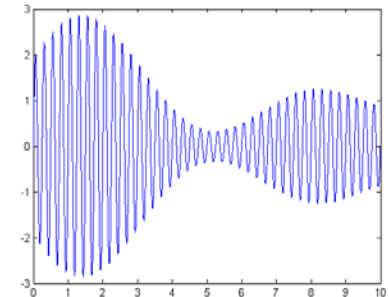


ECE3614

Introduction to Communications Systems

Fall 2007

Instructor: Dr. R. Michael Buehrer
Lecture #2: Review of the Fourier
Series



Overview

- The Objectives of Today's Lecture
 - Overview/Motivation for Fourier Theory
 - Review of the Fourier Series
 - Relationship to the Fourier Transform

- Reading
 - Appendix 2

Motivation

- If a system is linear, the response due to a sum of signals is the sum of the responses to each individual signal
- System analysis can be simplified by decomposing an input signal into a sum of simpler signals
 - The system output can then be found as the sum of the system responses to these simpler signals
- A physically meaningful way of decomposing signals is to represent them as a sum (or integral) of sinusoids
 - Periodic signals – Fourier Series
 - Aperiodic signals – Fourier Transform
 - Periodic signals can also be represented using the Fourier Transform
- This gives rise to the idea of the *frequency domain*

Fourier Theory

- Two basic types of signals
 - Power signals
 - Energy signals
- We can represent a signal in time or in frequency
- Fourier Representations
 - Fourier Series
 - Representation valid for all time if signal is periodic (i.e., power signals)
 - Representation is valid only over a certain interval for aperiodic signals
 - Fourier Transform
 - Applies directly to energy signals
 - Requires introduction of the impulse for application to power signals

Fourier Theory (cont.)

- Fourier Theory tells us that signals can be represented as weighted sums (or integrals) of sinusoids.
- The “amount” of each sinusoid is equivalent to the “frequency domain” information of a particular signal
- If the signal is periodic, the signal can be represented as an infinite sum of sinusoids whose frequencies are integer multiples of the fundamental frequency, f_0 .
- If a signal is aperiodic we can take the limit of the Fourier Series as the period goes to infinity. The result is the Fourier Transform
- The Fourier Transform doesn't technically apply to periodic signals.
 - However we can create a FT through the use of the delta function

Motivation (revisited)

- Assume that I want to transmit a signal through a linear system
- I know that the system output is
$$y(t) = x(t) \otimes h(t)$$
- If I know the time domain description of the signal, $x(t)$ and the impulse response of the channel $h(t)$ can I mentally determine $y(t)$?

$$x(t) = \cos(200\pi t)$$

$$h(t) = \text{sinc}(50t)$$

$$y(t) = ?$$

Motivation (cont.)

- Now assume that I know the frequency domain description of the signal and the channel.

$$Y(f) = X(f)H(f)$$

- Knowing $X(f)$ and $H(f)$ can I easily determine $Y(f)$?

$$X(f) = \frac{1}{2} \{ \delta(f - 100) + \delta(f + 100) \}$$

$$H(f) = \frac{1}{50} \text{rect}\left(\frac{f}{50}\right)$$

$$Y(f) = ?$$

In-class Drill

- Determine the output of a linear time-invariant system if the input and impulse response of the system are

Motivation (cont.)

- Many aspects of communication system analysis are simplified through the use of the frequency domain
- The Fourier Series represents any periodic signal as an infinite sum of sinusoids. The weights of the sinusoids provide “frequency domain” information of the signal
- Unfortunately, the Fourier Series is not applicable to energy signals (e.g., aperiodic signals)
- Thus, while we will review Fourier Series and the concept is important, we will typically use a more general technique of converting time domain signals into the frequency domain – the Fourier Transform

Continuous Time Fourier Series

- We can represent a periodic signal $x(t)$ with period T_0 exactly by the sum of complex sinusoids

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

where

$$c_n = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) e^{-j2\pi n f_0 t} dt$$

$$f_0 = \frac{1}{T_0}$$

- The above integral must converge
- This is termed the Continuous Time Fourier Series or simply the Fourier Series
- We can represent the relationship between $x(t)$ and c_n as

$$x(t) \overset{FS}{\leftrightarrow} c_n$$

Example 2.1

- Let's examine a square wave which can be written as

$$x(t) = 2 \sum_{i=-\infty}^{\infty} \text{rect}\left(2t - 2i - \frac{1}{2}\right) - 1$$

- Examining one period ($T_F = 1$) $0 < t < 1$ the Fourier Series can be written as

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

$$c_n = \frac{1}{T_o} \int_{t_o}^{t_o+T_o} x(t) e^{-j2\pi n f_0 t} dt = \frac{1}{T_o} \int_0^{T_o/2} e^{-j2\pi n f_0 t} dt - \frac{1}{T_o} \int_{T_o/2}^{T_o} e^{-j2\pi n f_0 t} dt$$

Example 2.1 (cont.)

□ Continuing ...

$$\begin{aligned}c_n &= \frac{1}{T_o} \int_0^{T_o/2} e^{-j2\pi n f_o t} dt - \frac{1}{T_o} \int_{T_o/2}^{T_o} e^{-j2\pi n f_o t} dt \\&= \frac{1}{T_o} \frac{1}{-j2\pi n f_o} \left\{ e^{-j2\pi n f_o T_o/2} - 1 \right\} - \frac{1}{T_o} \frac{1}{-j2\pi n f_o} \left\{ e^{-j2\pi n f_o T_o} - e^{-j2\pi n f_o T_o/2} \right\} \\&= \frac{1}{j2\pi n} \left\{ e^{-j2\pi n} - 2e^{-j\pi n} + 1 \right\}\end{aligned}$$

□ We can simplify this for n even and n odd...

$$c_n = \begin{cases} 0 & n \text{ even} \\ -\frac{2j}{\pi n} & n \text{ odd} \end{cases}$$

Example 2.1 (cont.)

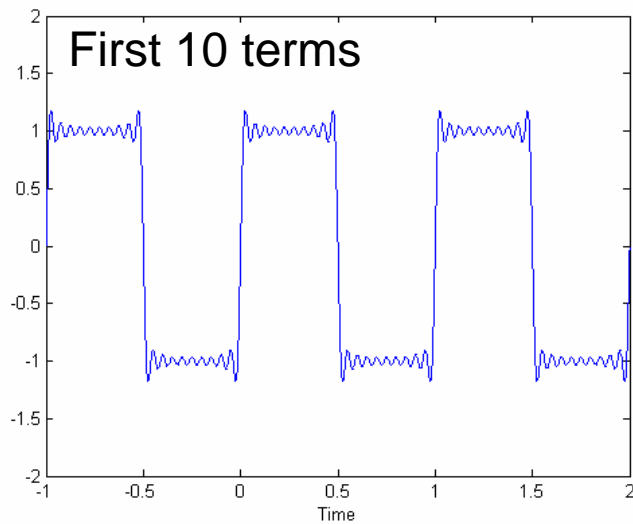
- Since the terms for positive and negative n can be combined using Euler's Identity we can write the equation for the original signal as

$$x(t) = \sum_{n=1}^{\infty} \frac{2 - 2\cos(\pi n)}{\pi n} \sin(2\pi n f_0 t)$$

- Since the even terms are zero we could also rewrite this as

$$x(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin(2\pi(2n-1)f_0 t)$$

Example 2.1 - Plots



- As we continue to add the sinusoidal terms our representation is closer and closer to the original signal *over the interval of interest*.
- When the number of terms goes to infinity the representation is *exact*

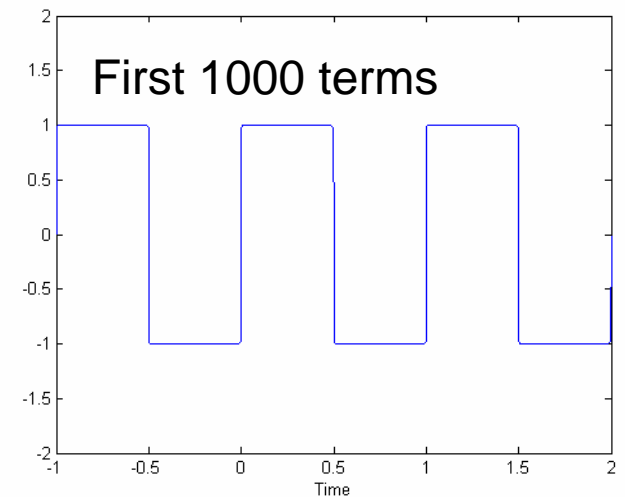
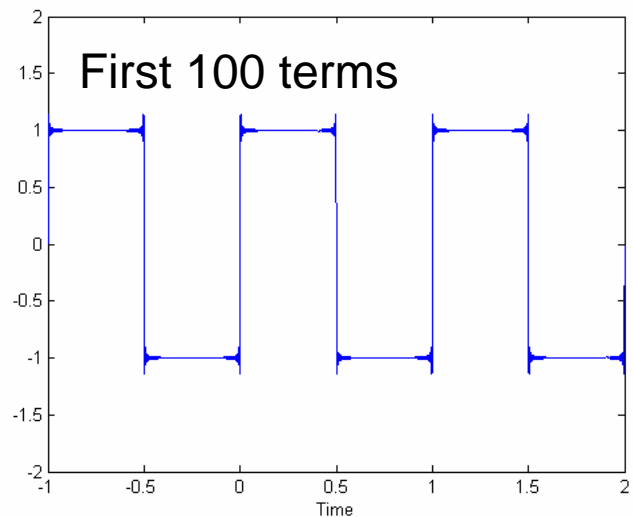


Table of Fourier Series Coefficients

Time Domain Function	Fourier Series Coefficients
$x(t) = e^{j2\pi f_o t}$	$c_n = \delta[n-1]$
$x(t) = \sin(2\pi f_o t)$	$c_n = \frac{j}{2} \{ \delta[n+1] - \delta[n-1] \}$
$x(t) = \cos(2\pi f_o t)$	$c_n = \frac{1}{2} \{ \delta[n+1] + \delta[n-1] \}$
$x(t) = 1$	$c_n = \delta[n]$
$x(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-n\tau}{T}\right)$	$c_n = \frac{T}{\tau} \text{sinc}\left(\frac{T}{\tau}n\right)$
$x(t) = \sum_{n=-\infty}^{\infty} \text{tri}\left(\frac{t-n\tau}{T}\right)$	$c_n = \frac{T}{\tau} \text{sinc}^2\left(\frac{T}{\tau}n\right)$

Properties of the Fourier Series

□ Important Properties of the Fourier Series

■ Linearity

- The FS of a sum of two functions is the sum of the individual FS

■ Time-shifting

- Time-shift corresponds to phase shift in FS coefficients

■ Frequency-shifting

- Frequency shift corresponds to multiplying the time signal by a complex sinusoid

■ Time reversal

■ Time scaling

Properties of the Fourier Series

- Time differentiation
 - Differentiating in time corresponds to multiplying the FS coefficients by f
- Time integration
 - Integrating in time corresponds to dividing the FS coefficients by f
- Multiplication-Convolution
 - Convoluting two functions in time corresponds to a convolution sum of Fourier Series coefficients
- Parseval's Theorem – Avg. power is the same in both domains

$$\underbrace{\frac{1}{T_0} \int_{T_0} |x(t)|^2 dt}_{\text{Average power of } x(t)} = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Average power of $x(t)$

Use of Tables

- In general we use properties and tables of pairs to find new pairs.

Property	
Conjugation	$x^*(t) \xleftrightarrow{FS} c_{-n}^*$
Linearity	$\alpha x(t) + \beta y(t) \xleftrightarrow{FS} \alpha c_n^x + \beta c_n^y$
Time-shifting	$x(t - t_o) \xleftrightarrow{FS} e^{-j2\pi n f_0 t_o} c_n$
Frequency-shifting	$e^{j2\pi n_o f_0 t} x(t) \xleftrightarrow{FS} c_{n-n_o}$
Time reversal	$x(-t) \xleftrightarrow{FS} c_{-n}$
Time-derivative	$\frac{d}{dt} \{x(t)\} \xleftrightarrow{FS} (j2\pi n f_0) c_n$
Time-integration	$\int_{-\infty}^t x(\tau) d\tau \xleftrightarrow{FS} \frac{1}{j2\pi n f_0} c_n$

Properties (cont.)

Property	
Time-scaling (integer a) $T_F = T_O$	$x(at) \xleftrightarrow{FS} \begin{cases} c_{n/a} & \frac{n}{a} = \text{integer} \\ 0 & \text{else} \end{cases}$
Multiplication	$x(t)y(t) \xleftrightarrow{FS} \sum_{q=-\infty}^{\infty} c_q^y c_{n-q}^x$
Convolution	$x(t) \otimes y(t) \xleftrightarrow{FS} T_0 c_n^x c_n^y$
Conjugation	$x^*(t) \xleftrightarrow{FS} c_{-n}^*$

Example 2.2

□ Find the Continuous Time Fourier Series of

$$x(t) = \cos(50\pi t - \pi/4)$$

for $T_0 = T_o = 1/f_o = 1/25$

We can use

$$\cos(2\pi f_o t) \xleftrightarrow{FS} \frac{1}{2} \delta[n-1] + \frac{1}{2} \delta[n+1]$$

and

$$x(t - t_o) \xleftrightarrow{FS} e^{-j2\pi n f_o t_o} C_n$$

Example 2.2 (cont.)

□ Re-writing the original equation

$$\begin{aligned}x(t) &= \cos(50\pi t - \pi/4) \\ &= \cos\left(50\pi\left(t - \frac{\pi/4}{50\pi}\right)\right) \\ &= \cos\left(50\pi\left(t - \frac{1}{200}\right)\right)\end{aligned}$$

$$\begin{aligned}c_n &= e^{-j2\pi n \cdot 25(1/200)} \left\{ \frac{1}{2} \delta[n-1] + \frac{1}{2} \delta[n+1] \right\} \\ &= e^{-j2\pi n/8} \left\{ \frac{1}{2} \delta[n-1] + \frac{1}{2} \delta[n+1] \right\}\end{aligned}$$

Example 2.2 (cont.)

□ Continuing

$$\begin{aligned}c_n &= e^{-j2\pi n/8} \left\{ \frac{1}{2} \delta[n-1] + \frac{1}{2} \delta[n+1] \right\} \\ &= e^{-j2\pi/8} \frac{1}{2} \delta[n-1] + e^{j2\pi/8} \frac{1}{2} \delta[n+1] \\ &= e^{-j\pi/4} \frac{1}{2} \delta[n-1] + e^{j\pi/4} \frac{1}{2} \delta[n+1] \\ &= \frac{(1-j)}{\sqrt{2}} \frac{1}{2} \delta[n-1] + \frac{(1+j)}{\sqrt{2}} \frac{1}{2} \delta[n+1]\end{aligned}$$

$$\cos(50\pi t - \pi/4) \xleftrightarrow{FS} \frac{(1-j)}{\sqrt{2}} \frac{1}{2} \delta[n-1] + \frac{(1+j)}{\sqrt{2}} \frac{1}{2} \delta[n+1]$$

Energy Signals

- ❑ The Fourier Series applies to *periodic signals* which are also *power signals*.
- ❑ However, we would like to analyze both power signals and *energy signals*.
- ❑ Thus, we need a more powerful analysis tool.
- ❑ The Fourier Transform is the answer

The Fourier Transform

- Let's examine the Fourier Series representation of a periodic signal with fundamental frequency

$$\Delta f = f_0 = 1/T_0$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n \Delta f t} \qquad c_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-j2\pi n \Delta f t} dt$$

- Substituting c_n into the equation for $x(t)$

$$x(t) = \sum_{n=-\infty}^{\infty} \left\{ \frac{1}{T_0} \int_{T_0} x(\tau) e^{-j2\pi n \Delta f \tau} d\tau \right\} e^{j2\pi n \Delta f t}$$

- Since the exact starting and stopping points for the integration are arbitrary (provided that we integrate over a period), let us define the integration period to be

$$-T_0/2 \leq t \leq T_0/2$$

The Fourier Transform (cont.)

- Using this region of integration

$$x(t) = \sum_{n=-\infty}^{\infty} \left\{ \Delta f \int_{-T_0/2}^{T_0/2} x(\tau) e^{-j2\pi n\Delta f\tau} d\tau \right\} e^{j2\pi n\Delta ft}$$

- Now in the limit as T_0 approaches infinity, the periodic signal becomes aperiodic, Δf approaches the differential df , $n\Delta f$ becomes a continuous variable f and

$$\begin{aligned} x(t) &= \lim_{T_F \rightarrow \infty} \left\{ \sum_{n=-\infty}^{\infty} \left\{ \Delta f \int_{-T_0/2}^{T_0/2} x(\tau) e^{-j2\pi n\Delta f\tau} d\tau \right\} e^{j2\pi n\Delta ft} \right\} \\ &= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} x(\tau) e^{-j2\pi f\tau} d\tau \right\} e^{j2\pi ft} df \end{aligned}$$

The Fourier Transform

- We then define the Continuous Time Fourier Transform as

$$\begin{aligned} X(f) &= \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \\ &= F\{x(t)\} \end{aligned}$$

and the original signal can be written in terms of the Fourier Transform as

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \\ &= F^{-1}\{X(f)\} \end{aligned}$$

This is typically called the *inverse Fourier Transform*

The Frequency Domain

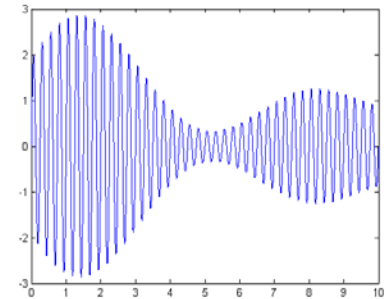
- The original signal $x(t)$ is said to be in the *time domain* since its argument represents time
- The Fourier Transform $X(f)$ representation is said to be in the *frequency domain* since its argument f represents frequency
- Notes:
 - Frequency is the reciprocal of time
 - The Fourier Transform is referred to as an *analysis* of the signal $x(t)$ since it extracts the components of $x(t)$ at each value of f
 - The Inverse Fourier Transform is referred to as *synthesis* since it recombines the components $X(f)$ to obtain the original signal $x(t)$
 - The physical meaning of $X(f)$ depends on the meaning of $x(t)$. If $x(t)$ has units of volts, $X(f)$ has units volts/Hz.
 - Thus it represents how much of the overall voltage signal is present at each frequency.

Summary

- Today we have begun our discussion of the frequency domain by briefly reviewing the Fourier Series
- The Fourier Series allows us to represent any periodic signal (power signals) as a weighted sum of sinusoids.
 - The weights associated with each sinusoid can be interpreted as the frequency domain information
- The Fourier Transform is a generalization of the Fourier Series that allows us to analyze aperiodic signals (such as energy signals)
 - We will spend more time on this tool next lecture.

Supplemental Material

Instructor: Dr. R. Michael Buehrer
Lecture #2: Review of the Fourier
Series



Trigonometric Fourier Series

- The form of the Fourier Series discussed in this lecture is termed the *complex form* of the Fourier Series which is the most general.
- For *real* functions, we can also define a *trigonometric form* of the Fourier Series. This is described in the following slide and demonstrated in the following example

Trigonometric Fourier Series

□ Recall that $\cos t = \frac{e^{jt} + e^{-jt}}{2}$ $\sin t = \frac{e^{jt} - e^{-jt}}{2j}$ $e^{jt} = \cos t + j \sin t$

□ Substituting these relationships into the previous equation (and assuming $x(t)$ is real) we have

$$x(t) = c_0 + \sum_{n=1}^{\infty} \left\{ 2 \operatorname{Re}(c_n) \cos(2\pi n f_0 t) - 2 \operatorname{Im}(c_n) \sin(2\pi n f_0 t) \right\}$$

which can be re-written as

$$x(t) = a_0 + 2 \sum_{n=1}^{\infty} \left\{ a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t) \right\}$$

with

$$a_n = \operatorname{Re}\{c_n\} = \operatorname{Re} \left\{ \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) e^{-j2\pi n f_0 t} dt \right\} \qquad b_n = -\operatorname{Im}\{c_n\} = -\operatorname{Im} \left\{ \frac{1}{T_F} \int_{t_0}^{t_0+T_0} x(t) e^{-j2\pi n f_0 t} dt \right\}$$
$$= \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) \cos(2\pi n f_0 t) dt \qquad = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) \sin(2\pi n f_0 t) dt$$

Example 2.3 – Trig. FS of Ex. 2.1

- Let's examine a square wave which can be written as

$$x(t) = 2 \sum_{i=-\infty}^{\infty} \text{rect}\left(2t - 2i - \frac{1}{2}\right) - 1$$

- Examining one period ($T_F = 1$) $0 < t < 1$ the Trigonometric Fourier Series can be written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\}$$

- First note that $f_0 = 1/T_0 = 1$
- The first term a_0 is

$$a_0 = \frac{1}{T_F} \int_{t_0}^{t_0+T_0} x(t) \cos(0) dt = \frac{1}{T_F} \int_0^{0.5} dt - \frac{1}{T_F} \int_{0.5}^1 dt = 0$$

Example 2.3 (cont.)

□ For the remaining cosine terms and the sine terms we have

$$\begin{aligned} a_n &= \int_0^{0.5} \cos(2\pi nt) dt - \int_{0.5}^1 \cos(2\pi nt) dt \\ &= \frac{1}{2\pi n} \sin(2\pi nt) \Big|_0^{0.5} - \frac{1}{2\pi n} \sin(2\pi nt) \Big|_{0.5}^1 \\ &= \frac{1}{2\pi n} (\sin(n\pi) - \sin(0) - \sin(2n\pi) + \sin(n\pi)) \\ &= 0 \end{aligned}$$

$$\begin{aligned} b_n &= \int_0^{0.5} \sin(2\pi nt) dt - \int_{0.5}^1 \sin(2\pi nt) dt \\ &= -\frac{1}{2\pi n} (\cos(2\pi nt)) \Big|_0^{0.5} + \frac{1}{2\pi n} (\cos(2\pi nt)) \Big|_{0.5}^1 \\ &= \frac{1}{2\pi n} (-\cos(\pi n) + \cos(0) + \cos(2\pi n) - \cos(\pi n)) \\ &= \frac{1}{\pi n} (1 - \cos(\pi n)) \end{aligned}$$

Example 2.3 (cont.)

□ Thus, we can write the sine terms as

$$b_n = \frac{1}{\pi n} (1 - \cos(\pi n))$$
$$= \begin{cases} 0 & n = \text{even} \\ \frac{2}{\pi n} & n = \text{odd} \end{cases}$$

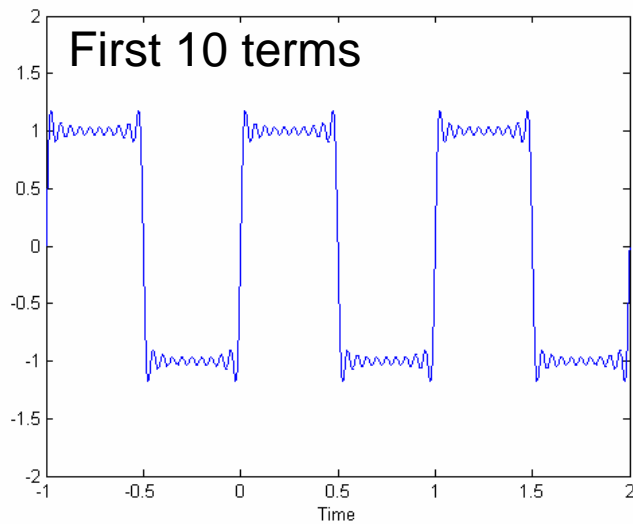
□ Thus we have

$$x(t) = \sum_{n=1}^{\infty} \frac{2 - 2\cos(\pi n)}{\pi n} \sin(2\pi n f_0 t)$$

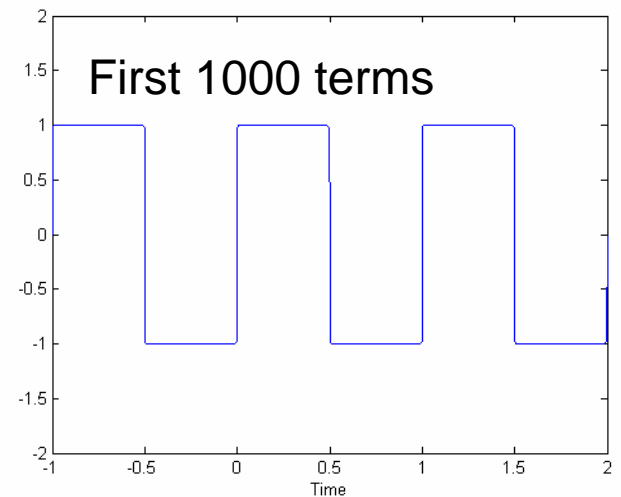
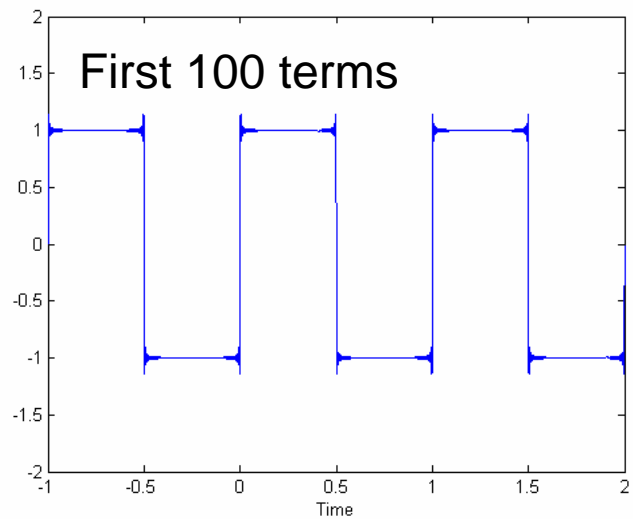
□ Since the even terms are zero we can rewrite this as

$$x(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} \sin(2\pi(2n-1)f_0 t)$$

Example 2.3 - Plots



- As we continue to add the sinusoidal terms our representation is closer and closer to the original signal *over the interval of interest*.
- When the number of terms goes to infinity the representation is *exact*



Example 2.3 - check

- Does this agree with our previous result?

$$a_0 = c_0$$

$$b_0 = 0$$

$$a_n = \frac{c_n + c_n^*}{2}$$

$$b_n = \frac{j(c_n - c_n^*)}{2}$$

$$n = 1, 2, 3, \dots$$

$$a_n = \begin{cases} 0 & n \text{ even} \\ \frac{1}{2} \left(-\frac{2j}{\pi n} + \frac{2j}{\pi n} \right) = 0 & n \text{ odd} \end{cases}$$

$$b_n = \begin{cases} 0 & n \text{ even} \\ \frac{j}{2} \left(-\frac{2j}{\pi n} - \frac{2j}{\pi n} \right) = \frac{2}{\pi n} & n \text{ odd} \end{cases}$$

Agrees!

Impact of the Signal Period

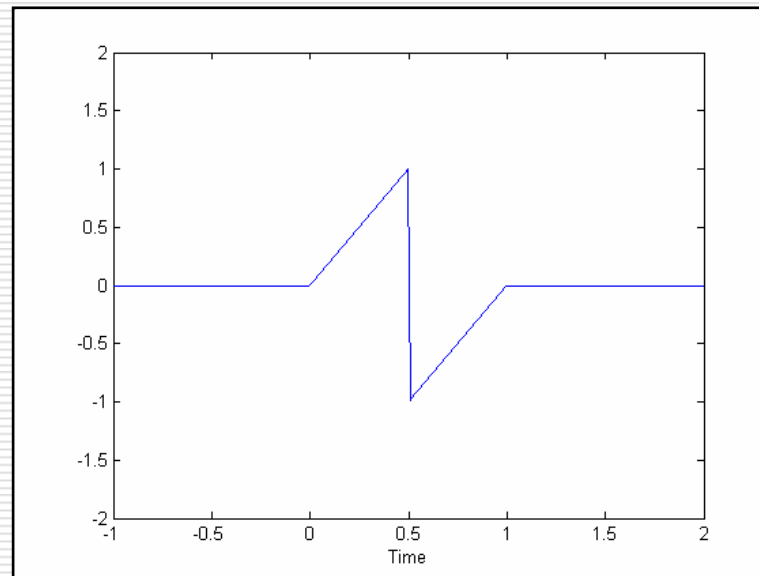
- Aperiodic signals
 - A Fourier Series representation can be calculated for an aperiodic signal, however it will only be valid over the interval of the original signal
 - See next example
- We have assumed thus far that we use the period of the signal as the fundamental interval of the Fourier Series
 - This is not necessary but is the most convenient
 - We can actually use any integer multiple of the period as the interval of interest
 - If we change the interval of interest from T_o to kT_o , for integer k , the Fourier Series coefficients are related to the original coefficients as

$$\tilde{c}_n = \begin{cases} c_{n/k} & n/k = \text{integer} \\ 0 & n/k \neq \text{integer} \end{cases}$$

Example 2.4 – Aperiodic signals

□ Let's consider a sawtooth waveform

$$x(t) = 2 * ramp(t) \left\{ u(t) - u\left(t - \frac{1}{2}\right) \right\} + \left(2 * ramp\left(t - \frac{1}{2}\right) - 2 \right) \left\{ u\left(t - \frac{1}{2}\right) - u(t - 1) \right\}$$



Note that this signal is NOT periodic

□ Define the time interval to be $0 < t < 1$

□ $T_0 = 1, f_0 = 1$

Example 2.4 (cont.)

□ The trigonometric Fourier Series is

$$x(t) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\}$$

□ Looking at the cosine terms

$$\begin{aligned} a_n &= \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) \cos(2\pi n f_0 t) dt \\ &= \int_0^{0.5} 2t \cos(2\pi n t) dt + \int_{0.5}^1 (2t-2) \cos(2\pi n t) dt \\ &= \frac{2}{(2\pi n)^2} \left[\cos(2\pi n t) + 2\pi n t \sin(2\pi n t) \right] \Big|_0^{0.5} + \frac{2}{(2\pi n)^2} \left[\cos(2\pi n t) + 2\pi n t \sin(2\pi n t) \right] \Big|_{0.5}^1 \\ &\quad - \frac{2}{2\pi n} \sin(2\pi n t) \Big|_{0.5}^1 \\ &= \frac{2}{(2\pi k)^2} \left[\cos(2\pi n) + 2\pi n \sin(2\pi n) - \cos(0) + \sin(0) \right] - \frac{2}{2\pi n} (\sin(2\pi n) - \sin(\pi n)) \\ &= 0 \end{aligned}$$

Example 2.4 (cont.)

□ Looking at the sin terms

$$\begin{aligned} b_n &= \frac{1}{T_F} \int_{t_o}^{t_o+T_0} x(t) \sin(2\pi n f_0 t) dt \\ &= \int_0^{0.5} 2t \sin(2\pi n t) dt + \int_{0.5}^1 (2t - 2) \sin(2\pi n t) dt \\ &= \frac{2}{(2\pi n)^2} \left[\sin(2\pi n t) - 2\pi n t \cos(2\pi n t) \right] \Big|_0^{0.5} + \frac{2}{(2\pi n)^2} \left[\sin(2\pi n t) - 2\pi n t \cos(2\pi n t) \right] \Big|_{0.5}^1 \\ &\quad + \frac{2}{2\pi n} \cos(2\pi n t) \Big|_{0.5}^1 \\ &= \frac{2}{(2\pi n)^2} \left[\sin(2\pi n) - 2\pi n \cos(2\pi n) - \sin(0) + \cos(0) \right] + \frac{2}{2\pi n} (\cos(2\pi n) - \cos(\pi n)) \\ &= -\frac{1}{\pi n} \cos(2\pi n) + \frac{1}{\pi n} \cos(2\pi n) - \frac{1}{\pi n} \cos(\pi n) \\ &= -\frac{1}{\pi n} \cos(\pi n) \\ &= \begin{cases} -\frac{1}{\pi n} & n = \text{even} \\ \frac{1}{\pi n} & n = \text{odd} \end{cases} \end{aligned}$$

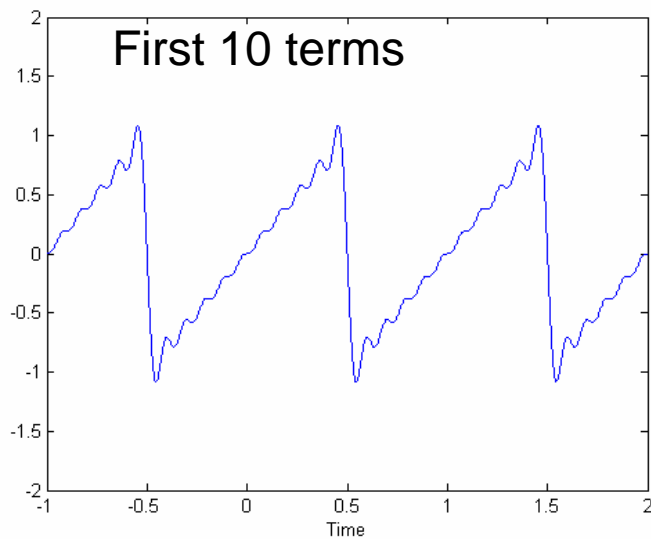
Example 2.4 (cont.)

- Summarizing we can write our Continuous Time Fourier Series as

$$x(t) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{\pi n} \sin(2\pi n t)$$

- Recalling that the signal is aperiodic, does this representation hold?
- From the plots on the next page, we see that the Fourier Series representation is *periodic* even though the original signal is not.
 - Thus, they are equivalent only over the interval defined for the original signal. i.e., $0 < t < 1$

Example 2.4 (final)



- As we continue to add the sinusoidal terms our representation is closer and closer to the original signal but only over the interval $0 < t < 1$.
- When the number of terms goes to infinity the representation is exact but only over the interval $0 < t < 1$.

