ECE3614
Introduction to Communications Systems
Fall 2007

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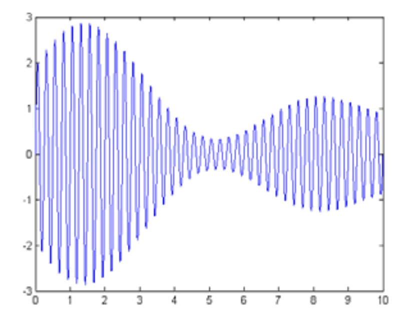
Lecture #19: Noise in AM Receivers

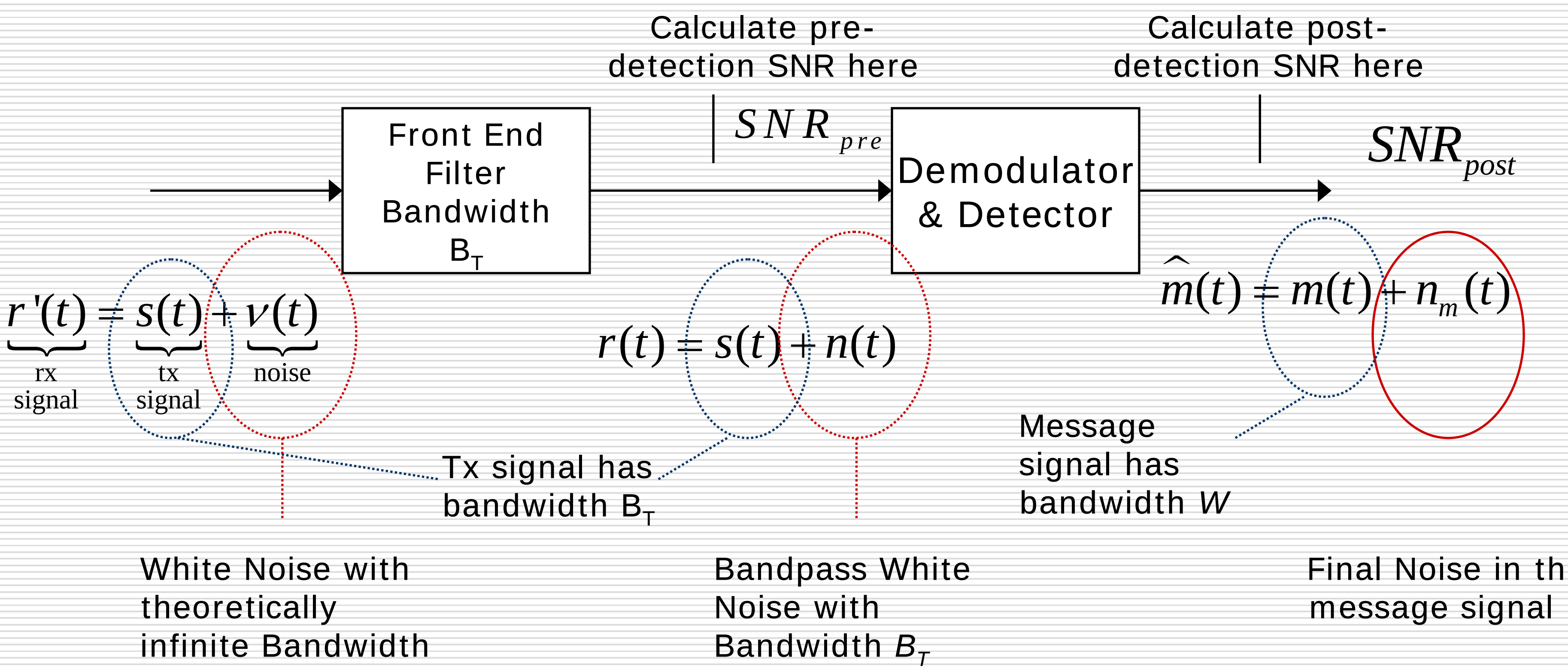


Overview

- We have studied the characteristics of AM and FM signals, but we haven't yet studied *performance.*
- Performance is measured in terms of the output (post-detection) signal-to-noise ratio
- Today we will examine the performance of AM systems as compared to a simple baseband reference systems
- Next class we will examine the performance of FM systems
- Reading
  - 9.5 – 9.6

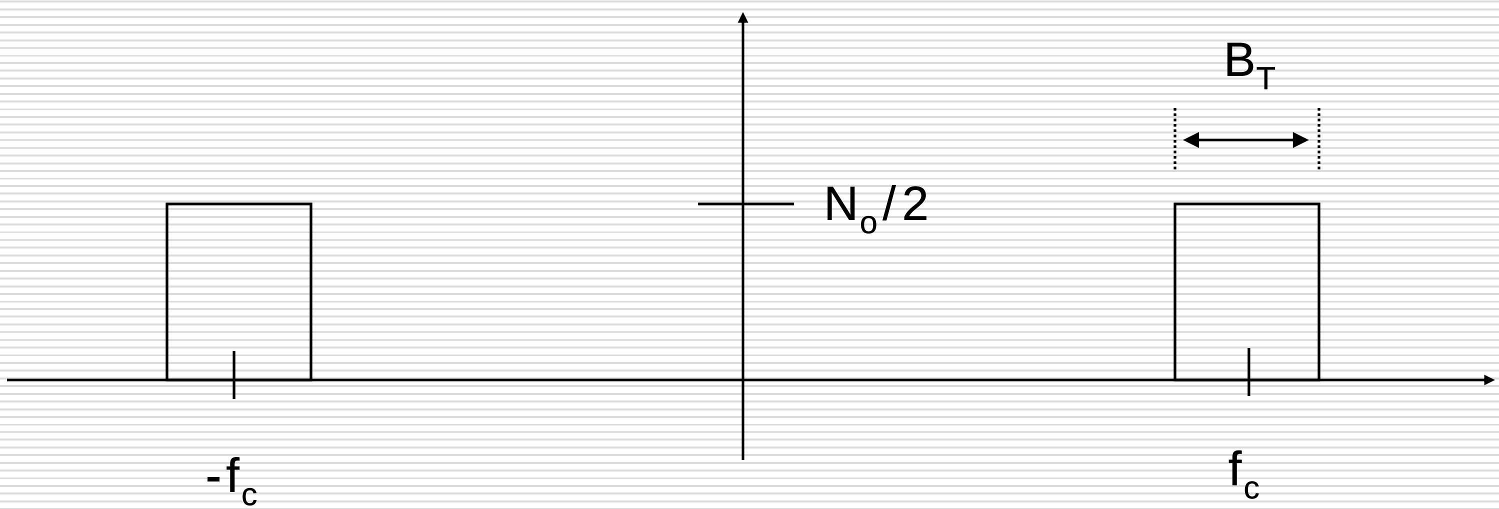
Pre-detection vs. Post-detection

- We are interested in the SNR both *before* and *after* the demodulator/detector



Bandpass Noise

- The noise immediately preceding the demodulator/detector is *bandpass noise*
- The power spectral density is:



- Thus, the total noise power is $N_o/2*2B_T = N_oB_T$

Bandpass Noise (2)

- Recall that any bandpass signal can be represented in In-phase and quadrature form.
- Thus

$$n(t) = n_I(t)\cos\left(2\pi f_c t\right) - n_Q(t)\sin\left(2\pi f_c t\right)$$

- Since the total power is $N_o B_T$:

$$N = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} n^2(t)dt$$

$$N = \mathbf{E}\left\{n^2(t)\right\} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} n^2(t)dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left(n_I(t)\cos\left(2\pi f_c t\right) - n_Q(t)\sin\left(2\pi f_c t\right)\right)^2 dt$$

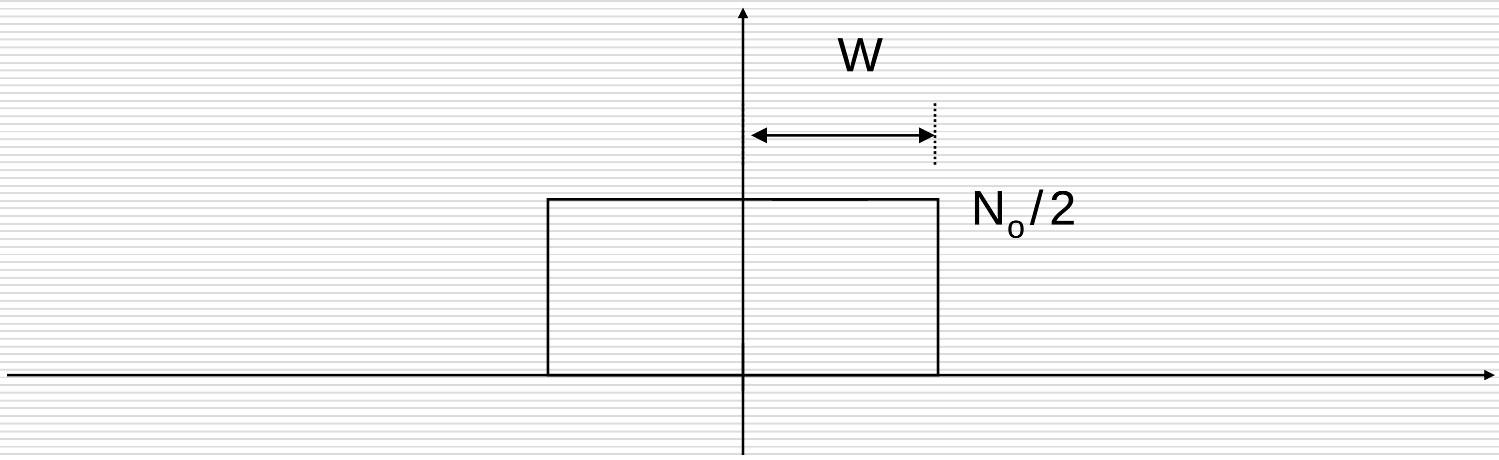
$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left(n_I^2(t)\cos^2\left(2\pi f_c t\right) - 2n_I(t)n_Q(t)\cos\left(2\pi f_c t\right)\sin\left(2\pi f_c t\right) + n_Q^2(t)\sin^2\left(2\pi f_c t\right)\right)dt$$

0

$$= \frac{1}{2}N_I + \frac{1}{2}N_Q \longrightarrow \boxed{N_I = N_Q = N_o B_T}$$

Lowpass Noise

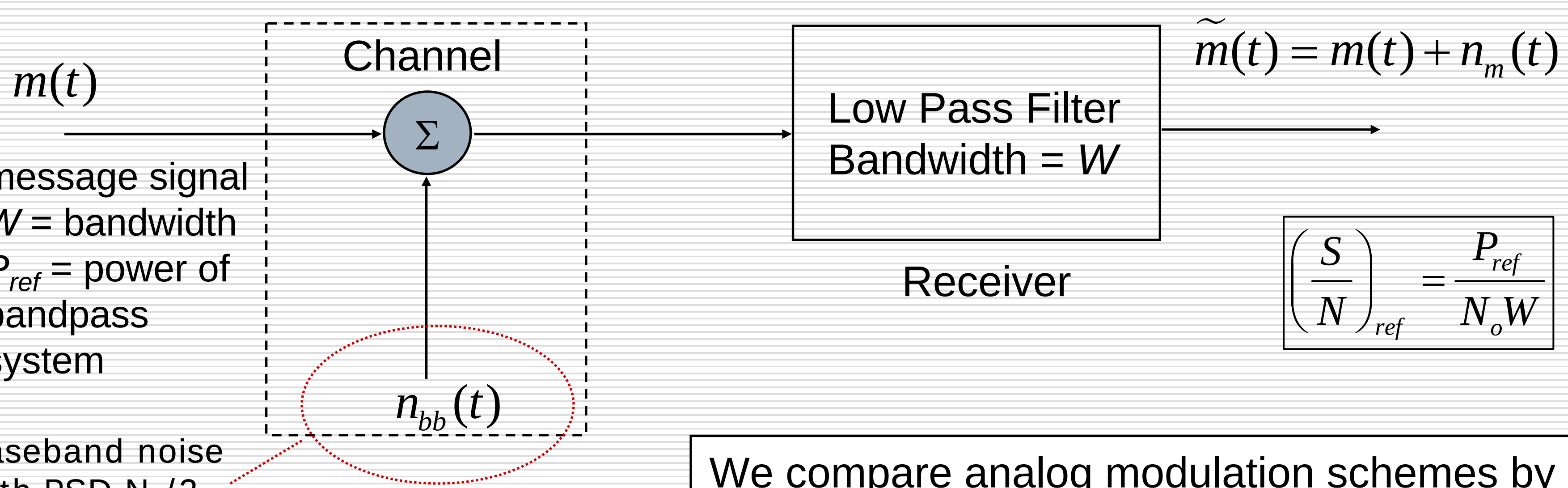
- The noise at the output of the baseband system with bandwidth *W* is *lowpass white noise*
- The power spectral density is:



- Thus, the total noise power is $N_o/2*2W = N_oW$

Baseband Reference System

- In order to compare various AM systems, we will compare each analog bandpass system to a baseband reference model



message signal
W = bandwidth
P_{ref} = power of bandpass system

Baseband noise with PSD $N_o/2$ and near infinite bandwidth

We compare analog modulation schemes by evaluating $(S/N)_{post}$ and comparing to $(S/N)_{ref}$

Calculating SNR – LC AM, DSBSC

- Power can be calculated either in the time domain or in the frequency domain
  - Both give the same result
- For mathematical convenience we typically calculate the noise power in the frequency domain and the signal power in the time domain
- Assuming that the message signal is zero mean, the signal power is at the input of the receiver is:

$$S = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s^2(t)\,dt$$
$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 \left(1 + k_a m(t)\right)^2 \cos^2\left(2\pi f_c t\right) dt$$
$$= \frac{A_c^2}{2}\left\{1 + k_a^2\,\mathbf{E}\left\{m^2(t)\right\}\right\}$$

Large Carrier AM

$$S = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s^2(t)\,dt$$
$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m^2(t) \cos^2\left(2\pi f_c t\right) dt$$
$$= \frac{A_c^2}{2}\,\mathbf{E}\left\{m^2(t)\right\}$$

Double Sideband Suppressed Carrier

Calc. SNR – LC AM, DSBSC (cont.)

- The noise power after front end filtering (assume an ideal bandpass filter with bandwidth B_T - the bandwidth of the AM signal) can most easily calculated in the frequency domain:

$$N = 2B_T N_0/2$$
$$= B_T N_0$$

- Since the transmit bandwidth of both LC AM and DSBSC AM is $B_T = 2W$:

$$N = N_0 B_T$$
$$= 2N_0 W$$

Input SNR of Large Carrier AM

- Thus, the input (i.e., pre-detection) SNR of Large Carrier AM is

$$\left(\frac{S}{N}\right)_{\text{pre}}^{\text{LC-AM}} = \frac{A_c^2\left[1+k_a^2\,\mathbf{E}\left\{m^2(t)\right\}\right]/2}{2N_0W}$$

$$= \frac{A_c^2\left[1+k_a^2\,\mathbf{E}\left\{m^2(t)\right\}\right]}{4N_0W}$$

$$= \frac{A_c^2\left[1+k_a^2P_m\right]}{4N_0W}$$

where P_m and W are the average power and bandwidth of the message respectively

Input SNR of DSBSC

- For Double Sideband Suppressed Carrier we have

$$\left(\frac{S}{N}\right)_{\text{pre}}^{\text{DSB}} = \frac{A_c^2\,\mathbf{E}\left\{m^2(t)\right\}/2}{2N_0W}$$

$$= \frac{A_c^2\,\mathbf{E}\left\{m^2(t)\right\}}{4N_0W}$$

$$= \frac{A_c^2P_m}{4N_0W}$$

where P_m and W are the average power and bandwidth of the message respectively

Calculating SNR – SSB

- The received power of a single sideband AM signal can be calculated as

$$S = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s^2(t)\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left\{\frac{A_c}{2}m(t)\cos(2\pi f_c t)\mp\frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\right\}^2 dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\left\{\frac{A_c^2}{4}m^2(t)\cos^2(2\pi f_c t)+\frac{A_c^2}{4}\hat{m}^2(t)\sin^2(2\pi f_c t)\right\}dt$$

where we have used the fact that cos(t) and sin(t) are orthogonal.

- Since the Hilbert transform does not change the power of a signal we have

$$S = \frac{A_c^2}{8}P_m + \frac{A_c^2}{8}P_m$$

$$= \frac{A_c^2}{4}P_m$$

Calculating SNR – SSB (cont.)

- The noise power at after front end filtering (assume an ideal bandpass filter with bandwidth B_T (the bandwidth of the AM signal) can most easily calculated in the frequency domain:

$$N = 2B_T N_0/2$$
$$= B_T N_0$$

- Since the transmit bandwidth of SSB AM is $B_T = W$:

$$N = N_0 B_T$$
$$= N_0 W$$

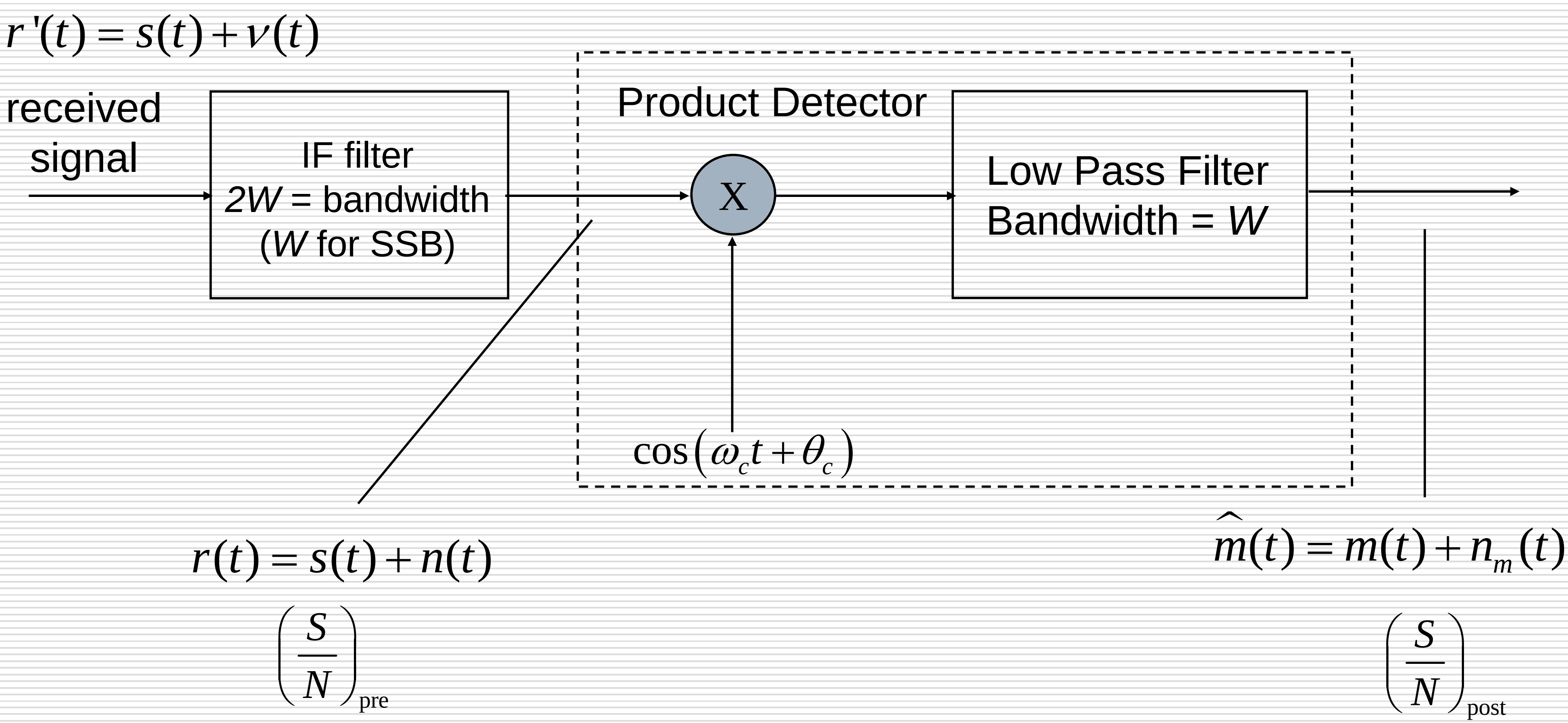
- Thus, the Input SNR is

$$\left(\frac{S}{N}\right)_{\text{pre}}^{\text{SSB}} = \frac{A_c^2 P_m/4}{N_0 W} = \frac{A_c^2 P_m}{4N_0 W}$$

Comparing Input SNR for AM Systems

- Pre-detection SNR (at front end of receiver):
  - DSB-SC: $$\left(\frac{S}{N}\right)_{\text{pre}}^{\text{DSB}} = \frac{A_c^2 P_m}{4N_0 B}$$
  - LC-AM: $$\left(\frac{S}{N}\right)_{\text{pre}}^{\text{LC-AM}} = \frac{A_c^2\left[1 + k_a^2 P_m\right]}{4N_0 B}$$
  - SSB: $$\left(\frac{S}{N}\right)_{\text{pre}}^{\text{SSB}} = \frac{A_c^2 P_m}{4N_0 B}$$

AM with Product Detector



The product detector is a coherent receiver

Product Detector SNR for DSB-SC

- After the mixer we have

$$r(t)\cos\left(2\pi f_c t\right) = \left\{s(t) + n(t)\right\}\cos\left(2\pi f_c t\right)$$

$$= A_c m\left(t\right)\cos^2\left(2\pi f_c t\right) + \left\{n_I(t)\cos\left(2\pi f_c t\right) - n_Q(t)\sin\left(2\pi f_c t\right)\right\}\cos\left(2\pi f_c t\right)$$

$$= \frac{A_c}{2} m\left(t\right)\left(1 + \cos\left(4\pi f_c t\right)\right) + n_I(t)\frac{1}{2}\left(1 + \cos\left(4\pi f_c t\right)\right) - n_Q(t)\frac{1}{2}\sin\left(4\pi f_c t\right)$$

- After the baseband filter we have

$$\widehat{m}(t) = \frac{A_c}{2} m\left(t\right) + \frac{1}{2} n_I(t)$$

Output SNR - DSBSC

- The signal power and noise power at the output of the lowpass filter are

$$S = \frac{A_c^2}{4} P_m$$

$$N = \frac{1}{4} N_o B_T$$

- The output SNR is then

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{DSB}} = \frac{\frac{A_c^2}{4} P_m}{\frac{N_o W}{2}} = \frac{A_c^2 P_m}{2 N_o W}$$

Relative SNR - DSBSC

$$\frac{\left(\frac{S}{N}\right)_{post}^{DSB}}{\left(\frac{S}{N}\right)_{pre}^{DSB}} = \frac{\left(\frac{A_c^2 P_m}{2N_o B}\right)}{\left(\frac{A_c^2 P_m}{4N_o B}\right)} = 2$$

Comparing the pre and post detection SNRs we see that there is a gain of 2 in the processing since the Q channel noise is eliminated.

$$\frac{\left(\frac{S}{N}\right)_{post}^{DSB}}{\left(\frac{S}{N}\right)_{ref}^{DSB}} = \frac{\left(\frac{A_c^2 P_m}{2N_o W}\right)}{\left(\frac{\frac{A_c^2}{2} P_m}{N_o W}\right)} = 1$$

If we used the same transmit power in a baseband system, we see that we would achieve the same SNR. Thus, there is no wasted power in the DSB-SC transmit scheme (but no gain either). The book calls this the *Figure of Merit* for the modulation scheme

Product Detector SNR – LC AM

- At the output of the product detector when the input is a large carrier AM signal:

$$LPF\left\{r(t)\cos\left(2\pi f_c t\right)\right\} = LPF\left\{\left[s(t)+n(t)\right]\cos\left(2\pi f_c t\right)\right\}$$

$$= LPF\left\{\left[A_c\left(1+k_a m(t)\right)\cos\left(2\pi f_c t\right)+n(t)\right]\cos\left(2\pi f_c t\right)\right\}$$

$$= LPF\left\{\left[A_c\left(1+k_a m(t)\right)\cos^2\left(2\pi f_c t\right)+n(t)\cos\left(2\pi f_c t\right)\right]\right\}$$

$$= \frac{A_c}{2}\left(1+k_a m(t)\right)+\frac{1}{2}n_I(t)$$

- The output SNR (after removing the constant) is then

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}} = \frac{\left(A_c^2/4\right)k_a^2 P_m}{N_o W/2} = \frac{A_c^2 k_a^2 P_m}{2 N_o W}$$

Relative SNRs – LC AM

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}}}{\left(\frac{S}{N}\right)_{\text{pre}}^{\text{LC-AM}}} = \frac{\left(\frac{A_c^2 k_a^2 P_m}{2N_o W}\right)}{\left(\frac{A_c^2\left[1+k_a^2 P_m\right]}{4N_0 W}\right)} = \frac{2k_a^2 P_m}{1+k_a^2 P_m}$$

Comparing pre and post detection SNRs we see that there is a loss of SNR due to the unmodulated carrier. However, there is a gain of 2 since the Q channel noise is eliminated.

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}}}{\left(\frac{S}{N}\right)_{\text{ref}}^{\text{LC-AM}}} = \frac{\left(\frac{A_c^2 k_a^2 P_m}{2N_o W}\right)}{\left(\frac{A_c^2\left[1+k_a^2 P_m\right]}{2N_0 W}\right)} = \frac{k_a^2 P_m}{1+k_a^2 P_m}$$

Comparing post detection SNR to a baseband system with the same transmit power, we see that there is again a loss due to the unmodulated carrier.

NOTE: An envelope detector gives the same performance in the high SNR region (normal operating range for AM). The performance of the envelope detector is worse than the product detector at low pre-detection SNR's.

Product Detector - SSB

- After the mixer we have

$$r(t)\cos\left(2\pi f_c t\right)=\left\{s(t)+n(t)\right\}\cos\left(2\pi f_c t\right)$$

$$=\left\{\frac{A_c}{2}m\left(t\right)\cos\left(2\pi f_c t\right)\mp\frac{A_c}{2}\widehat{m}\left(t\right)\sin\left(2\pi f_c t\right)\right\}\cos\left(2\pi f_c t\right)...$$

$$+\left\{n_I(t)\cos\left(2\pi f_c t\right)-n_Q(t)\sin\left(2\pi f_c t\right)\right\}\cos\left(2\pi f_c t\right)$$

$$=\frac{A_c}{4}m\left(t\right)\left(1+\cos\left(4\pi f_c t\right)\right)\mp\frac{A_c}{4}\widehat{m}\left(t\right)\sin\left(4\pi f_c t\right)+...$$

$$n_I(t)\frac{1}{2}\left(1+\cos\left(4\pi f_c t\right)\right)-n_Q(t)\frac{1}{2}\sin\left(4\pi f_c t\right)$$

- After the baseband filter we have

$$\widehat{m}(t)=\frac{A_c}{4}m\left(t\right)+\frac{1}{2}n_I(t)$$

Product Detector (SSB)

- At the output of the detector we have

$$\widehat{m}(t) = \frac{A_c}{4} m(t) + \frac{1}{2} n_I(t)$$

- The SNR is then

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{SSB}} = \frac{\mathrm{E}\left\{\frac{A_c^2}{16} m^2(t)\right\}}{\mathrm{E}\left\{\frac{1}{4} n_I{}^2(t)\right\}} = \frac{A_c^2 P_m}{4 N_o W}$$

where

$$\mathrm{E}\left\{x^2(t)\right\} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

Relative SNRs – SSB

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}}{\left(\frac{S}{N}\right)_{\text{pre}}} = \frac{\left(\frac{A_c^2 P_m}{4N_o W}\right)}{\left(\frac{A_c^2 P_m}{4N_o W}\right)} = 1$$

Comparing pre and post detection SNRs we see that there is no gain since even though the Q channel noise is eliminated, ½ the desired signal power is lost in the Hilbert Transform signal.

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}}{\left(\frac{S}{N}\right)_{\text{ref}}} = \frac{\left(\frac{A_c^2 P_m}{4N_o W}\right)}{\left(\frac{A_c^2 P_m}{4N_o W}\right)} = 1$$

Comparing post detection SNR to a baseband system with the same transmit power, we see that there is no loss or gain.

Comparing Post Detection SNRs

- Post-detection SNR (at the output of receiver):
  - DSB-SC: $$\left(\frac{S}{N}\right)_{\text{post}}^{\text{DSB}} = \frac{A_c^2 P_m}{2N_0 W}$$
  - LC- AM: $$\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}} = \frac{A_c^2 k_a^2 P_m}{2N_0 W}$$
  - SSB $$\left(\frac{S}{N}\right)_{\text{post}}^{\text{SSB}} = \frac{A_c^2 P_m}{4N_0 W}$$

SNR Comparisons

- DSB-SC

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{DSB}}}{\left(\frac{S}{N}\right)_{\text{pre}}^{\text{DSB}}} = 2 \qquad \frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{DSB}}}{\left(\frac{S}{N}\right)_{\text{ref}}^{\text{DSB}}} = 1$$

- SSB

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{SSB}}}{\left(\frac{S}{N}\right)_{\text{pre}}^{\text{SSB}}} = 1 \qquad \frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{SSB}}}{\left(\frac{S}{N}\right)_{\text{ref}}^{\text{SSB}}} = 1$$

- LC - AM

$$\frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}}}{\left(\frac{S}{N}\right)_{\text{pre}}^{\text{LC-AM}}} = \frac{2k_a^2 P_m}{1+k_a^2 P_m} \qquad \frac{\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}}}{\left(\frac{S}{N}\right)_{\text{ref}}^{\text{LC-AM}}} = \frac{k_a^2 P_m}{1+k_a^2 P_m}$$

Example 19.1

- An AM radio station transmits a Large Carrier AM signal with a total power of 1kW. Assuming the power loss versus distance is d^3, what is the post detection SNR at a distance of 100km if the receiver noise temperature is 1000K? (Assume that the message has a peak value of 1, an average power of -10dBW and a bandwidth of $W = 10kHz$, and k_a = 0.25)

- **SOLUTION:** In order to determine the post detection SNR, we need to know the received signal power and the portion dedicated to the message.

$$S = \frac{A_c^2}{2}\left\{1 + k_a^2 P_m\right\} = \frac{P_{tx}}{PL} = \frac{P_{tx}}{d^3}$$

$$= \frac{1000W}{(100000)^3} = 10^{-12} W$$

Example 19.1 – cont.

- Since we know that k_a = 0.25, P_m = 0.1:

$$\frac{A_c^2}{2}\left\{1+k_a^2P_m\right\}=10^{-12}W$$

$$A_c^2=\frac{2*10^{-12}}{1+k_a^2P_m}$$

$$=\frac{2*10^{-12}}{1.006}$$

- The post detection SNR is

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}}=\frac{A_c^2k_a^2P_m}{2N_0W}$$

Example 19.1 – cont.

- The noise power spectral density is calculated as

$$N_0 = kT = 1.38*10^{-23}*1000$$
$$= 1.38*10^{-20}$$

- The post detection SNR is then:

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}} = \frac{A_c^2 k_a^2 P_m}{2N_0 W}$$
$$= \frac{2*10^{-12}*0.25^2*0.1}{2*1.38*10^{-20}*10000}$$
$$= 45.3$$

- Or in dB we have

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}} = 10\log_{10}(45.3) = 16dB$$

Example 19.2

- What is the maximum SNR we could achieve with LC-AM using the parameters from Example 19.1? What about DSB-SC?
- **SOLUTION:** The best SNR we can achieve with Large Carrier AM is to let $k_a = 1$:

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{LC-AM}} = \frac{A_c^{\,2} k_a^2 P_m}{2 N_0 W}$$

$$= \frac{1.8*10^{-12}*1*0.1}{2*1.38*10^{-20}*10000}$$

$$= 658$$

$$A_c^2 = \frac{2*10^{-12}}{1+k_a^2 P_m} = 1.8*10^{-12}$$

- Which is 28dB

Example 19.2 – cont.

- For DSBSC the received signal power is

$$\frac{A_c^2}{2} P_m = 10^{-12} W$$

$$A_c^2 = \frac{2 * 10^{-12}}{P_m}$$

$$= \frac{2 * 10^{-12}}{0.1} = 2 * 10^{-11}$$

- The post detection SNR is then

$$\left(\frac{S}{N}\right)_{\text{post}}^{\text{DSB}} = \frac{A_c^2 P_m}{2 N_0 W}$$

$$= \frac{2 * 10^{-11} * 0.1}{2 * 1.38 * 10^{-20} * 10000}$$

$$= 7246$$

or 38dB

Summary

- In this lecture we have studied the impact of noise on various AM radio systems.
- When using a coherent receiver (e.g., a product detector) a gain of 2 is achieved in post-detection SNR as compared to the pre-detection SNR
- Large Carrier AM, while providing for a very simple receiver, is not particularly power efficient.
  - This is reflected in the poor SNR for the same receiver noise as compared to DSBSC or SSB.
  - It can also be seen by comparing the post-detection SNR to that of a baseband reference system