ECE3614
Introduction to Communications Systems
Fall 2007

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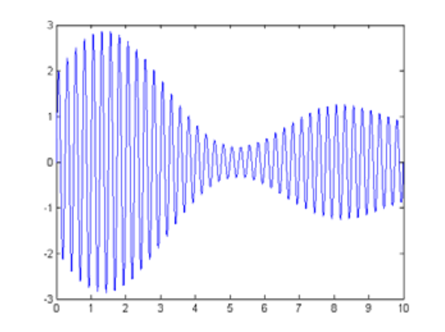
Lecture #18: Noise in Analog Communications



Overview

- Today we examine the main degradation in communication systems – thermal noise
  - The term "thermal" is typically used to represent the fact that noise is due to the non-zero temperature of physical devices
- Noise is usually modeled as a additive white Gaussian random process
- Today we will examine noise in general
- In the coming classes we will examine noise in AM and FM systems
- What to Read
  - 8.10, 9.1 – 9.4

Noise

- What do we mean by the term "noise" in communications systems?
  - In later courses you will learn about probabilistic descriptions of noise, but for this course you can think of noise as simply "the unwanted signal" at the receiver
- This unwanted signal is
  - Random (i.e., we don't know what it is ahead of time)
  - Additive (i.e., it simply creates an added voltage to the signal of interest)
  - The main problem facing the receiver
    - Since the message and the noise are both unknown *a priori* we have no way of distinguishing between the message and noise at the receiver
    - If there were no noise, communications would be trivial

Thermal Noise

- All objects with physical temperature greater than 0 Kelvin generate random electromagnetic noise.
- This noise power is given by

$$P_n = kT_nB$$

Botzman's Constant 1.38 x 10^{-23} J/K

Noise temperature of device

Bandwidth of device

$$kT_n = \text{Noise Spectral Density}$$

AWGN

- Noise in the system contributes an additional random voltage on top of the received signal. Thus, it is **additive**.
- The noise is uncorrelated from one sample to the next (i.e., $R_x(\tau)=\delta(\tau)$). A delta function in the correlation function means that the PSD is a constant, thus it is **white.**
- Noise in the system originates from components in the receiver and from signals captured by the antenna. Since there are many random signals all contributing some small amount, the sum tends to a Gaussian process (Central Limit Theorem). Thus, noise samples have a **Gaussian** probability distribution.
- **A**dditive **W**hite **G**aussian **N**oise (AWGN)

AWGN Channels

- The term 'AWGN Channel' is something of a misnomer.
- The channel doesn't necessarily add noise (at least not all of the noise). It attenuates the signal to such a degree that the internal noise of the receiver (as well as the noise observed by the antenna in wireless systems) is comparable to that of the received signal.
- Usually, we assume that the received signal has normalized average received power, while the noise has some power σ^2 where σ is the standard deviation of the thermal noise.
- In "AWGN Channels" we assume that the only distortion to the signal is the AWGN. Normalizing the powers:

$$r(t) = x(t) + n(t)$$

Noise Power

- A noise signal is assumed to be a *power signal*
  - It has infinite time duration
- Thus, a noise signal has
  - Infinite energy
  - Finite power
- We typically use the variable *N* to represent noise power which is defined as

$$N = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} n^2(t)\,dt$$

Signal-to-Noise Ratio

- The ratio of the desired signal's power to the noise signal's power is termed the *signal-to-noise ratio*

$$\frac{S}{N}=\frac{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}s^2(t)\,dt}{\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}n^2(t)\,dt}$$

- Signal-to-Noise ratio (SNR) is a measure of the quality of the received signal
  - Low SNR (say below 5) indicates poor quality
  - High SNR (say above 20) indicated good quality

Improving SNR through Filtering

- It is often difficult to distinguish between the message and noise in the time domain since both signals are unknown *a priori*
- However, noise has a very wide bandwidth as compared to the message
- Thus we can reduce the amount of noise in the receiver through filtering
  - By restricting the received signal to a bandwidth commensurate with the signal, we can reduce noise power without significantly impacting the power of the desired signal

AWGN – White RP

- A 'white' random process is one whose power spectral density (PSD) is a constant:

$$S_n(f) = \frac{N_o}{2} \quad \forall f$$

- The autocorrelation of any random process is simply the inverse Fourier Transform of the PSD

$$R_n(\tau) = F^{-1}\{S_n(f)\}$$

$$= \frac{N_o}{2}\delta(\tau)$$

Important Note

- While we often consider white noise with a flat (i.e., constant) power spectral density, in actuality this type of process is not realistic
  - A process with a truly constant PSD has infinite power

$$P_{tot} = \int_{-\infty}^{\infty} S_n(f)\,df$$

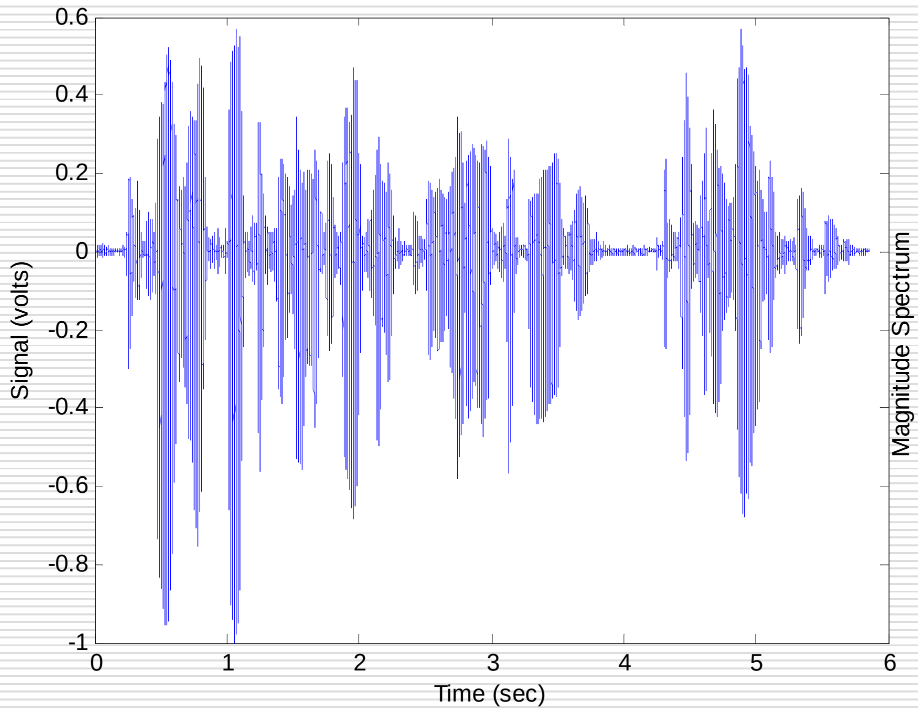
$$= \int_{-\infty}^{\infty} \frac{N_o}{2}\,df$$

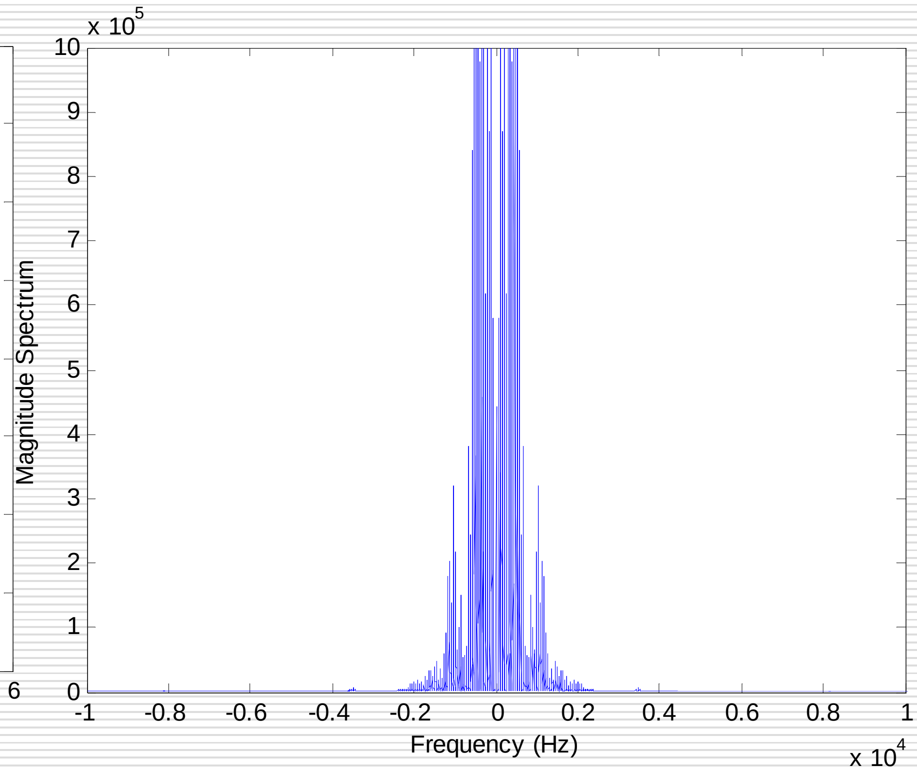
$$= \infty$$

- However, noise which is white over the bandwidth *B* of the receiver (i.e., band-limited white noise) does not have infinite power and is realistic.
- If such a process is sampled at $T_s = 1/B$ consecutive samples have zero correlation ($R_n(\tau) = N_o/2 \,\mathrm{sinc}(\tau B)$)

Example 18.1

- Message signal – voice sample

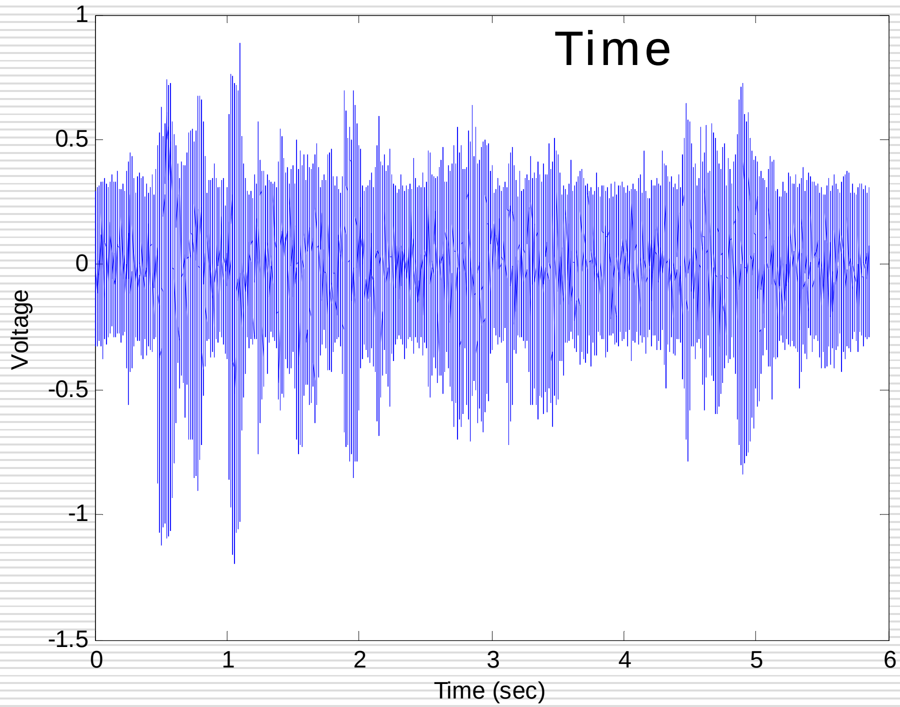


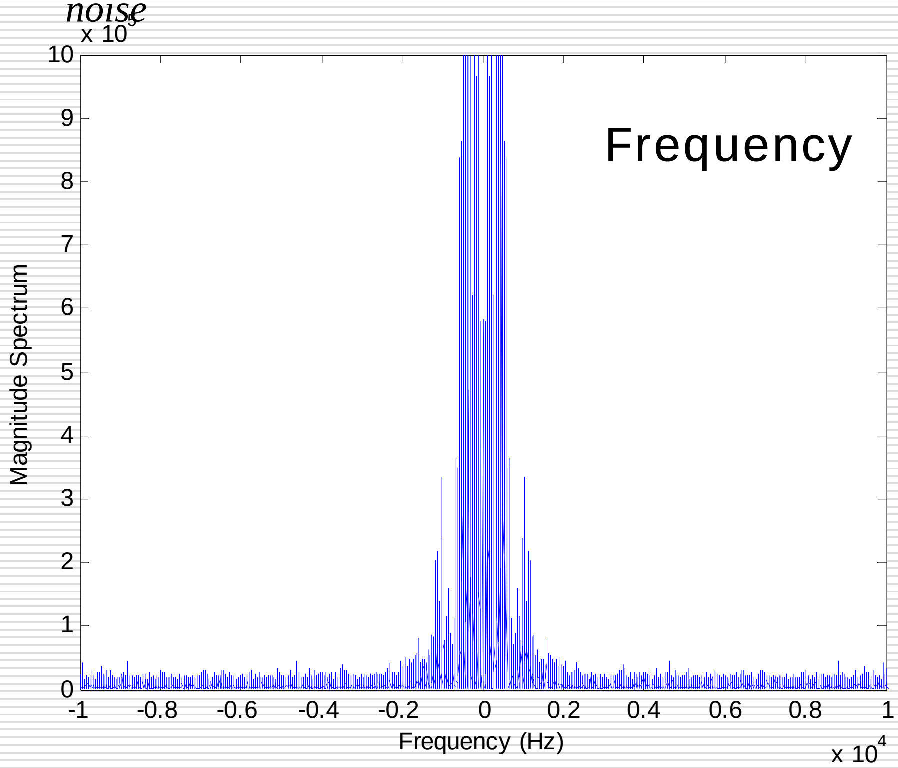


Example 18.1 – cont.

- Message – SNR = 0dB

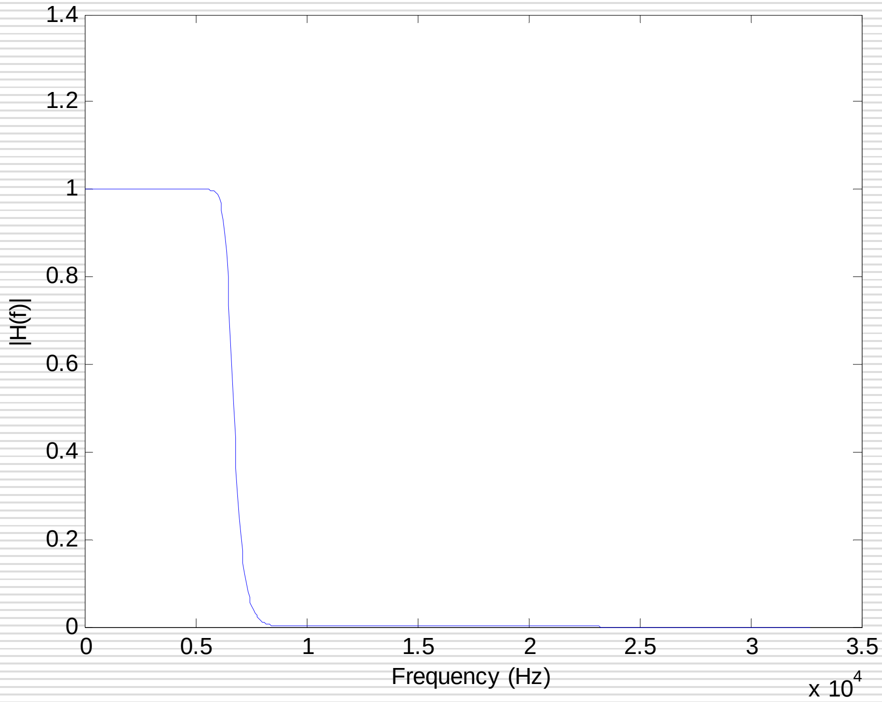
$$r(t) = \underbrace{m(t)}_{message} + \underbrace{n(t)}_{noise}$$





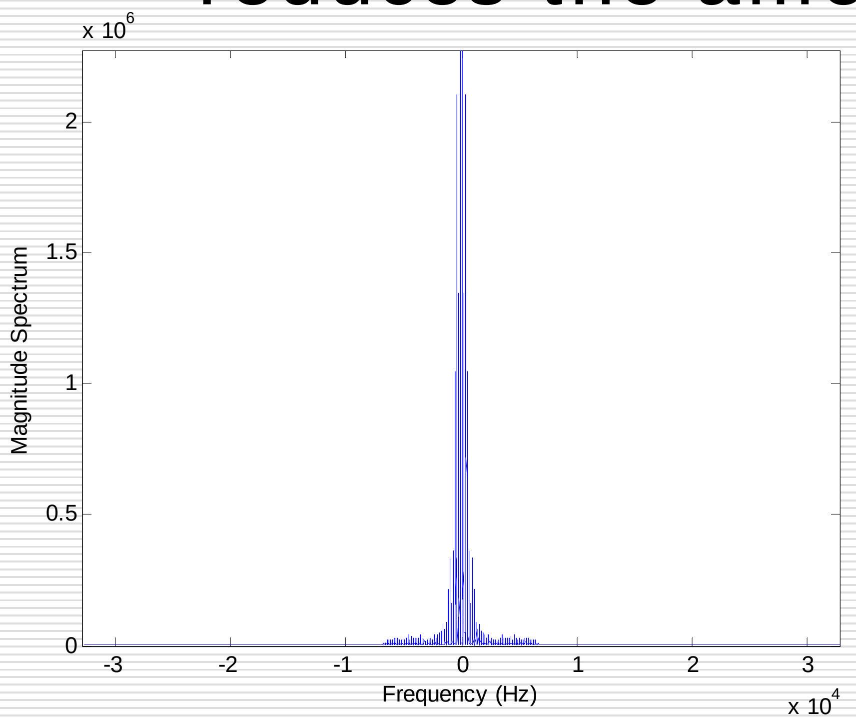
Filter – Frequency domain

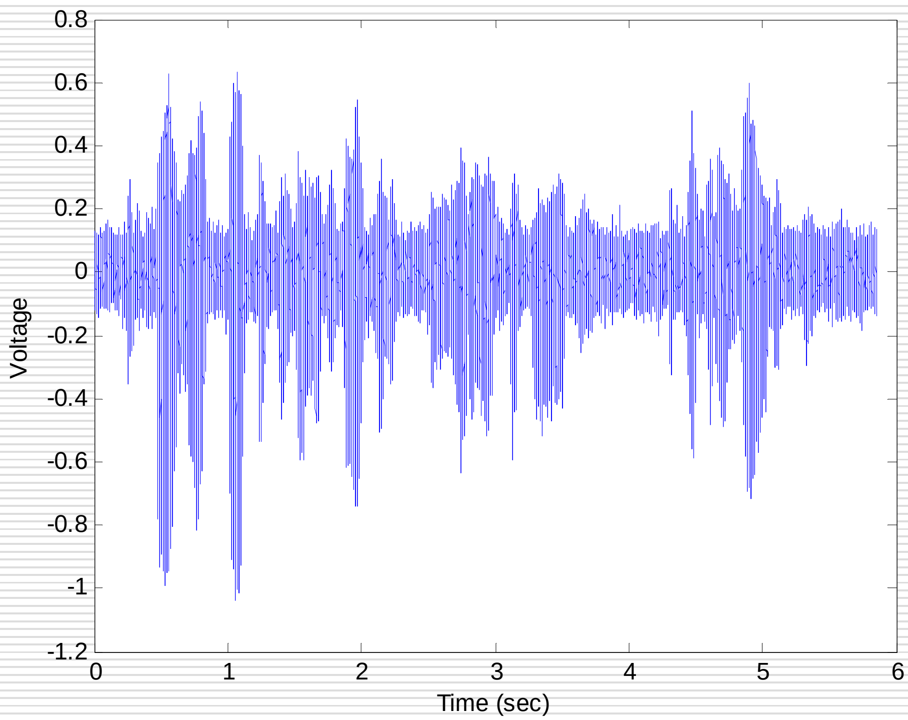
- Filter has a bandwidth of approximately 7kHz
- Signal has a bandwidth of roughly 4kHz
- Noise has much larger bandwidth (>> 10kHz)



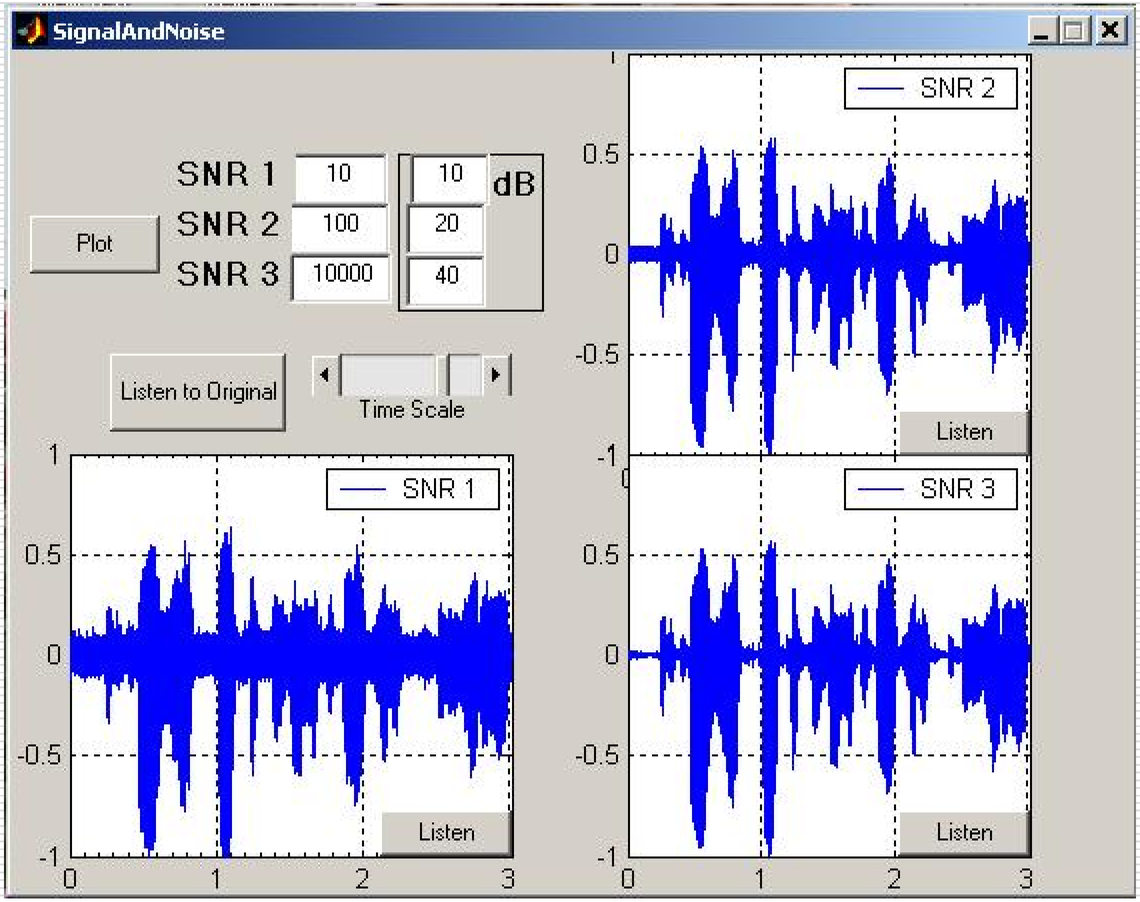
Filter Output

- The filter passes very nearly the entire desired signal, but substantially reduces the amount of noise





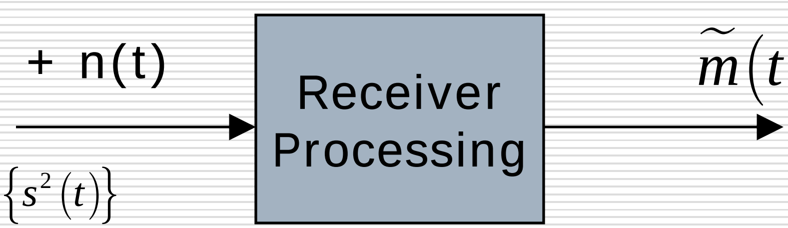
SNR – Interactive Example



SNR – Input versus Output

- Throughout our discussion of the performance of analog modulation performance, we will distinguish between the SNR at the input to the receiver and the SNR at the output (i.e., in the final estimated message)
- The two are closely related, but we care much more about the receiver's output SNR

r(t) = s(t) + n(t)



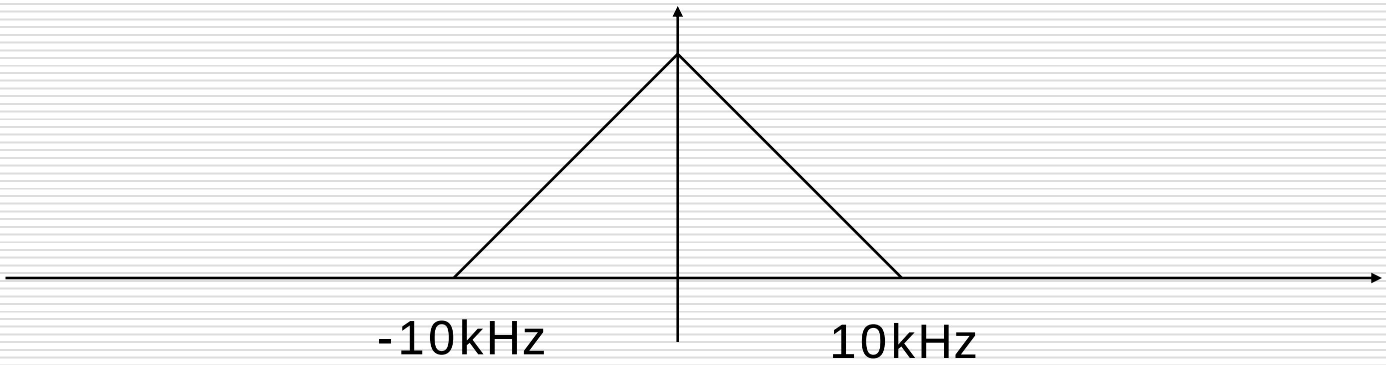
$$\tilde{m}(t) = m(t) + v(t)$$

$$\left(\frac{S}{N}\right)_{in} = \frac{E\{s^2(t)\}}{E\{n^2(t)\}}$$

$$\left(\frac{S}{N}\right)_{out} = \frac{E\{m^2(t)\}}{E\{v^2(t)\}}$$

Example 18.2

- Consider a received desired signal with power spectral density shown below



- If the desired signal's received power is -105dBm and the receiver front end has an ideal response of bandwidth 1MHz and a noise temperature of 350K, what is the received signal-to-noise ratio (SNR)? If the received signal is then passed through an ideal low pass filter with bandwidth 10kHz, what is the resulting SNR?

Example 18.2 – Noise calculation

- The SNR requires us to know the desired received signal power as well as the noise power in the receiver. We can calculate this in either the time or frequency domains.
- The desired signal power is given in dBm (decibels relative to 1mW). We simply need the noise power at the receiver.
- The noise power can be determined by integrating the power spectral density of the noise over all frequencies:

$$N = \int_{-\infty}^{\infty} S_n(f)\,df$$

- Since the PSD is white over the band of interest (1MHz):

$$N = \int_{-B}^{B} \frac{N_o}{2}\,df$$

$$= \frac{N_o}{2} 2B = N_o B$$

Noise calculation – cont.

- Since the front end of the receiver has a bandwidth of 1MHz:

$$N = N_o B$$
$$= 10^6 N_o$$

- The noise power spectral density N_o is calculated

$$N_o = kT_o$$
$$= 1.23 * 10^{-23} * 350$$
$$= 4.8 * 10^{-21} W / Hz$$

- The noise power is then $N = 10^6 * 4.8 * 10^{-21} W$

$$= 4.8 * 10^{-15} W$$

Example 18.2 - SNR

- To calculate the SNR in dB, we first must convert the received signal power from dBm to dBW. After converting the noise power to dBW, we simply subtract the noise power from the signal power

$$S(dBW) = -105dBm - 30dB$$

$$= -135dBW$$

$$N(dBW) = 10 * \log_{10}\left(4.8 * 10^{-15}\right)$$

$$= -143.2dBW$$

$$\frac{S}{N} = -135dBW - \left(-143.2dBW\right)$$

$$= 8.2dB$$

Example 18.2 – cont.

- Filtering the signal can improve the SNR if it doesn't distort the desired signal.
- Since the original signal has a bandwidth of 10kHz, an ideal LPF with bandwidth 10kHz will not affect the desired signal, but will reduce the noise power.
- Thus,

$$S(dBW) = -135dBW$$

$$N(dBW) = 10*\log_{10}\left(4.8*10^{-21}*10000\right)$$

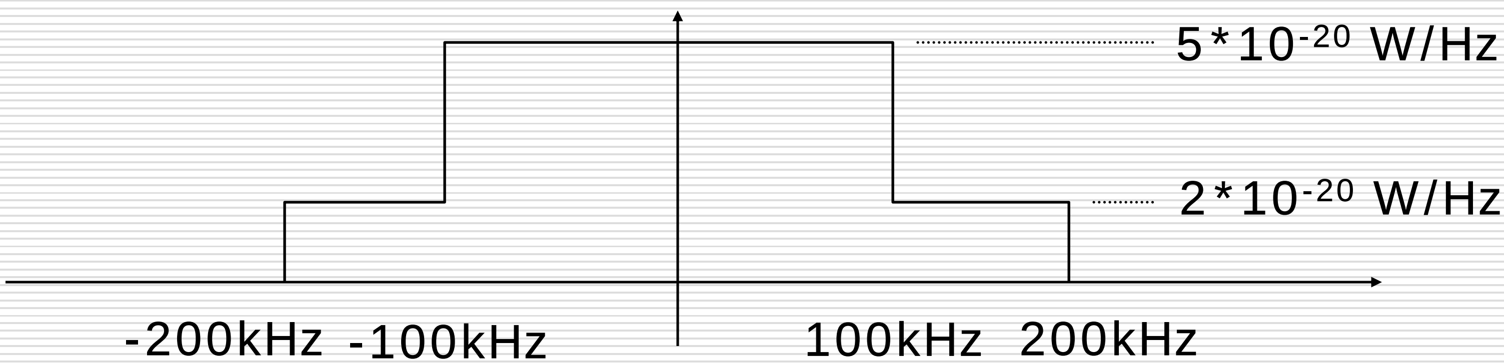
$$= -163.2dBW$$

$$\frac{S}{N} = -135dBW - \left(-163.2dBW\right)$$

$$= 28.2dB$$

Example 18.3

- Consider a received desired signal with power spectral density shown below



- If the receiver front end has an ideal response of bandwidth 500kHz and a noise temperature of 375K, what is the received signal-to-noise ratio (SNR)? If the received signal is then passed through an ideal low pass filter with bandwidth 125kHz, what is the resulting SNR? What bandwidth should be used to maximize SNR?

Example 18.3 – Signal Power

- The front end bandwidth is larger than the received signal so the received power is

$$S = 2\left(100kHz * 5 * 10^{-20} W/Hz + 100kHz * 2 * 10^{-20} W/Hz\right)$$

$$= 1.4 * 10^{-14} W$$

- The noise power is then

$$N = N_o B$$

$$= kT_o B$$

$$= 1.23 * 10^{-23} W/Hz/K * 375K * 500kHz$$

$$= 2.6 * 10^{-15} W$$

Example 18.3 - SNR

- The resulting SNR (in dB) is

$$\frac{S}{N}(dB) = 10\log_{10}\left\{1.4*10^{-14}\right\}dBW - 10\log_{10}\left\{2.6*10^{-15}\right\}dBW$$

$$= -138.6dBW - \left(-146dBW\right)$$

$$= 7.4dB$$

- Now, if we reduce the bandwidth to 125kHz, we can substantially reduce the noise:

$$N = 1.23*10^{-23}W/Hz/K*375K*125kHz$$

$$= 6.5*10^{-16}W$$

Example 18.3 – Signal Power

- However, the signal power is also reduced:

$$S = 2\left(100kHz * 5*10^{-16} W / Hz + 25kHz * 2*10^{-16} W / Hz\right)$$

$$= 1.1*10^{-14} W$$

- The resulting SNR is

$$\frac{S}{N}(dB) = 10\log_{10}\left\{1.1*10^{-14}\right\} dBW - 10\log_{10}\left\{6.5*10^{-16}\right\} dBW$$

$$= 12.3dB$$

Example 18.3 – optimizing BW

- The optimal bandwidth of the filter can be found by examining the SNR as a function of *B.*
- Further, there are three regions to examine: *B*<100kHz, 100kHz<*B*<200kHz and B>200kHz. For *B*<100kHz, the SNR can be written as

$$\frac{S}{N}(dB) = 10\log_{10}\left\{2*5*10^{-20}B\right\}dBW - 10\log_{10}\left\{5.2*10^{-21}*B\right\}dBW$$

$$= -190dBW/Hz + 10\log_{10}\left\{B\right\}dBHz + 202.8dBW/Hz...$$

$$-10\log_{10}\left\{B\right\}dBHz$$

$$= 12.8dB$$

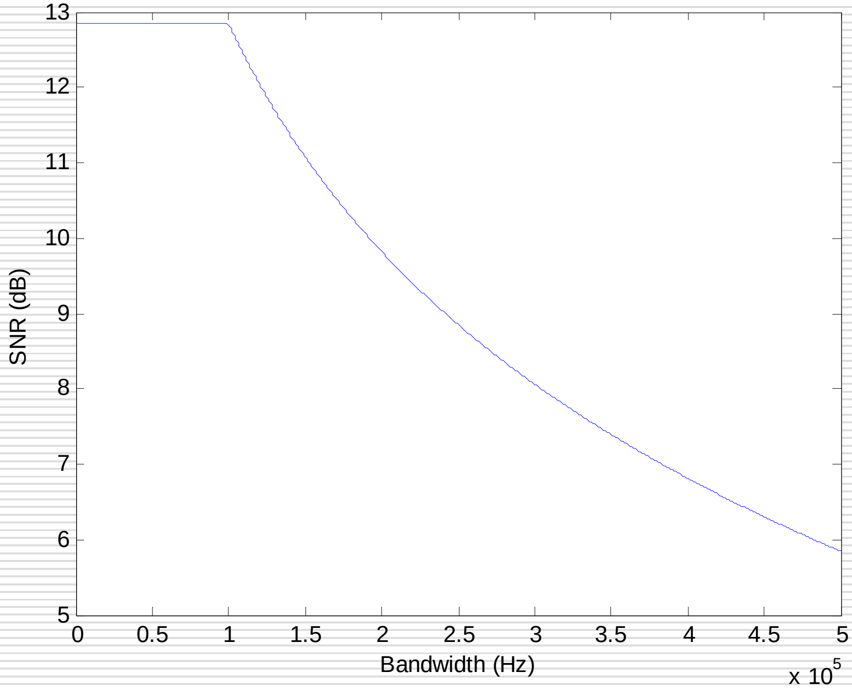
In other words, SNR is independent of *B* for values less than 100kHz.

Example 18.3 – optimizing BW

- For 100kHz<*B*<200kHz, the SNR can be written as

$$\frac{S}{N}(dB) = 10\log_{10}\left\{2*5*10^{-20}(100kHz) + 2*2*10^{-20}(B-100kHz)\right\}dBW \ldots$$

$$-10\log_{10}\left\{5.2*10^{-21}*B\right\}dBW$$



Thus, the optimal bandwidth is any value less than or equal to 100kHz. For values of B > 100kHz, the SNR degrades.

Noise Equivalent Bandwidth

- To this point we have assumed that all of the filters we used are ideal filters with a square frequency response.
- For non-ideal filters we can define a bandwidth for an ideal filter that allows the same amount of noise power to pass through.
- This is termed the *noise-equivalent bandwidth*
- The power passed by a filter with frequency response *H(f)* can be written as

$$P_N = \int_{-\infty}^{\infty} \left|H(f)\right|^2 S_n(f)\,df$$

$$= \int_{-\infty}^{\infty} \left|H(f)\right|^2 \frac{N_o}{2}\,df$$

$$= N_o \int_{0}^{\infty} \left|H(f)\right|^2 df$$

Noise Equivalent Bandwidth

- For an ideal filter with bandwidth B_N and gain *H(0)*, the noise power passed is

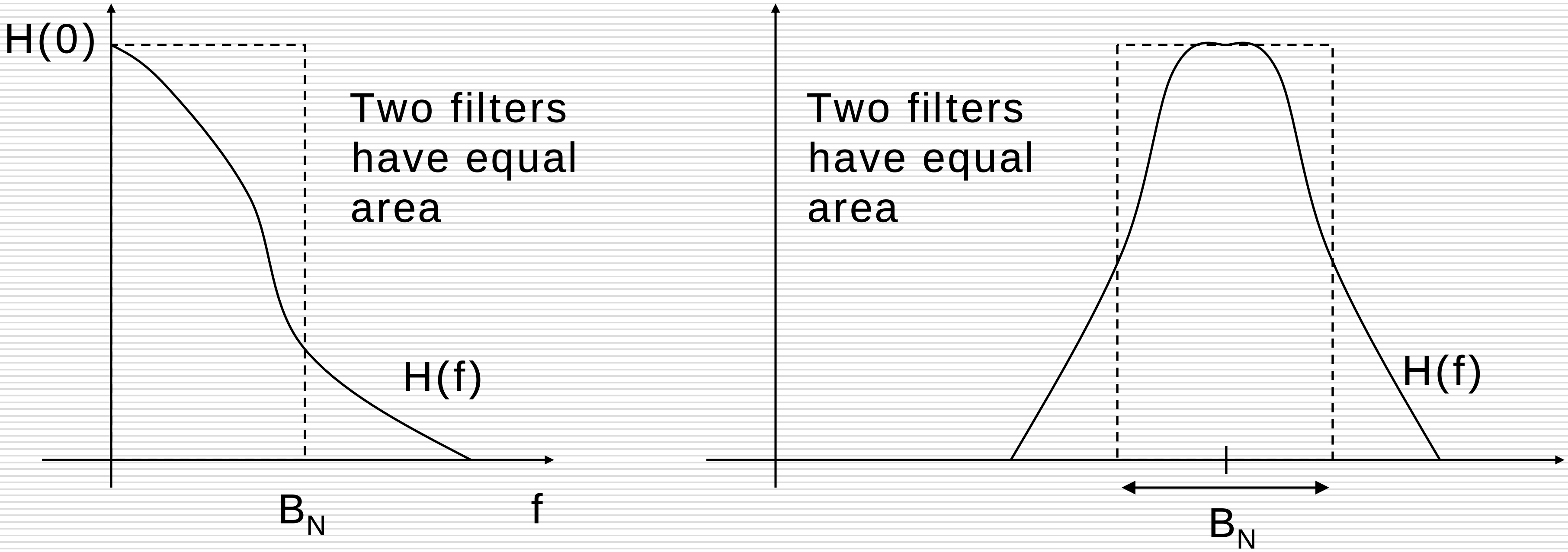
$$P_N = N_o B_N \ H(0)$$

- Thus, we can define the noise equivalent bandwidth of any filter by equating the noise passed by the two filters:

$$N_o B_N \ H(0) = N_o \int_0^\infty |H(f)|^2 \, df$$

$$B_N = \frac{\int_0^\infty |H(f)|^2 \, df}{H(0)}$$

Graphical Illustration



- The Noise Equivalent Bandwidth B_N is the bandwidth of a rectangular filter with equivalent area (i.e., passes an equivalent amount of noise)

Summary

- Today we have examined one of the dominant degradations in communication systems, additive white Gaussian noise (AWGN)
- The power spectral density of noise is a constant with an extremely large
- Since the bandwidth of noise is extremely large, the impact of noise can be reduced by filtering
- *Noise equivalent bandwidth* is a measure of a filter which provides a means for determining the amount of noise passed by the filter