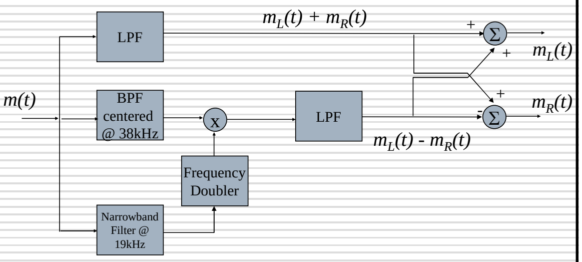
Broadcast FM - Superheterodyne Receiver



IF frequency = 10.7MHz
IF bandwidth = 200kHz

- •Limiter removes amplitude modulation
- •Discriminator performs frequency demodulation
- •De-emphasis removes high band gain and improves SNR
- •Demux network recovers right and left channels for stereo systems
- •(Monphonic receivers do not have the this part. They use the $m_L(t)$ + $m_R(t)$ signal)

FM Broadcast - Demux Network


$m(t)$

LPF

BPF centered @ 38kHz

X

Frequency Doubler

Narrowband Filter @ 19kHz

LPF

$m_L(t) + m_R(t)$

$m_L(t) - m_R(t)$

Σ

Σ

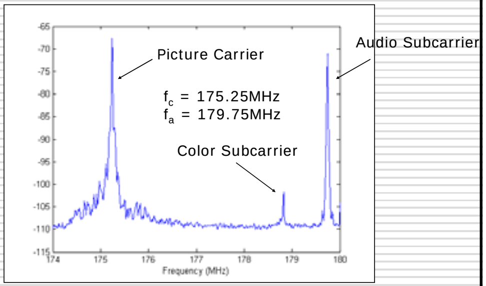
$m_L(t)$

$m_R(t)$

Broadcast TV - Audio Signal

- FM signal with carrier located 4.5MHz above the picture carrier
- Δf = 25kHz
- W = 15kHz
- B ~ $2\Delta f + 2W$ = 80kHz
- D = Δf / B =1.67
  - wideband but not real wide

Example – Analog TV (Channel 7)


Picture Carrier

Audio Subcarrier

f_c = 175.25MHz
f_a = 179.75MHz

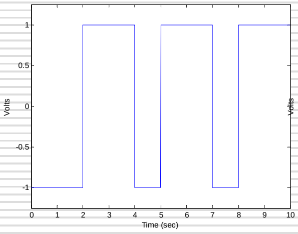
Color Subcarrier

Digital Communications

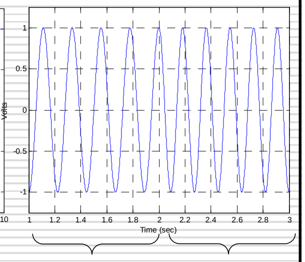
- While FM is an analog communications technique, the basic idea can also be used with digital modulation
- If we modulate the signal with a digital waveform (+1/-1 modulated square pulses representing data bits 1/0) and use $\Delta f = R_b/2$ where R_b is the bit rate we have a digital modulation scheme known as Binary Frequency Shift Keying (BFSK)

Example

$R_b = R_s = 1\text{bps}$



$f_c = 5\text{Hz} \rightarrow f_1 = 4.5\text{Hz}, f_2 = 5.5\text{Hz}$



$f_1 = 4.5\text{Hz}$, $f_2 = 5.5\text{Hz}$

Summary

- In this lecture we have presented a few examples of FM modulation
- We have now discussed AM and FM modulation schemes fairly thoroughly, particularly in terms of their spectral properties, transmitter design and receiver design.
- In the coming classes we will turn to performance
  - FM generally outperforms AM (i.e., achieves a higher SNR at the output of the system) in exchange for higher bandwidth