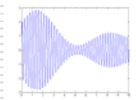
ECE3614
Introduction to
Communications Systems
Fall 2007

Instructor: Dr. R. Michael Buehrer

Lecture #2: Review of the Fourier Series



Overview

- The Objectives of Today's Lecture
  - Overview/Motivation for Fourier Theory
  - Review of the Fourier Series
  - Relationship to the Fourier Transform
- Reading
  - Appendix 2

Motivation

- If a system is linear, the response due to a sum of signals is the sum of the responses to each individual signal
- System analysis can be simplified by decomposing an input signal into a sum of simpler signals
  - The system output can then be found as the sum of the system responses to these simpler signals
- A physically meaningful way of decomposing signals is to represent them as a sum (or integral) of sinusoids
  - Periodic signals – Fourier Series
  - Aperiodic signals – Fourier Transform
    - Periodic signals can also be represented using the Fourier Transform
- This gives rise to the idea of the *frequency domain*

Fourier Theory

- Two basic types of signals
  - Power signals
  - Energy signals
- We can represent a signal in time or in frequency
- Fourier Representations
  - Fourier Series
    - Representation valid for all time if signal is periodic (i.e., power signals)
    - Representation is valid only over a certain interval for aperiodic signals
  - Fourier Transform
    - Applies directly to energy signals
    - Requires introduction of the impulse for application to power signals

Fourier Theory (cont.)

- Fourier Theory tells us that signals can be represented as weighted sums (or integrals) of sinusoids.
- The "amount" of each sinusoid is equivalent to the "frequency domain" information of a particular signal
- If the signal is periodic, the signal can be represented as an infinite sum of sinusoids whose frequencies are integer multiples of the fundamental frequency, f_o.
- If a signal is aperiodic we can take the limit of the Fourier Series as the period goes to infinity. The result is the Fourier Transform
- The Fourier Transform doesn't technically apply to periodic signals.
  - However we can create a FT through the use of the delta function

Motivation (revisited)

- Assume that I want to transmit a signal through a linear system
- I know that the system output is

$$y(t) = x(t) \otimes h(t)$$

- If I know the time domain description of the signal, *x(t)* and the impulse response of the channel *h(t)* can I mentally determine *y(t)*?

$$x(t) = \cos(200\pi t)$$

$$h(t) = \text{sinc}(50t)$$

$$y(t) = ?$$

Motivation (cont.)

- Now assume that I know the frequency domain description of the signal and the channel.

$$Y(f) = X(f)H(f)$$

- Knowing *X(f)* and *H(f)* can I easily determine *Y(f)*?

$$X(f) = \frac{1}{2}\{\delta(f-100) + \delta(f+100)\}$$

$$H(f) = \frac{1}{50}rect\left(\frac{f}{50}\right)$$

$$Y(f) = ?$$

In-class Drill

- Determine the output of a linear time-invariant system if the input and impulse response of the system are

Motivation (cont.)

- Many aspects of communication system analysis are simplified through the use of the frequency domain
- The Fourier Series represents any periodic signal as an infinite sum of sinusoids. The weights of the sinusoids provide "frequency domain" information of the signal
- Unfortunately, the Fourier Series is not applicable to energy signals (e.g., aperiodic signals)
- Thus, while we will review Fourier Series and the concept is important, we will typically use a more general technique of converting time domain signals into the frequency domain – the Fourier Transform

Continuous Time Fourier Series

- We can represent a periodic signal $x(t)$ with period T_0 exactly by the sum of complex sinusoids

$$x(t)=\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

where

$$c_n=\frac{1}{T_0}\int_{t_o}^{t_o+T_0} x(t)e^{-j2\pi n f_0 t}dt \qquad \boxed{f_0=\frac{1}{T_0}}$$

- The above integral must converge
- This is termed the Continuous Time Fourier Series or simply the Fourier Series
- We can represent the relationship between $x(t)$ and c_n as

$$x(t)\overset{FS}{\leftrightarrow} c_n$$

Example 2.1

- Let's examine a square wave which can be written as

$$x(t)=2\sum_{i=-\infty}^{\infty} rect\left(2t-2i-\frac{1}{2}\right)-1$$

- Examining one period (T_F = 1) 0<t<1 the Fourier Series can be written as

$$x(t)=\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

$$c_n=\frac{1}{T_o}\int_{t_o}^{t_o+T_0} x(t)e^{-j2\pi n f_o t}dt=\frac{1}{T_o}\int_{0}^{T_o/2} e^{-j2\pi n f_o t}dt-\frac{1}{T_o}\int_{T_o/2}^{T_o} e^{-j2\pi n f_o t}dt$$

Example 2.1 (cont.)

- Continuing ...

$$c_n=\frac{1}{T_o}\int_{0}^{T_o/2} e^{-j2\pi n f_o t}dt-\frac{1}{T_o}\int_{T_o/2}^{T_o} e^{-j2\pi n f_o t}dt$$

$$=\frac{1}{T_o}\frac{1}{-j2\pi n f_o}\left\{e^{-j2\pi n f_o T_o/2}-1\right\}-\frac{1}{T_o}\frac{1}{-j2\pi n f_o}\left\{e^{-j2\pi n f_o T_o}-e^{-j2\pi n f_o T_o/2}\right\}$$

$$=\frac{1}{j2\pi n}\left\{e^{-j2\pi n}-2e^{-j\pi n}+1\right\}$$

- We can simplify this for n even and n odd...

$$c_n=\begin{cases}0 & n \text{ even}\\ -\dfrac{2j}{\pi n} & n \text{ odd}\end{cases}$$

Example 2.1 (cont.)

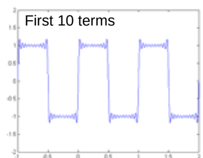
- Since the terms for positive and negative *n* can be combined using Euler's Identity we can write the equation for the original signal as

$$x(t)=\sum_{n=1}^{\infty}\frac{2-2\cos(\pi n)}{\pi n}\sin(2\pi n f_0 t)$$

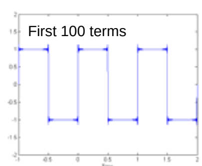
- Since the even terms are zero we could also rewrite this as

$$x(t)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin(2\pi(2n-1)f_0 t)$$

Example 2.1 - Plots



- As we continue to add the sinusoidal terms our representation is closer and closer to the original signal *over the interval of interest.*
- When the number of terms goes to infinity the representation is *exact*



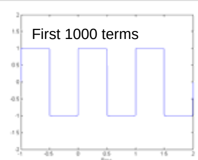


Table of Fourier Series Coefficients

Time Domain Function	Fourier Series Coefficients
$x(t)=e^{j2\pi f_o t}$	$c_n=\delta[n-1]$
$x(t)=\sin(2\pi f_o t)$	$c_n=\frac{j}{2}\{\delta[n+1]-\delta[n-1]\}$
$x(t)=\cos(2\pi f_o t)$	$c_n=\frac{1}{2}\{\delta[n+1]+\delta[n-1]\}$
$x(t)=1$	$c_n=\delta[n]$
$x(t)=\sum_{n=-\infty}^{\infty} rect\left(\frac{t-n\tau}{T}\right)$	$c_n=\frac{T}{\tau}\text{sinc}\left(\frac{T}{\tau}n\right)$
$x(t)=\sum_{n=-\infty}^{\infty} tri\left(\frac{t-n\tau}{T}\right)$	$c_n=\frac{T}{\tau}\text{sinc}^2\left(\frac{T}{\tau}n\right)$

Properties of the Fourier Series

- Important Properties of the Fourier Series
  - Linearity
    - The FS of a sum of two functions is the sum of the individual FS
  - Time-shifting
    - Time-shift corresponds to phase shift in FS coefficients
  - Frequency-shifting
    - Frequency shift corresponds to multiplying the time signal by a complex sinusoid
  - Time reversal
  - Time scaling

Properties of the Fourier Series

- Time differentiation
  - Differentiating in time corresponds to multiplying the FS coefficients by *f*
- Time integration
  - Integrating in time corresponds to dividing the FS coefficients by *f*
- Multiplication-Convolution
  - Convolving two functions in time corresponds to a convolution sum of Fourier Series coefficients
- Parseval's Theorem – Avg. power is the same in both domains

$$\underbrace{\frac{1}{T_0}\int_{T_0}\left|x(t)\right|^2 dt}_{\text{Average power of } x(t)} = \sum_{n=-\infty}^{\infty}\left|c_n\right|^2$$

Use of Tables

- In general we use properties and tables of pairs to find new pairs.

Property	
Conjugation	$x^*(t) \xleftrightarrow{FS} c_{-n}{}^*$
Linearity	$\alpha x(t)+\beta y(t) \xleftrightarrow{FS} \alpha c_n^x + \beta c_n^y$
Time-shifting	$x(t-t_o) \xleftrightarrow{FS} e^{-j2\pi n f_0 t_o} c_n$
Frequency-shifting	$e^{j2\pi n_o f_0 t} x(t) \xleftrightarrow{FS} c_{n-n_o}$
Time reversal	$x(-t) \xleftrightarrow{FS} c_{-n}$
Time-derivative	$\frac{d}{dt}\{x(t)\} \xleftrightarrow{FS} (j2\pi n f_0) c_n$
Time-integration	$\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{FS} \frac{1}{j2\pi n f_0} c_n$

Properties (cont.)

Property	
Time-scaling (integer a) $T_F = T_o$	$x(at) \overset{FS}{\leftrightarrow} \begin{cases} c_{n/a} & \frac{n}{a} = \text{integer} \\ 0 & \text{else} \end{cases}$
Multiplication	$x(t)\,y(t) \xleftrightarrow{FS} \sum_{q=-\infty}^{\infty} c_q^y c_{n-q}^x$
Convolution	$x(t) \otimes y(t) \xleftrightarrow{FS} T_0 c_n^x c_n^y$
Conjugation	$x^*(t) \xleftrightarrow{FS} c_{-n}^*$

Example 2.2

- Find the Continuous Time Fourier Series of

$$x(t) = \cos(50\pi t - \pi/4)$$

for $T_0 = T_o = 1/f_o = 1/25$

We can use

$$\cos(2\pi f_o t) \overset{FS}{\leftrightarrow} \frac{1}{2}\delta[n-1] + \frac{1}{2}\delta[n+1]$$

and $$x(t - t_o) \xleftrightarrow{FS} e^{-j2\pi n f_0 t_o} c_n$$

Example 2.2 (cont.)

- Re-writing the original equation

$$\begin{aligned} x(t) &= \cos(50\pi t - \pi/4) \\ &= \cos\left(50\pi\left(t - \frac{\pi/4}{50\pi}\right)\right) \\ &= \cos\left(50\pi\left(t - \frac{1}{200}\right)\right) \end{aligned}$$

$$\begin{aligned} c_n &= e^{-j2\pi n * 25(1/200)}\left\{\frac{1}{2}\delta[n-1] + \frac{1}{2}\delta[n+1]\right\} \\ &= e^{-j2\pi n/8}\left\{\frac{1}{2}\delta[n-1] + \frac{1}{2}\delta[n+1]\right\} \end{aligned}$$

Example 2.2 (cont.)

- Continuing $c_n = e^{-j2\pi n/8}\left\{\frac{1}{2}\delta[n-1]+\frac{1}{2}\delta[n+1]\right\}$

$$= e^{-j2\pi/8}\frac{1}{2}\delta[n-1]+e^{j2\pi/8}\frac{1}{2}\delta[n+1]$$

$$= e^{-j\pi/4}\frac{1}{2}\delta[n-1]+e^{j\pi/4}\frac{1}{2}\delta[n+1]$$

$$= \frac{(1-j)}{\sqrt{2}}\frac{1}{2}\delta[n-1]+\frac{(1+j)}{\sqrt{2}}\frac{1}{2}\delta[n+1]$$

$$\cos(50\pi t-\pi/4)\xleftrightarrow{FS}\frac{(1-j)}{\sqrt{2}}\frac{1}{2}\delta[n-1]+\frac{(1+j)}{\sqrt{2}}\frac{1}{2}\delta[n+1]$$

Energy Signals

- The Fourier Series applies to *periodic signals* which are also *power signals.*
- However, we would like to analyze both power signals and *energy signals.*
- Thus, we need a more powerful analysis tool.
- The Fourier Transform is the answer

The Fourier Transform

- Let's examine the Fourier Series representation of a periodic signal with fundamental frequency

$$\Delta f = f_0 = 1/T_0$$

$$x(t)=\sum_{n=-\infty}^{\infty} c_n e^{j2\pi n\Delta f t} \qquad c_n = \frac{1}{T_o}\int_{T_o} x(t)e^{-j2\pi n\Delta f t}dt$$

- Substituting c_n into the equation for $x(t)$

$$x(t)=\sum_{n=-\infty}^{\infty}\left\{\frac{1}{T_0}\int_{T_0} x(\tau)e^{-j2\pi n\Delta f\tau}d\tau\right\}e^{j2\pi n\Delta f t}$$

- Since the exact starting and stopping points for the integration are arbitrary (provided that we integrate over a period), let us define the integration period to be

$$-T_0/2 \le t \le T_0/2$$

The Fourier Transform (cont.)

- Using this region of integration

$$x(t)=\sum_{n=-\infty}^{\infty}\left\{\Delta f\int_{-T_0/2}^{T_0/2}x(\tau)e^{-j2\pi n\Delta f\tau}d\tau\right\}e^{j2\pi n\Delta ft}$$

- Now in the limit as T_0 approaches infinity, the periodic signal becomes aperiodic, Δf approaches the differential df, $n\Delta f$ becomes a continuous variable f and

$$x(t)=\lim_{T_F\to\infty}\left\{\sum_{n=-\infty}^{\infty}\left\{\Delta f\int_{-T_0/2}^{T_0/2}x(\tau)e^{-j2\pi n\Delta f\tau}d\tau\right\}e^{j2\pi n\Delta ft}\right\}$$

$$=\int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty}x(\tau)e^{-j2\pi f\tau}d\tau\right\}e^{j2\pi ft}df$$

The Fourier Transform

- We then define the Continuous Time Fourier Transform as

$$X(f)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt$$

$$=F\{x(t)\}$$

and the original signal can be written in terms of the Fourier Transform as

$$x(t)=\int_{-\infty}^{\infty}X(f)e^{j2\pi ft}df$$

$$=F^{-1}\{X(f)\}$$

This is typically called the *inverse Fourier Transform*

The Frequency Domain

- The original signal *x(t)* is said to be in the *time domain* since its argument represents time
- The Fourier Transform *X(f)* representation is said to be in the *frequency domain* since its argument *f* represents frequency
- Notes:
  - Frequency is the reciprocal of time
  - The Fourier Transform is referred to as an *analysis* of the signal *x(t)* since it extracts the components of *x(t)* at each value of *f*
  - The Inverse Fourier Transform is referred to as *synthesis* since it recombines the components *X(f)* to obtain the original signal *x(t)*
  - The physical meaning of X(f) depends on the meaning of x(t). If x(t) has units of volts, X(f) has units volts/Hz.
    - Thus it represents how much of the overall voltage signal is present at each frequency.

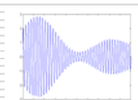
Summary

- Today we have begun our discussion of the frequency domain by briefly reviewing the Fourier Series
- The Fourier Series allows us to represent any periodic signal (power signals) as a weighted sum of sinusoids.
  - The weights associated with each sinusoid can be interpreted as the frequency domain information
- The Fourier Transform is a generalization of the Fourier Series that allows us to analyze aperiodic signals (such as energy signals)
  - We will spend more time on this tool next lecture.

Supplemental Material

Instructor: Dr. R. Michael Buehrer

Lecture #2: Review of the Fourier Series



Trigonometric Fourier Series

- The form of the Fourier Series discussed in this lecture is termed the *complex form* of the Fourier Series which is the most general.
- For *real* functions, we can also define a *trigonometric form* of the Fourier Series. This is described in the following slide and demonstrated in the following example

Trigonometric Fourier Series

- Recall that $\cos t = \frac{e^{jt} + e^{-jt}}{2}$ $\sin t = \frac{e^{jt} - e^{-jt}}{2j}$ $e^{jt} = \cos t + j \sin t$
- Substituting these relationships into the previous equation (and assuming *x(t)* is real) we have

$$x(t) = c_0 + \sum_{n=1}^{\infty} \left\{ 2\,\mathrm{Re}(c_n) \cos(2\pi n f_0 t) - 2\,\mathrm{Im}(c_n) \sin(2\pi n f_0 t) \right\}$$

which can be re-written as

$$x(t) = a_0 + 2\sum_{n=1}^{\infty} \left\{ a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t) \right\}$$

with

$$a_n = \mathrm{Re}\{c_n\} = \mathrm{Re}\left\{ \frac{1}{T_0} \int_{t_o}^{t_o+T_0} x(t) e^{-j2\pi n f_0 t} dt \right\} = \frac{1}{T_0} \int_{t_o}^{t_o+T_0} x(t) \cos(2\pi n f_0 t) dt$$

$$b_n = -\mathrm{Im}\{c_n\} = -\mathrm{Im}\left\{ \frac{1}{T_F} \int_{t_o}^{t_o+T_0} x(t) e^{-j2\pi n f_0 t} dt \right\} = \frac{1}{T_0} \int_{t_o}^{t_o+T_0} x(t) \sin(2\pi n f_0 t) dt$$

Example 2.3 – Trig. FS of Ex. 2.1

- Let's examine a square wave which can be written as

$$x(t) = 2\sum_{i=-\infty}^{\infty} rect\left(2t - 2i - \frac{1}{2}\right) - 1$$

- Examining one period (T_F = 1) 0<t<1 the Trigonometric Fourier Series can be written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \cos(2\pi k f_0 t) + b_n \sin(2\pi n f_0 t) \right\}$$

- First note that $f_0 = 1/T_0 = 1$
- The first term a_0 is

$$a_0 = \frac{1}{T_F} \int_{t_o}^{t_o+T_0} x(t) \cos(0) dt = \frac{1}{T_F} \int_0^{0.5} dt - \frac{1}{T_F} \int_{0.5}^{1} dt = 0$$

Example 2.3 (cont.)

- For the remaining cosine terms and the sine terms we have

$$\begin{aligned} a_n &= \int_0^{0.5} \cos(2\pi n t) dt - \int_{0.5}^{1} \cos(2\pi n t) dt \\ &= \frac{1}{2\pi n} \sin(2\pi n t)\Big|_0^{0.5} - \frac{1}{2\pi n} \sin(2\pi n t)\Big|_{0.5}^{1} \\ &= \frac{1}{2\pi n}\left(\sin(n\pi) - \sin(0) - \sin(2n\pi) + \sin(n\pi)\right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} b_n &= \int_0^{0.5} \sin(2\pi n t) dt - \int_{0.5}^{1} \sin(2\pi n t) dt \\ &= -\frac{1}{2\pi n}\left(\cos(2\pi n t)\right)\Big|_0^{0.5} + \frac{1}{2\pi n}\left(\cos(2\pi n t)\right)\Big|_{0.5}^{1} \\ &= \frac{1}{2\pi n}\left(-\cos(\pi n) + \cos(0) + \cos(2\pi n) - \cos(\pi n)\right) \\ &= \frac{1}{\pi n}\left(1 - \cos(\pi n)\right) \end{aligned}$$

Example 2.3 (cont.)

- Thus, we can write the sine terms as

$$b_n = \frac{1}{\pi n}\left(1-\cos(\pi n)\right) = \begin{cases} 0 & n = even \\ \frac{2}{\pi n} & n = odd \end{cases}$$

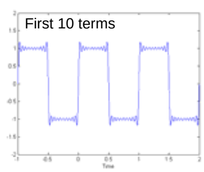
- Thus we have

$$x(t) = \sum_{n=1}^{\infty} \frac{2-2\cos(\pi n)}{\pi n} \sin(2\pi n f_0 t)$$

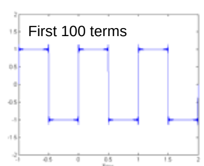
- Since the even terms are zero we can rewrite this as

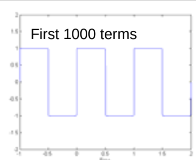
$$x(t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin(2\pi(2n-1) f_0 t)$$

Example 2.3 - Plots



- As we continue to add the sinusoidal terms our representation is closer and closer to the original signal *over the interval of interest.*
- When the number of terms goes to infinity the representation is *exact*





Example 2.3 - check

- Does this agree with our previous result?

$$a_0 = c_0 \qquad b_0 = 0$$

$$a_n = \frac{c_n + c_n^*}{2} \qquad b_n = \frac{j\left(c_n - c_n^*\right)}{2} \qquad n = 1,2,3,\ldots$$

$$a_n = \begin{cases} 0 & n \text{ even} \\ \frac{1}{2}\left(-\frac{2j}{\pi n} + \frac{2j}{\pi n}\right) = 0 & n \text{ odd} \end{cases}$$

$$b_n = \begin{cases} 0 & n \text{ even} \\ \frac{j}{2}\left(-\frac{2j}{\pi n} - \frac{2j}{\pi n}\right) = \frac{2}{\pi n} & n \text{ odd} \end{cases}$$

Agrees!

Impact of the Signal Period

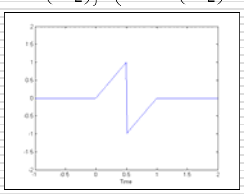
- Aperiodic signals
  - A Fourier Series representation can be calculated for an aperiodic signal, however it will only be valid over the interval of the original signal
  - See next example
- We have assumed thus far that we use the period of the signal as the fundamental interval of the Fourier Series
  - This is not necessary but is the most convenient
  - We can actually use any integer multiple of the period as the interval of interest
  - If we change the interval of interest from T_o to kT_o, for integer k, the Fourier Series coefficients are related to the original coefficients as

$$\tilde{c}_n = \begin{cases} c_{n/k} & n/k = \text{integer} \\ 0 & n/k \neq \text{integer} \end{cases}$$

Example 2.4 – Aperiodic signals

- Let's consider a sawtooth waveform

$$x(t) = 2*ramp(t)\left\{u(t) - u\left(t-\frac{1}{2}\right)\right\} + \left(2*ramp\left(t-\frac{1}{2}\right)-2\right)\left\{u\left(t-\frac{1}{2}\right) - u(t-1)\right\}$$



Note that this signal is NOT periodic

- Define the time interval to be 0<t<1
- $T_0 = 1$, $f_0 = 1$

Example 2.4 (cont.)

- The trigonometric Fourier Series is

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left\{a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\right\}$$

- Looking at the cosine terms

$$a_n = \frac{1}{T_0}\int_{t_o}^{t_o+T_0} x(t)\cos(2\pi n f_0 t)\,dt$$
$$= \int_0^{0.5} 2t\cos(2\pi nt)\,dt + \int_{0.5}^{1}(2t-2)\cos(2\pi nt)\,dt$$
$$= \frac{2}{(2\pi n)^2}\left[\cos(2\pi nt) + 2\pi kt\sin(2\pi nt)\right]\Big|_0^{0.5} + \frac{2}{(2\pi n)^2}\left[\cos(2\pi nt) + 2\pi nt\sin(2\pi nt)\right]\Big|_{0.5}^{1}$$
$$-\frac{2}{2\pi n}\sin(2\pi nt)\Big|_{0.5}^{1}$$
$$= \frac{2}{(2\pi k)^2}\left[\cos(2\pi n) + 2\pi n\sin(2\pi n) - \cos(0) + \sin(0)\right] - \frac{2}{2\pi n}\left(\sin(2\pi n) - \sin(\pi n)\right)$$
$$= 0$$

0 0 0 0

Example 2.4 (cont.)

- Looking at the sin terms

$$
\begin{aligned}
b_n &= \frac{1}{T_F}\int_{t_o}^{t_o+T_0} x(t)\sin(2\pi n f_0 t)\,dt \\
&= \int_0^{0.5} 2t\sin(2\pi nt)\,dt + \int_{0.5}^{1}(2t-2)\sin(2\pi nt)\,dt \\
&= \frac{2}{(2\pi n)^2}\left[\sin(2\pi nt) - 2\pi nt\cos(2\pi nt)\right]\Big|_0^{0.5} + \frac{2}{(2\pi n)^2}\left[\sin(2\pi nt) - 2\pi nt\cos(2\pi nt)\right]\Big|_{0.5}^{1} \\
&\quad + \frac{2}{2\pi n}\cos(2\pi nt)\Big|_{0.5}^{1} \\
&= \frac{2}{(2\pi n)^2}\left[\sin(2\pi n) - 2\pi n\cos(2\pi n) - \sin(0) + \cos(0)\right] + \frac{2}{2\pi n}\left(\cos(2\pi n) - \cos(\pi n)\right) \\
&= -\frac{1}{\pi n}\cos(2\pi n) + \frac{1}{\pi n}\cos(2\pi n) - \frac{1}{\pi n}\cos(\pi n) \\
&= -\frac{1}{\pi n}\cos(\pi n) \\
&= \begin{cases} -\dfrac{1}{\pi n} & n = even \\ \dfrac{1}{\pi n} & n = odd \end{cases}
\end{aligned}
$$

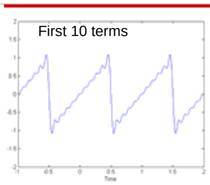
Example 2.4 (cont.)

- Summarizing we can write our Continuous Time Fourier Series as

$$x(t) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{\pi n}\sin(2\pi nt)$$

- Recalling that the signal is aperiodic, does this representation hold?
- From the plots on the next page, we see that the Fourier Series representation is *periodic* even though the original signal is not.
  - Thus, they are equivalent only over the interval defined for the original signal. i.e., 0<t<1

Example 2.4 (final)



- As we continue to add the sinusoidal terms our representation is closer and closer to the original signal but only *over the interval 0<t<1.*
- When the number of terms goes to infinity the representation is exact but only *over the interval 0<t<1.*