# ECE3614
# Introduction to Communications Systems
# Fall 2007

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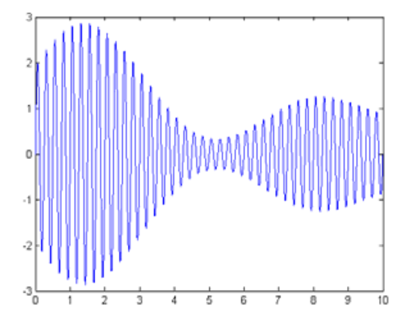
Lecture #22: Digital Communications – Quantization and Waveform Encoding

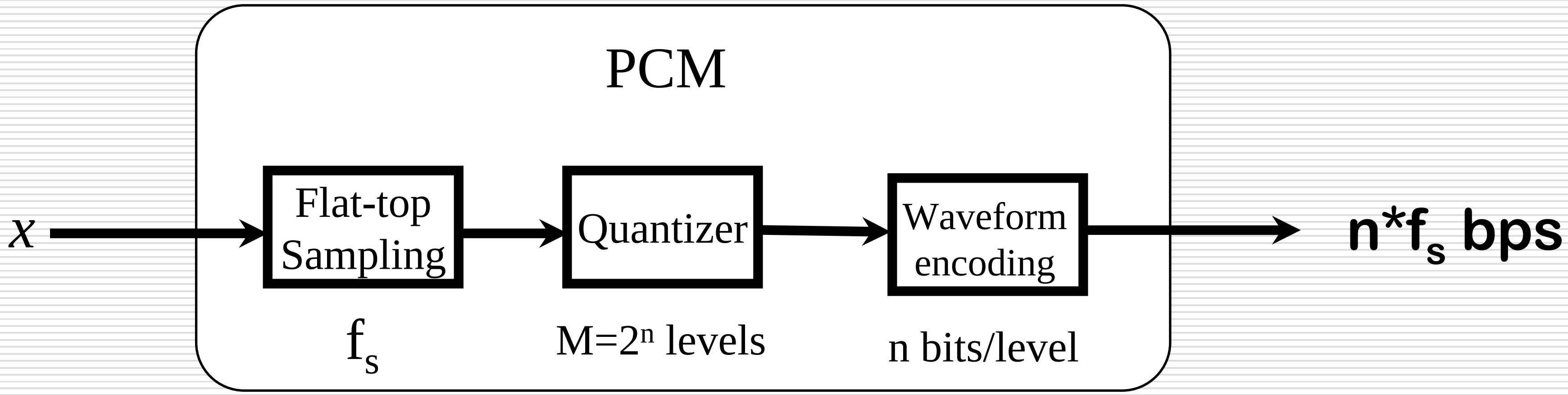


# Overview

- Last class we introduced the concept of digital communications
- Digital communication of analog message signals requires analog-to-digital conversion which includes
  - Sampling
  - Quantization
  - Waveform encoding
- Today we discuss quantization and waveform encoding
- Reading
  - 5.4 – 5.5

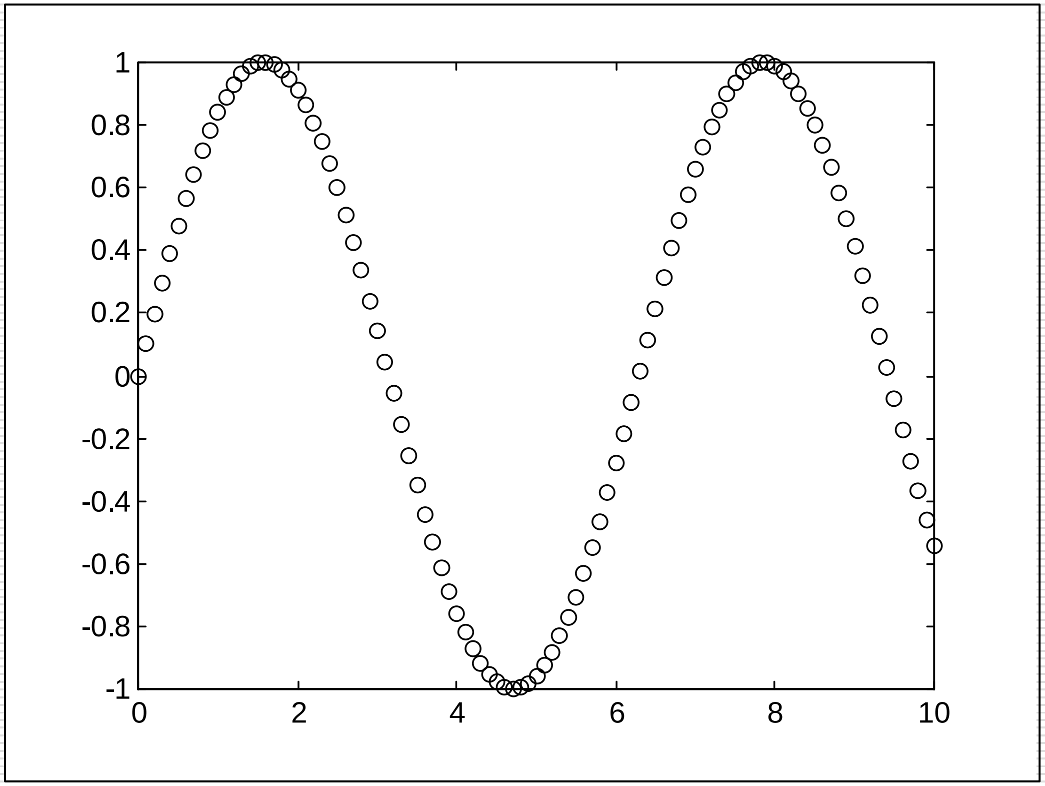
# Pulse Code Modulation (PCM)

- Pulse Code Modulation refers to a system that creates a digital baseband signal from an analog signal using sampling, quantization and waveform encoding.



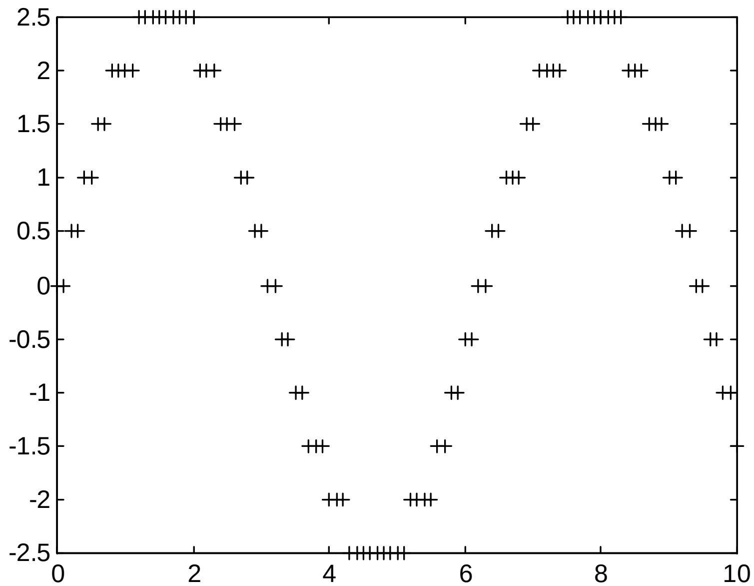
# Digital Representation of Analog Signals

- Sampling analog signals makes them discrete in time:



# Digital Representation of Analog Signals

- <u>Quantization</u> of sampled analog signals makes the samples discrete in amplitude:



# Quantization

- Continuous time signals are sampled at discrete time intervals
- Sampling may be performed without distortion provided signal is sampled at Nyquist rate
- Continuous-valued samples of data require an infinite # of bits to represent with perfect precision.
- <u>Quantization</u> is the process of approximating continuous-valued samples with a finite # of bits.
- Quantization always introduces some distortion.

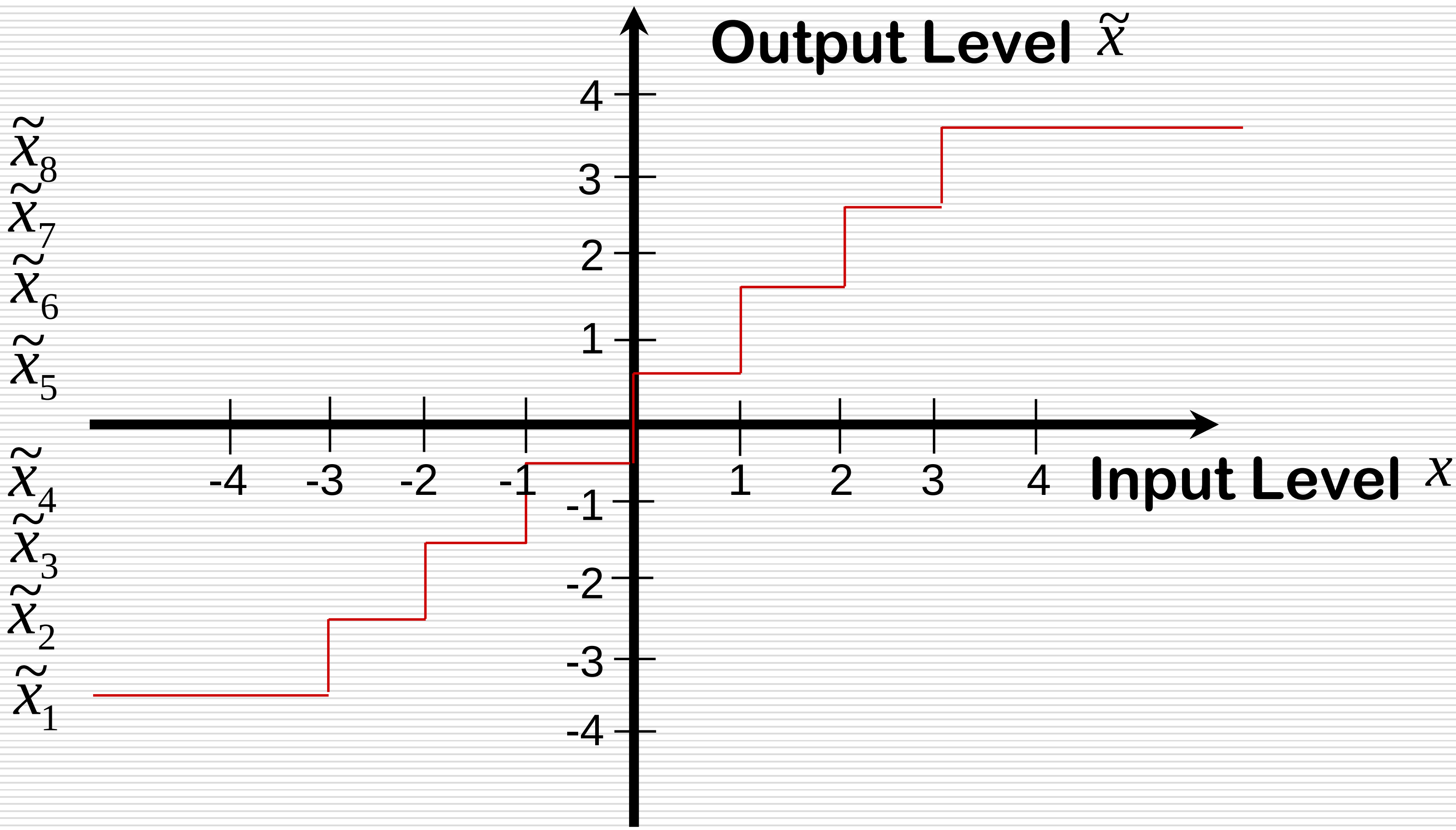
# Notation Associated with Quantization

- Let *X* be a random variable representing a sample of data.
- Then $\tilde{X} = f_Q(X)$ is the quantized value of *X*.
- A quantizer has *M* quantization levels:

$$\tilde{X} \in \{\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_M\}$$

- The *M* levels correspond to *M* quantization regions.
- The endpoints of the quantization regions are specified by $M+1$ values: $\{x_0, x_1, \ldots, x_M\}$ , where $x_0 = -\infty, x_M = \infty$
- Then: $x_{k-1} \le x < x_k \Rightarrow \tilde{x} = f_Q(x) = \tilde{x}_k$

# Graphical Description of Quantization



# Table Representation of Quantizer

| $k$ | $x_{k-1}$ | $x_k$ | $\tilde{x}_k$ | Output Bits |
|---|---|---|---|---|
| 1 | $-\infty$ | -3 | -3.5 | 000 |
| 2 | -3 | -2 | -2.5 | 001 |
| 3 | -2 | -1 | -1.5 | 010 |
| 4 | -1 | 0 | -0.5 | 011 |
| 5 | 0 | 1 | 0.5 | 100 |
| 6 | 1 | 2 | 1.5 | 101 |
| 7 | 2 | 3 | 2.5 | 110 |
| 8 | 3 | $\infty$ | 3.5 | 111 |

# Concise Representation of Quantizer

- Usually, it is sufficient just to list the quantization levels.
- Example: {-3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5}
- Why?
  - We assume that all points are quantized to the nearest quantization level
  - This determines where the borders of the quantization regions are
  - Any other borders would increase the error introduced by the quantizer

# Practical Methods for Implementing Analog to Digital Converters

- Counting or Ramp ADC
  - test value is incremented in equal steps until it is greater than input sample
- Serial or Successive Approximation ADC
  - uses binary search to narrow range of input sample until desired accuracy is reached
- Parallel or Flash ADC
  - input sample is compared with all possible quantization levels at once

Lower Complexity

Faster

# Distortion

- Quantization introduces distortion into a signal.
- We want to minimize average distortion $D$, where

$$D = E\left[\left(X - \tilde{X}\right)^2\right]$$

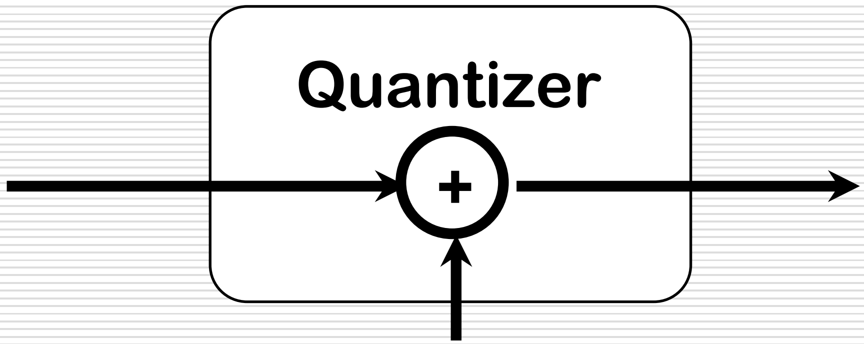
$$= \int_{-\infty}^{\infty} \left(x - \tilde{x}\right)^2 f\left(x\right) dx$$

$$= \sum_{k=1}^{M} \int_{x_{k-1}}^{x_k} \left(x - \tilde{x}_k\right)^2 f\left(x\right) dx$$

- This measure of distortion is sometimes also called mean square error (MSE)
- MSE penalizes large errors more than small errors

# Another Way of Viewing Quantization

- Quantization adds a random "noise" to the true value of the sample point
- Then $\text{MSE} = E\left[n_Q^2\right]$ may be thought of as noise power
- We can define a signal-to-noise ratio (SNR) to measure performance




$x$

$\tilde{x} = f_Q(x) = x + n_Q$

$n_Q = x - \tilde{x}$

# Signal to Noise Ratio Calculations for Quantizers

- Average SNR

$$\left(\frac{S}{N}\right)_{\text{avg}} = \frac{E\left[X^2\right]}{E\left[n_Q^2\right]} = \frac{E\left[X^2\right]}{D} = \frac{\int_{-\infty}^{\infty} x^2 f_X(x)\,dx}{\int_{-\infty}^{\infty} (x-\tilde{x})^2 f_X(x)\,dx}$$

# Example of SNR Calculation

- Let: $\{\tilde{x}_1 = -3.5, \tilde{x}_2 = -2.5, \tilde{x}_3 = -1.5, \tilde{x}_4 = -0.5, \tilde{x}_5 = 0.5, \tilde{x}_6 = 1.5, \tilde{x}_7 = 2.5, \tilde{x}_8 = 3.5\}$

- Let: $$f(x) = \begin{cases} 1/8, & -4 \le x \le 4 \\ 0, & \text{else} \end{cases}$$

$$E\left[X^2\right] = \int_{-4}^{4} x^2 \cdot \frac{1}{8} dx = \left.\frac{x^3}{24}\right|_{-4}^{4} = \frac{128}{24} = \frac{16}{3}$$

# Example of SNR Calculation (cont.)

- Distortion: $D = MSE = \sum_{k=1}^{M} \int_{x_{k-1}}^{x_k} \left(x - \tilde{x}_k\right)^2 f_X(x)\,dx$

$$= \sum_{k=1}^{8} \int_{-5+k}^{-4+k} \left(x - \left(-4.5 + k\right)\right)^2 \cdot \frac{1}{8} dx$$

- Examine the $k = 5$ term:

$$\int_0^1 (x - 0.5)^2 \cdot \frac{1}{8} dx = \frac{1}{8} \cdot \int_0^1 x^2 - x + 0.25 dx = \frac{1}{8} \cdot \left[ \frac{x^3}{3} - \frac{x^2}{2} + 0.25x \right]_0^1$$

$$= \frac{1}{8} \cdot \left[ \frac{1}{3} - \frac{1}{2} + \frac{1}{4} \right] = \frac{1}{8} \cdot \frac{1}{12}$$

# Example of SNR Calculation (continued)

- All 8 terms are identical. $\therefore \mathrm{MSE} = 1/12$

$$\left(\frac{S}{N}\right)_{\text{avg}} = 10\log_{10}\frac{E\left[X^2\right]}{D} = 10\log_{10}\left(\frac{16/3}{1/12}\right) = 18.1\text{dB}$$

- A standard rule of thumb is that each additional bit adds 6 dB to the SNR of uniform quantizers operating on a uniform pdf.

# SNR for Uniform Quantization

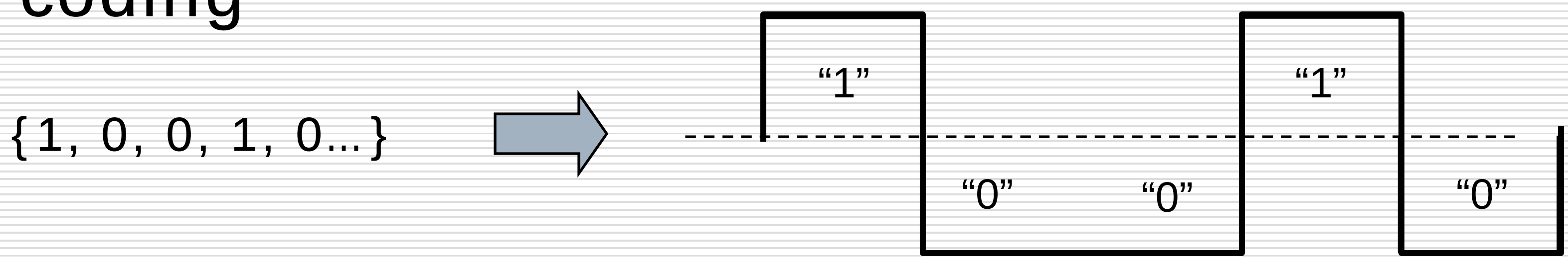
- General result: $$\left(\frac{S}{N}\right)_{\text{avg}} = M^2$$
  - Assumes uniform quantizer with *M* levels
  - Assumes that input samples have uniform distribution with identical range as quantizer
  - Though some texts do not make it clear, this result applies only when these special set of conditions hold. Otherwise, we have to use integral formula
- However, a useful Rule of Thumb:
  - Each additional bit (doubling *M*) increases SNR by 6 dB

$$\left(\frac{S}{N}\right)_{avg} = 6.02n + \alpha$$

  - Where $\alpha$ depends on the distribution of the signal

# Waveform Encoding

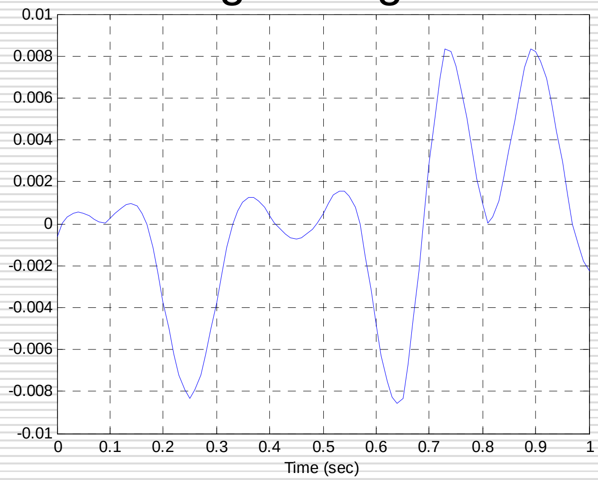
- Once the information is converted to bits, it must be mapped onto waveforms
- If each bit is mapped to one of two different waveforms, we term this *binary encoding*
- If $m$ bits are mapped to $M = 2^m$ waveforms, we term this $M$-ary encoding
- The mapping of bits to pulse-based waveforms (i.e., pulse trains) is termed "line coding"



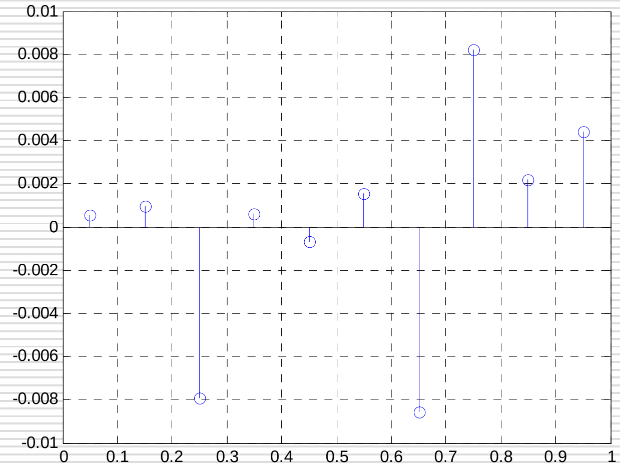

Example: Non-return to zero polar signaling

# PCM – Example

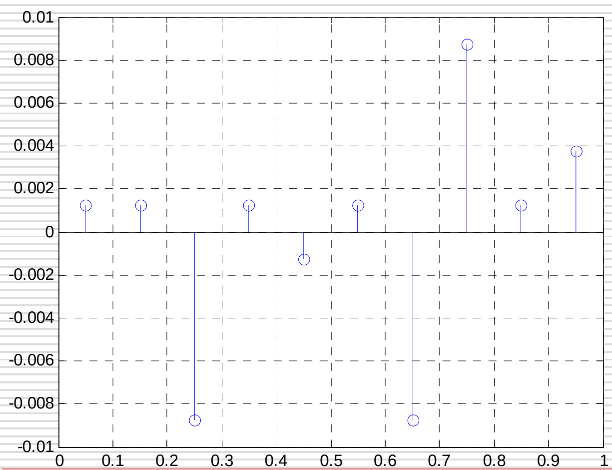
Original Signal



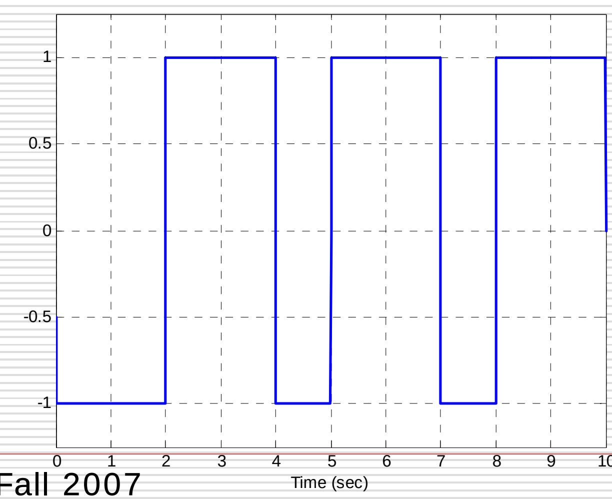

Sampled Signal




Quantized Signal




Resulting Digital Signal



# Bandwidth of PCM Signals

- Sample rate: $f_s$ samples/second.
- Bit rate out of the quantizer:

$$f_s \log_2 M = f_s \cdot n \text{ bits / second}$$

- Bandwidth of the resulting digital signal depends on the waveform encoding used
- Minimum theoretical bandwidth with optimal waveform (baseband waveforms) : $f_s \cdot n/2$ Hz
- First null bandwidth (with rectangular pulse waveforms) for baseband waveforms: $f_s \cdot n$ Hz

Note that the resulting bandwidth depends on the digital waveform that is used.

# Example PCM Calculation

- **Problem:**
  - Suppose that an analog music signal is found to have a bandwidth of 15 kHz and that samples of the signal may be modeled as having a uniform distribution.
  - Find the minimum first-null bandwidth (assuming the use of square pulses as the waveform) at which it would be possible to transmit a PCM signal while maintaining an average SNR of at least 58 dB.

# Example PCM Calculation (cont.)

- **Solution:**

$$f_S \geq 2B = 30{,}000 \text{ samples / sec}$$

$$10\log_{10} M^2 \geq 58\text{dB} \Rightarrow M \geq 794 \Rightarrow M \geq 1024$$

$$\Rightarrow n \geq 10 \text{ bits/sample}$$

  - We will assume *n* must be an integer, thus *n* = 10 $R = f_s n \geq 300kbps$
  - Minimum data rate:
  - First null BW: $B = f_s n = 300kHz$
  - Assumes rectangular pulses are used as the waveforms

Note: Bandwidth comes into play twice here. Once for the original analog signal and once for the resulting digital signal.

# Bandpass Digital Modulation

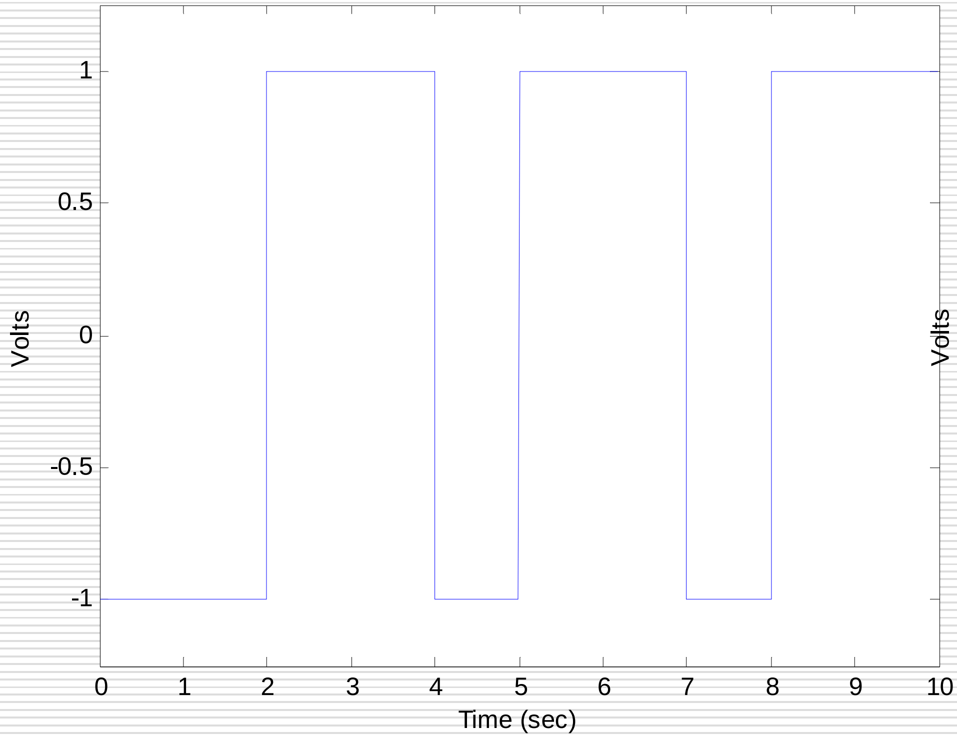
- Baseband digital maps bits to pulses
- Bandpass digital maps bits to sinusoids with different
  - phases
  - frequencies
  - amplitudes
- Bandpass digital techniques can be viewed as AM or FM with digital baseband waveforms as the message signal

# Binary Frequency Shift Keying

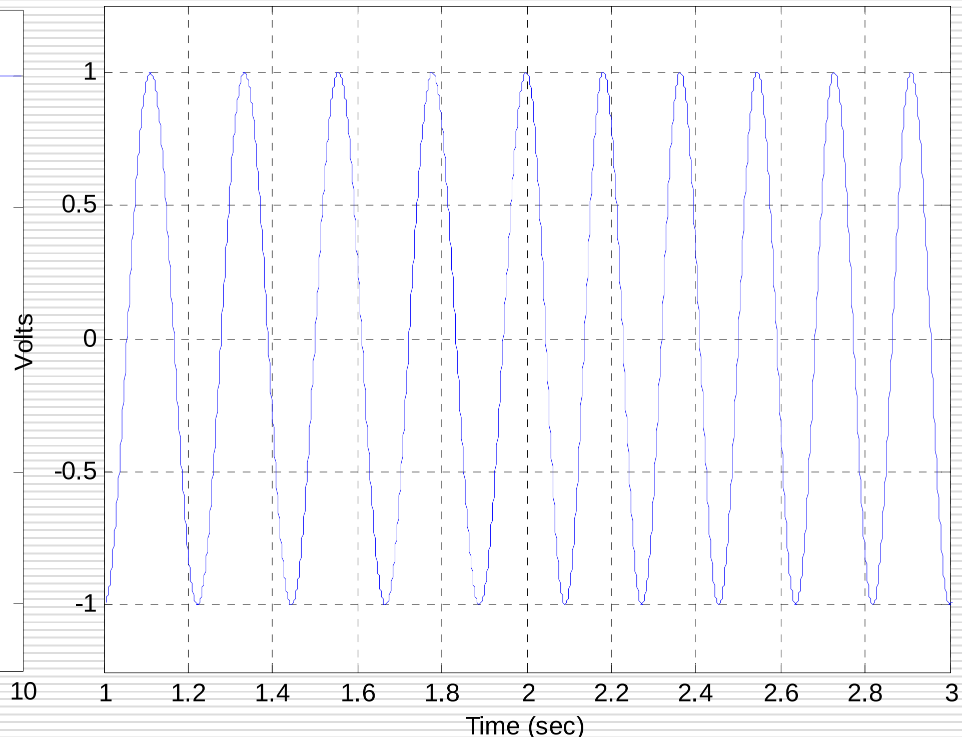
- While FM is an analog communications technique, the basic idea can also be used with digital modulation
- First we first map bits to square pulses with amplitudes +1/-1 representing data bits 1/0.
- Second, we use the square pulse baseband signal as our message and use $k_f = \Delta f = R_b/2$ where $R_b$ is the bit rate.
- The result is a digital modulation scheme known as Binary Frequency Shift Keying (BFSK)

# Example

$R_b = R_s = 1\text{bps}$




$f_c = 5\text{Hz} \rightarrow f_1 = 4.5\text{Hz},\ f_2 = 5.5\text{Hz}$



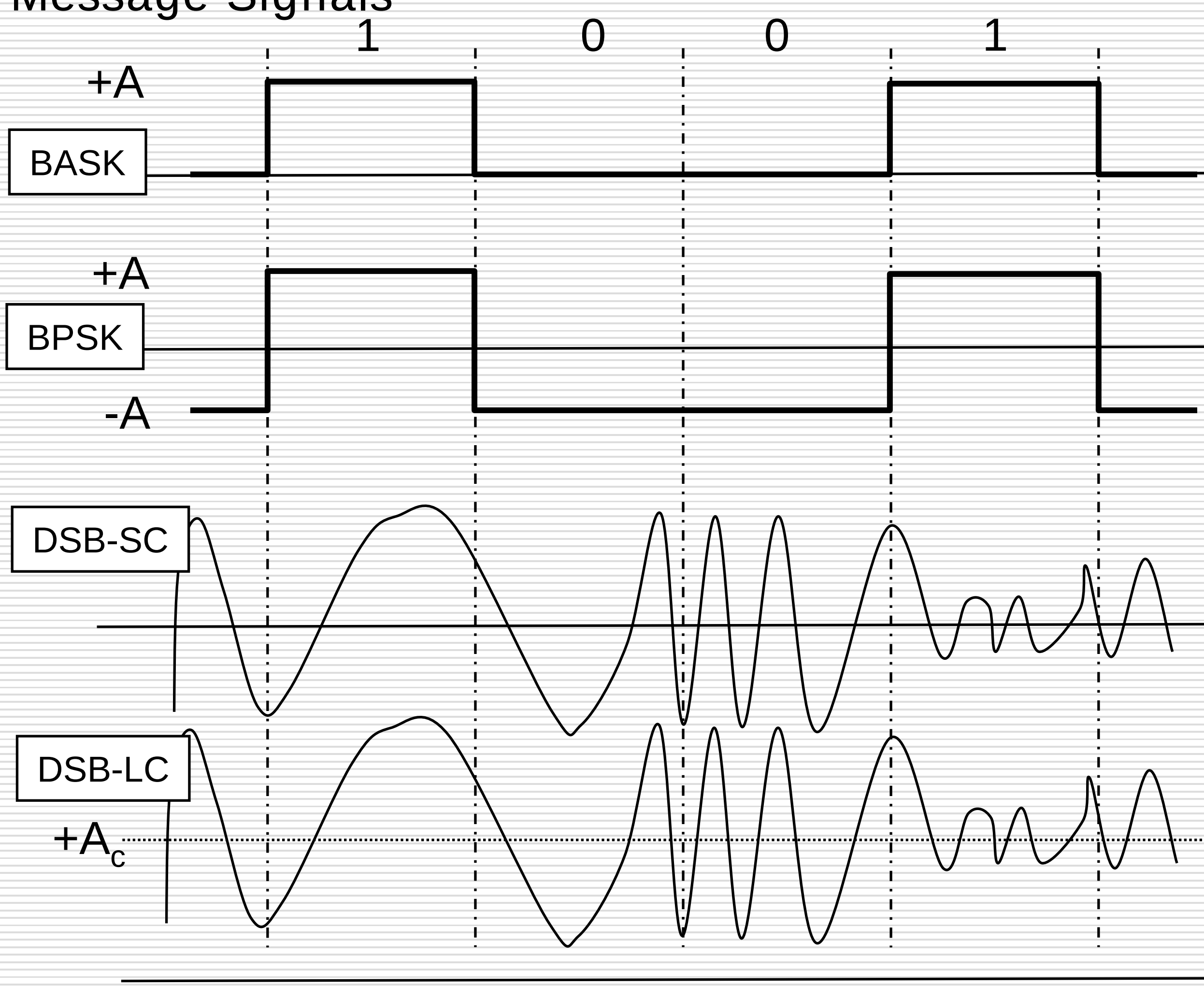

$f_1 = 4.5\text{Hz}$ , $f_2 = 5.5\text{Hz}$

# Relationship between AM and Digital Modulation

- DSB-LC AM → BASK
  - Message is entirely in the amplitude
- DSB-SC AM → BPSK, BASK
  - Message is in the phase and amplitude
  - In BPSK, amplitude is always 1 since the message is square wave.
  - In BASK the message signal doesn't go negative so there are no phase changes.
- SSB AM → QPSK
  - Since message is all real, we can eliminate ½ of spectrum
  - QPSK is based on fact that since a real message only needs ½ of the spectrum, 2 messages can be sent in quadrature in original BW

# Relationship Between Analog and Digital Modulation Schemes

Message Signals




| Message | What is modulated? |
|---|---|
| Unipolar NRZ | Amplitude Only |
| Polar NRZ | Phase Only |
| Arbitrary Analog signal | Amplitude and Phase |
| Arbitrary Analog signal with offset | Amplitude Only |

# Summary

- Transmitting an analog signal over a Digital Communication system involves
  - Sampling
  - Quantizing
  - Waveform encoding (i.e., mapping the bits to transmit waveforms)
- Today we examined the last two items
  - Sampling/Quantization convert analog signals to a stream of 1's and 0's
  - Waveform encoding maps bits by modulating
    - A pulse train in baseband digital communications
    - A sinusoid in bandpass digital communicaitons