ECE3614
Introduction to Communications Systems
Fall 2007

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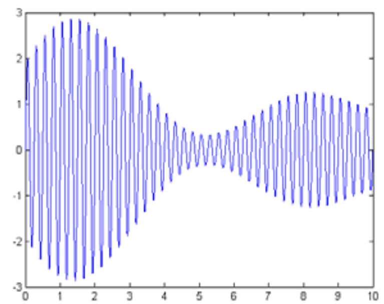
Lecture #7: Filtering



Overview

- In the previous classes we have discussed using the Fourier Transform to characterize systems.
- In this lecture we investigate an important sub-system of communications systems – filters
- It is important to view filters in the frequency domain in order to more fully understand their characteristics.
- Reading
  - Sections 2.6 and 2.7

Filters

- A filter is a system which passes certain frequencies and rejects other frequencies
- Types of filters
  - Low pass filter
  - High pass filter
  - Bandpass filter
  - Bandstop filter
- Ideal filter
  - An ideal filter is one which perfectly passes frequencies in a certain range (termed the *pass band*) and perfectly rejects frequencies in another range termed the *stop band*
  - An ideal filter doesn't *distort* the signal in the pass band

The Ideal Lowpass Filter

- The ideal lowpass filter passes without distortion all frequencies inside the passband of the filter (i.e., frequencies below frequency *B*) and completely rejects all frequencies inside the stop band
- Frequency response

$$H(f) = \begin{cases} e^{-j2\pi f t_o} & |f| \le B \\ 0 & |f| > B \end{cases}$$

$$= \underbrace{rect\left(\frac{f}{2B}\right)}_{\substack{magnitude\\response}} \underbrace{e^{-j2\pi f t_o}}_{\substack{time\\delay}}$$

Magnitude/Phase Responses

$$\left|H(f)\right| = rect\left(\frac{f}{2B}\right)$$

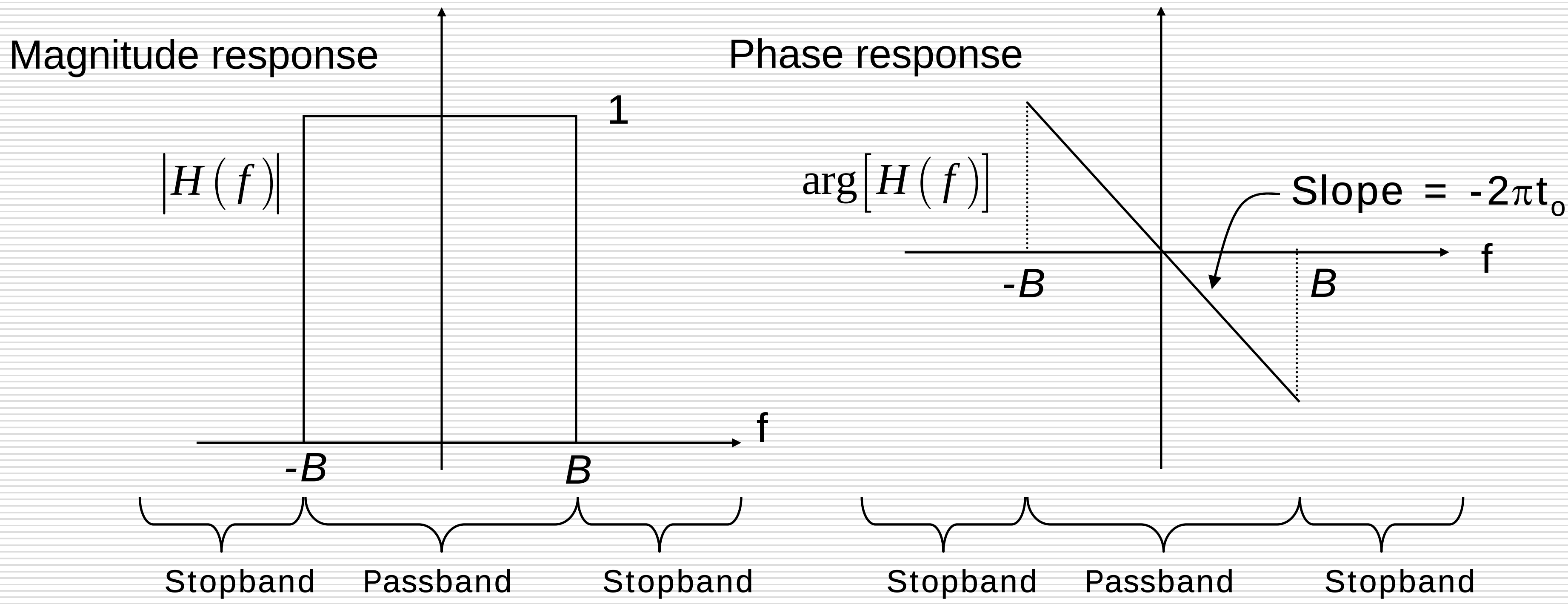
$$\arg\left[H(f)\right] = \arg\left[rect\left(\frac{f}{2B}\right)e^{-j2\pi f t_o}\right]$$

$$= \begin{cases} \arg\left[\cos(2\pi f t_o) + j\sin(2\pi f t_o)\right] & |f| \le B \\ 0 & else \end{cases}$$

$$= \begin{cases} \tan^{-1}\left[\frac{\sin(2\pi f t_o)}{\cos(2\pi f t_o)}\right] & |f| \le B \\ 0 & else \end{cases}$$

$$= \begin{cases} 2\pi f t_o & |f| \le B \\ 0 & else \end{cases}$$

Illustration – Ideal LPF



Linear Phase

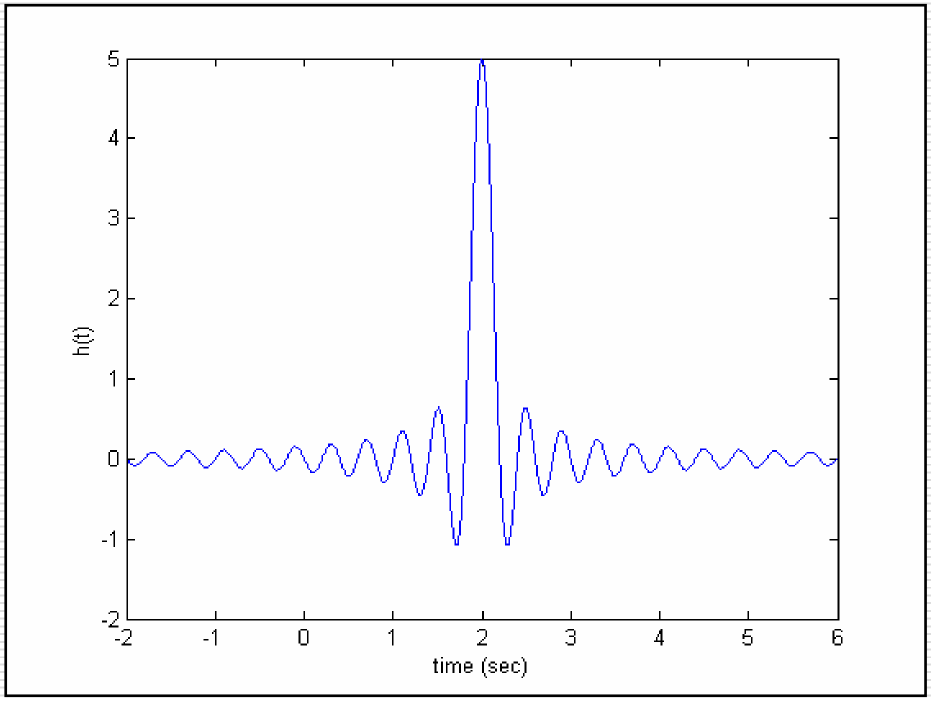
- What does it mean to have a *linear* phase response?

$$\arg\left[H(f)\right] = 2\pi f t_o \rightarrow \quad \begin{aligned} H(f) &= \left|H(f)\right| e^{-j2\pi f t_o} \\ &= \mathrm{A}(f) e^{-j2\pi f t_o} \end{aligned}$$

- Linear phase corresponds to all frequencies being delayed by the same amount. In other words, there is no phase distortion imparted to the input signal.

Impulse response of the Ideal LPF

- The impulse response of the ideal LPF can be found by taking the inverse Fourier Transform of the ideal LPF frequency response



$$h(t) = F^{-1}\{H(f)\}$$

$$= F^{-1}\left\{rect\left(\frac{f}{2B}\right)e^{-j2\pi f t_o}\right\}$$

$$= 2B\text{sinc}\left(2B\left(t - t_o\right)\right)$$

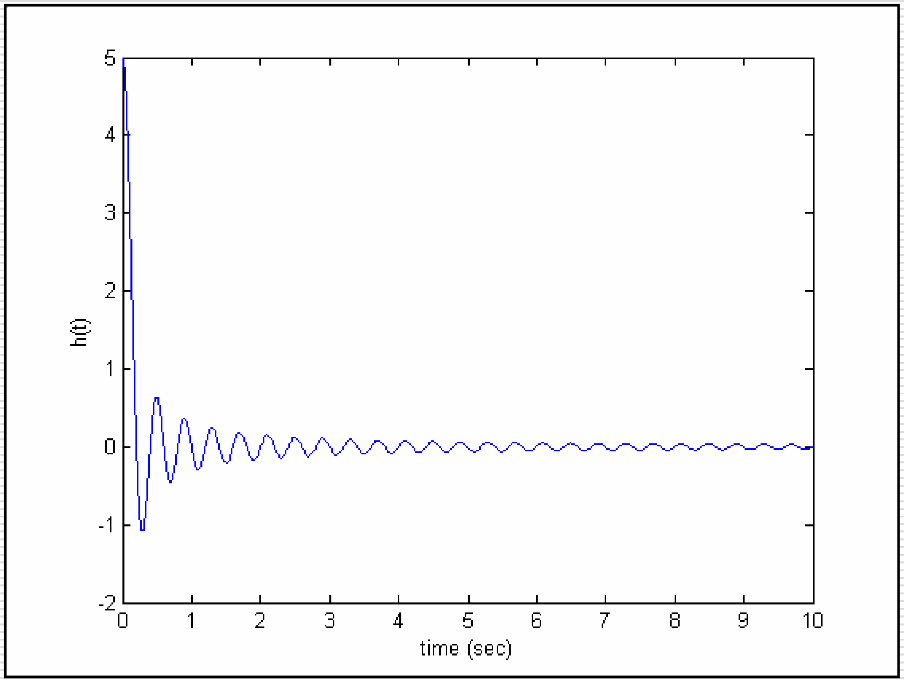
- Example: $B = 5$
- $t_o = 2$
- Is there any problem with this impulse response?

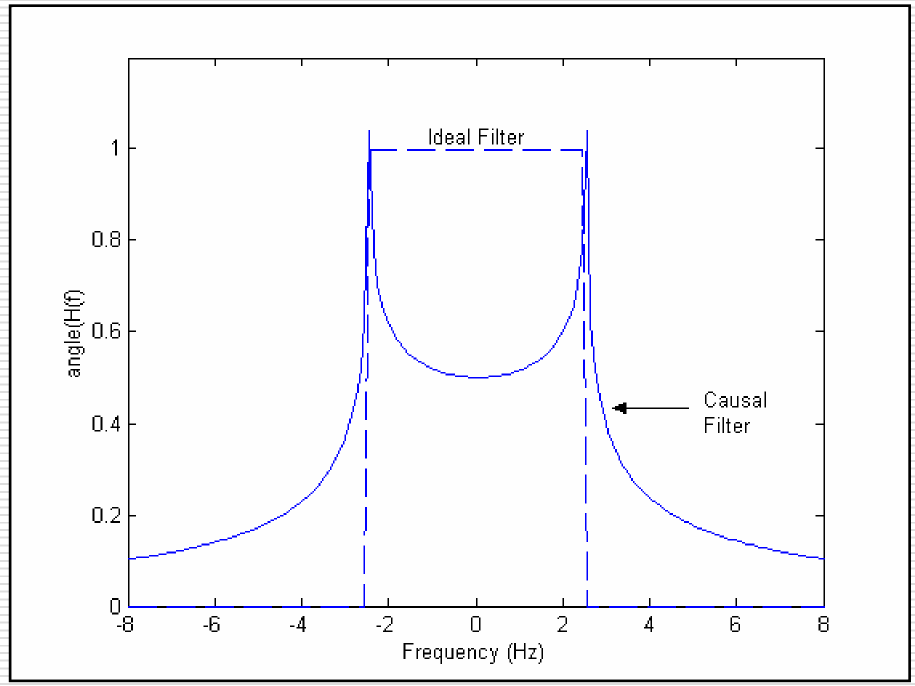
Causality

- A causal system is one that does not have a response prior to the time of the applied input
- The impulse response is the response of a system to an impulse applied at time $t=0$.
- A system whose impulse response is nonzero for $t<0$ is thus *non-causal.*
- The ideal lowpass filter is thus non-causal and for that reason is not physically realizable.
- We can attempt to approximate the ideal LPF by introducing a delay into our system

Causal LPF

- We can make the filter causal by simply truncating the impulse response before *t=0.*

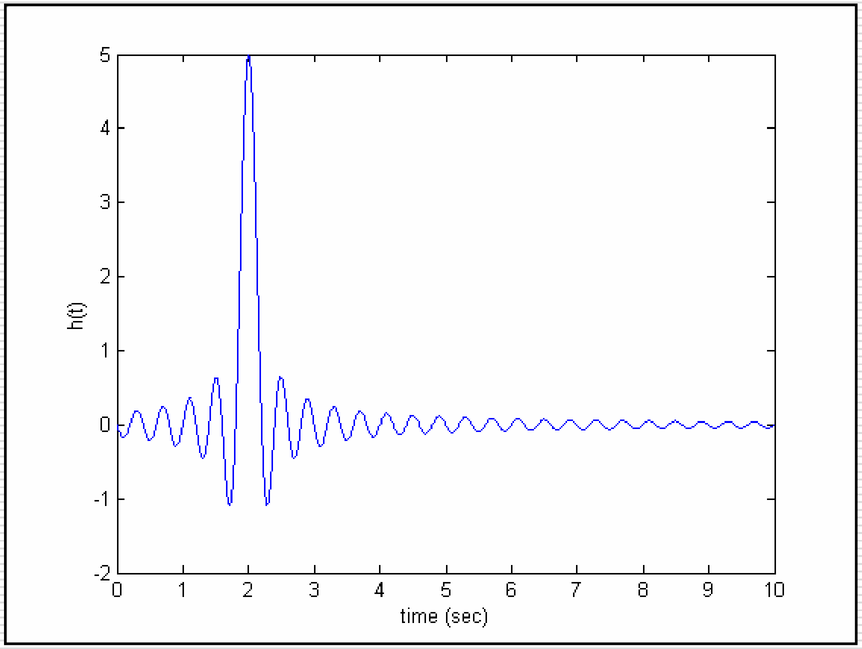


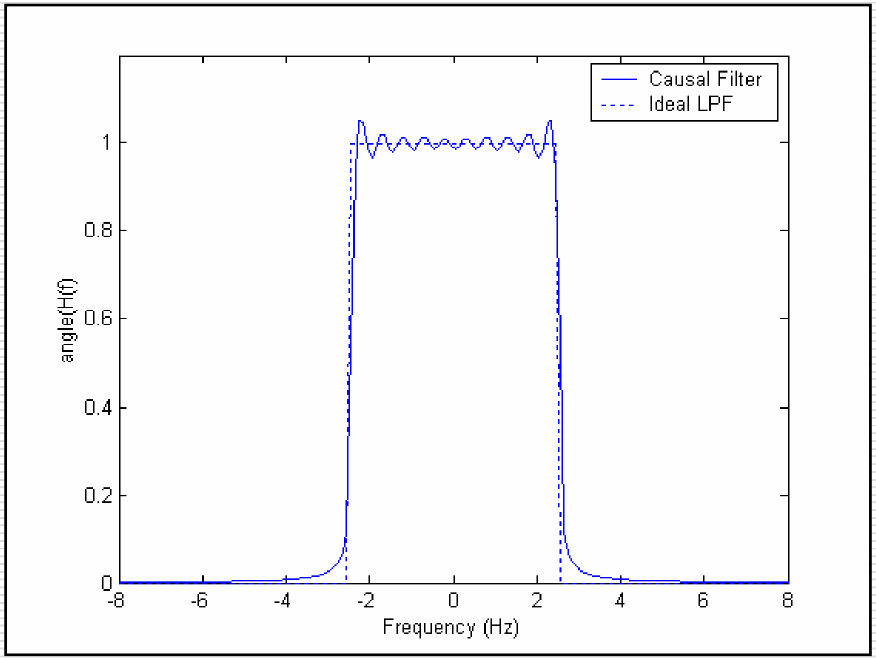


- The resulting filter is far from ideal.

Causal LPF

- A second option is to delay the impulse response and truncate it.
- Delay = 2

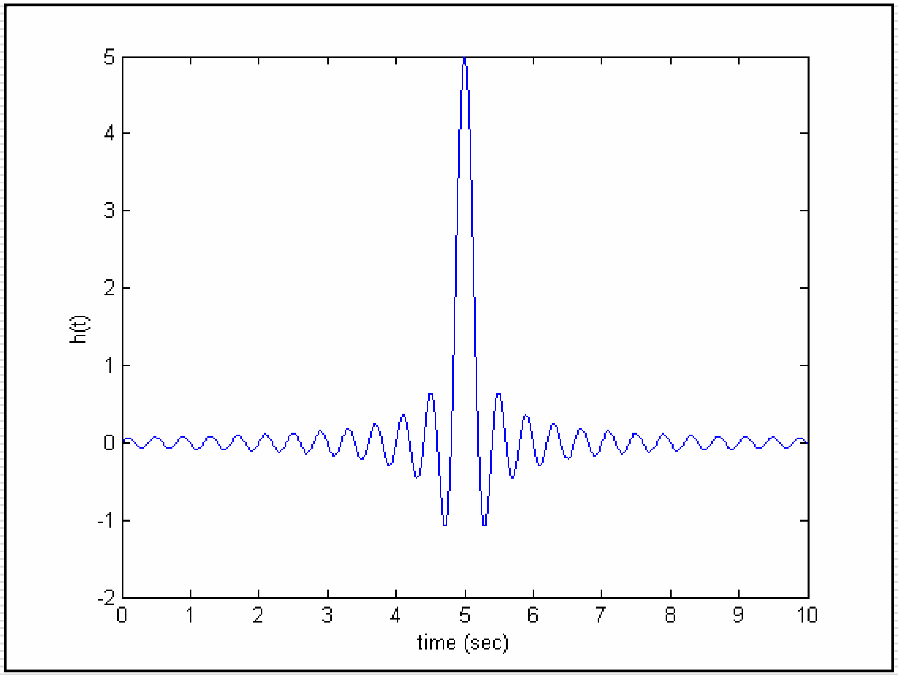


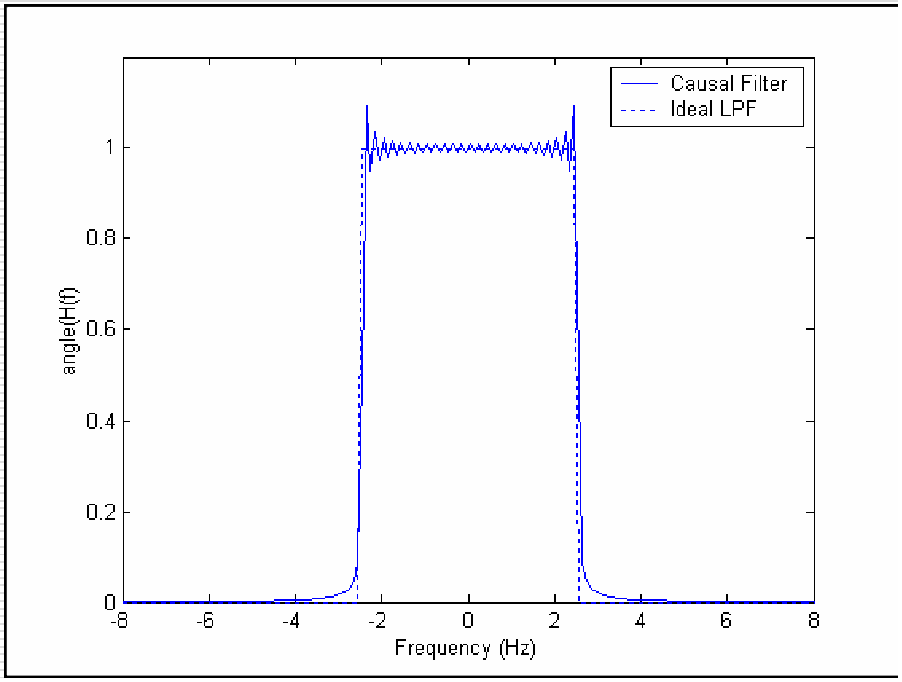


- This makes the system closer to ideal, but requires a delay which some applications may not tolerate

Causal LPF

- A second option is to delay the impulse response and truncate it.
- Delay = 5

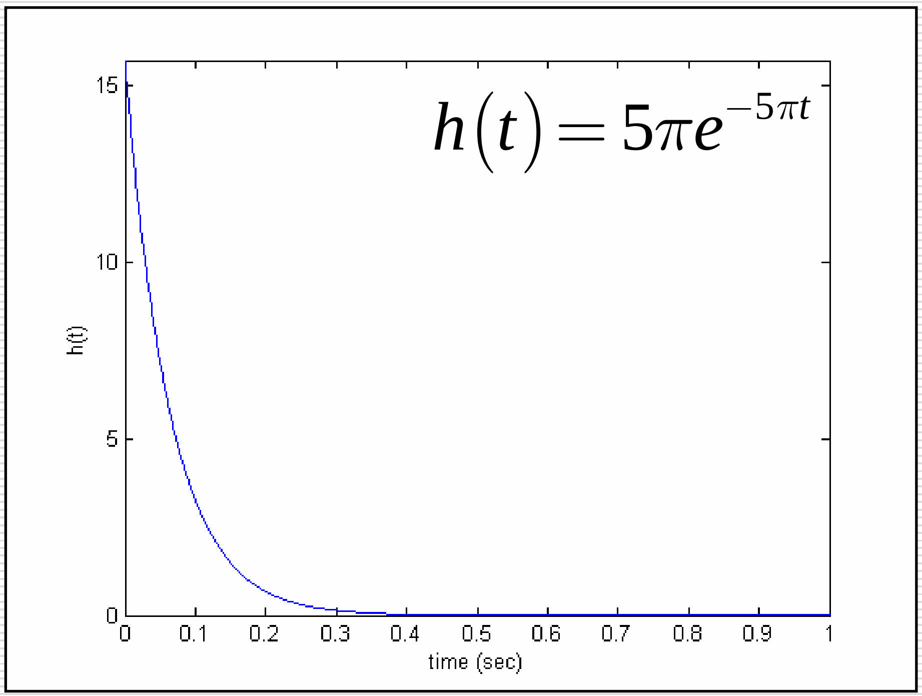


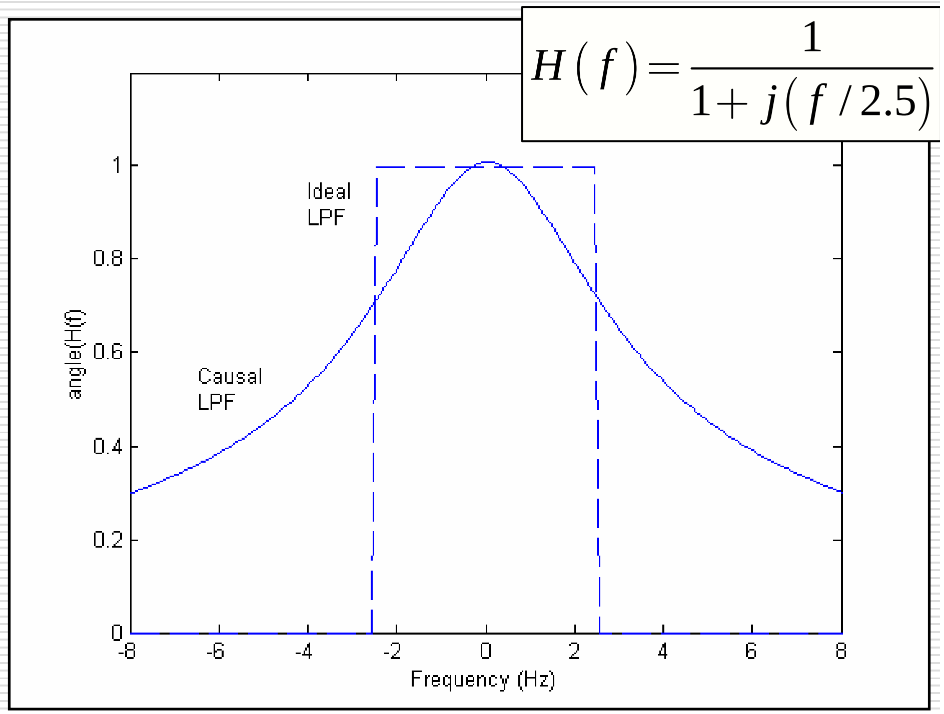


- Larger delay leads to better approximation of the ideal LPF

Causal LPF

- RC filter
- Causal, exponential impulse response
- Non-ideal LPF





Pulse Response of Ideal LPF

- Let us apply a rectangular pulse *x(t) = rect(t/T)* to an ideal low-pass filter (impulse response *h(t) = 2B sinc(2πBt)*

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} rect(\tau/T)\, 2B\mathrm{sinc}\left(2B(t-\tau)\right) d\tau$$

$$= \int_{-T/2}^{T/2} 2B\mathrm{sinc}\left(2B(t-\tau)\right) d\tau$$

$$= \int_{-T/2}^{T/2} 2B \frac{\sin\left(2\pi B(t-\tau)\right)}{2\pi B(t-\tau)} d\tau$$

$$= \frac{1}{\pi} \int_{2\pi B(t-T/2)}^{2\pi B(t+T/2)} \frac{\sin(\lambda)}{\lambda} d\lambda$$

Pulse response (cont.)

- Continuing...

$$y(t) = \frac{1}{\pi}\int_{2\pi B(t-T/2)}^{2\pi B(t+T/2)} \frac{\sin(\lambda)}{\lambda} d\lambda$$

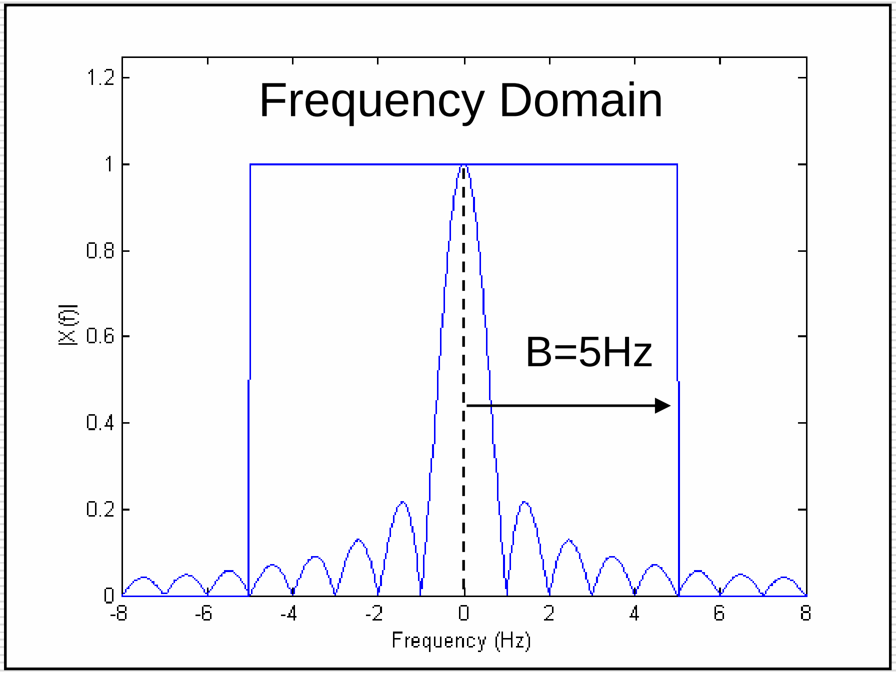
$$= \frac{1}{\pi}\int_{0}^{2\pi B(t+T/2)} \frac{\sin(\lambda)}{\lambda} d\lambda - \frac{1}{\pi}\int_{0}^{2\pi B(t-T/2)} \frac{\sin(\lambda)}{\lambda} d\lambda$$

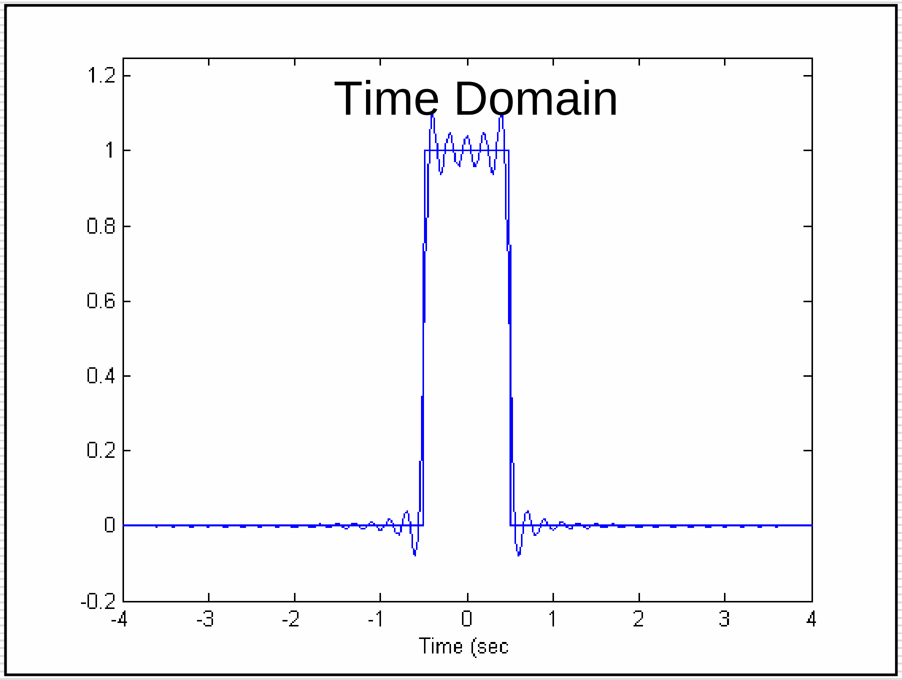
$$= \frac{1}{\pi}\left[Si\left(2\pi B\left(t+T/2\right)\right) - Si\left(2\pi B\left(t-T/2\right)\right)\right]$$

$$Si(x) = \int_{0}^{x} \frac{\sin(\lambda)}{\lambda} d\lambda$$

Example 7.1

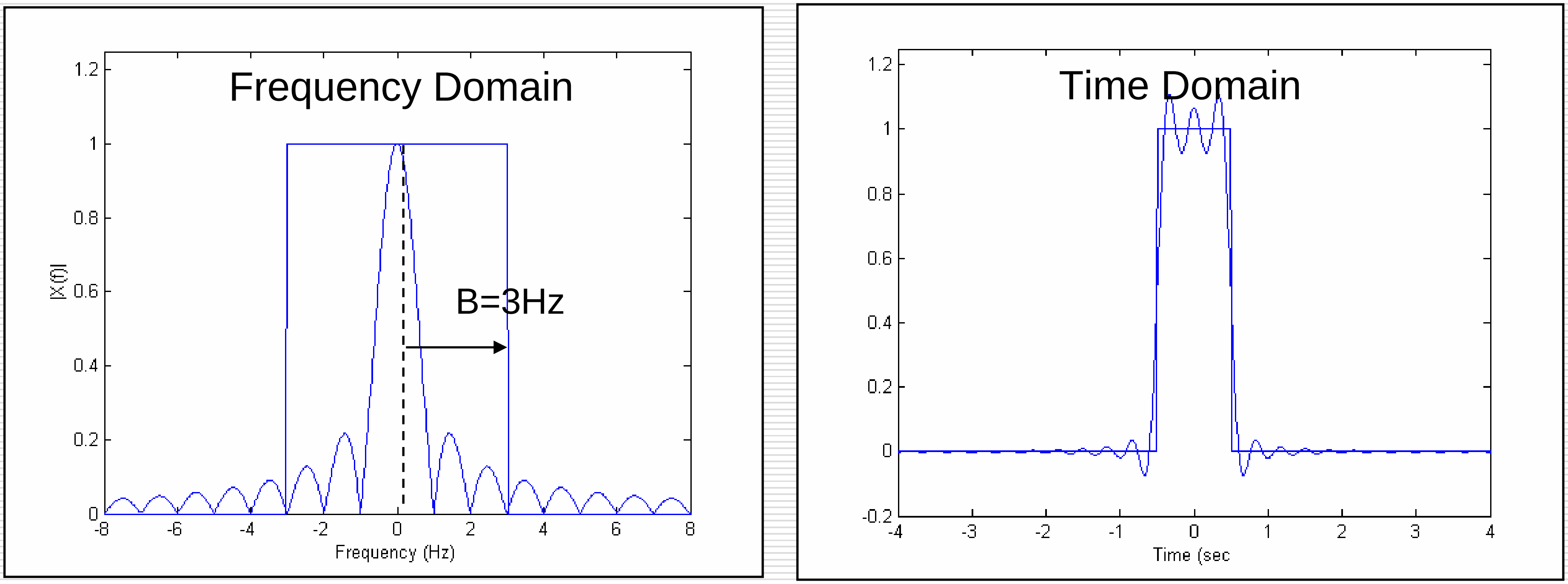
- Consider a time-domain square pulse of width 1 second which is passed through a filter with a bandwidth of 5Hz – BT = 5
- The bandwidth restriction does not cause a substantial change in the pulse shape





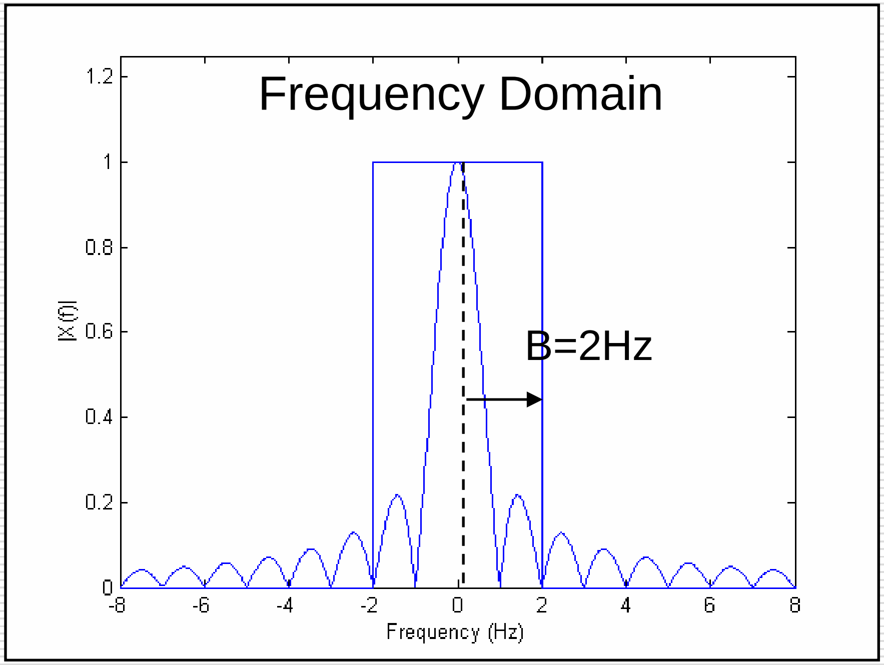
Example 7.1 – cont.

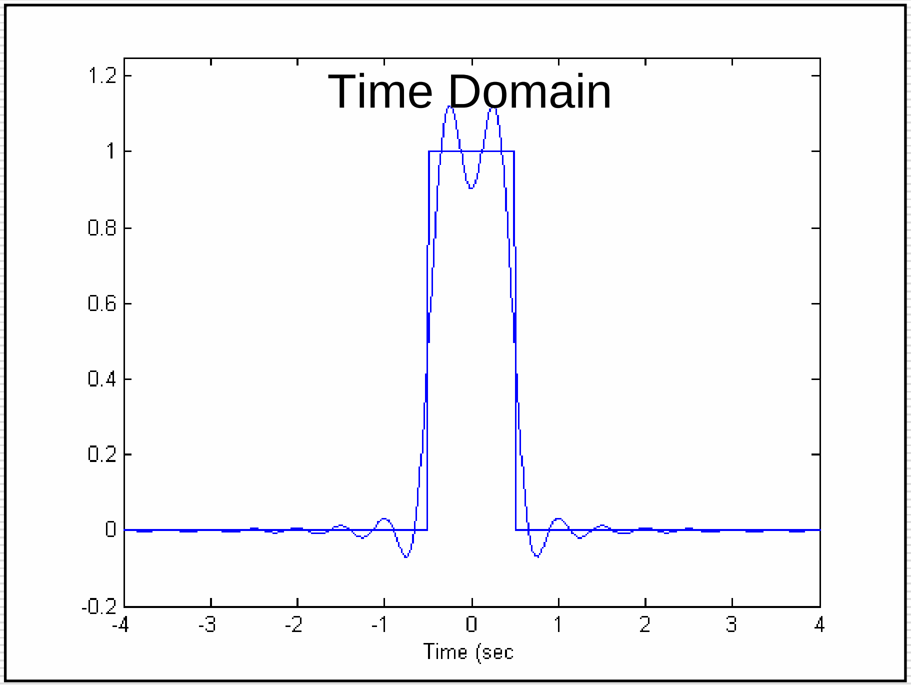
- Now consider a a filter with a bandwidth of 3Hz – BT = 3
- The bandwidth restriction still does not cause a substantial change in the pulse shape
- Ringing occurs near sharp transitions



Example 7.1 – cont.

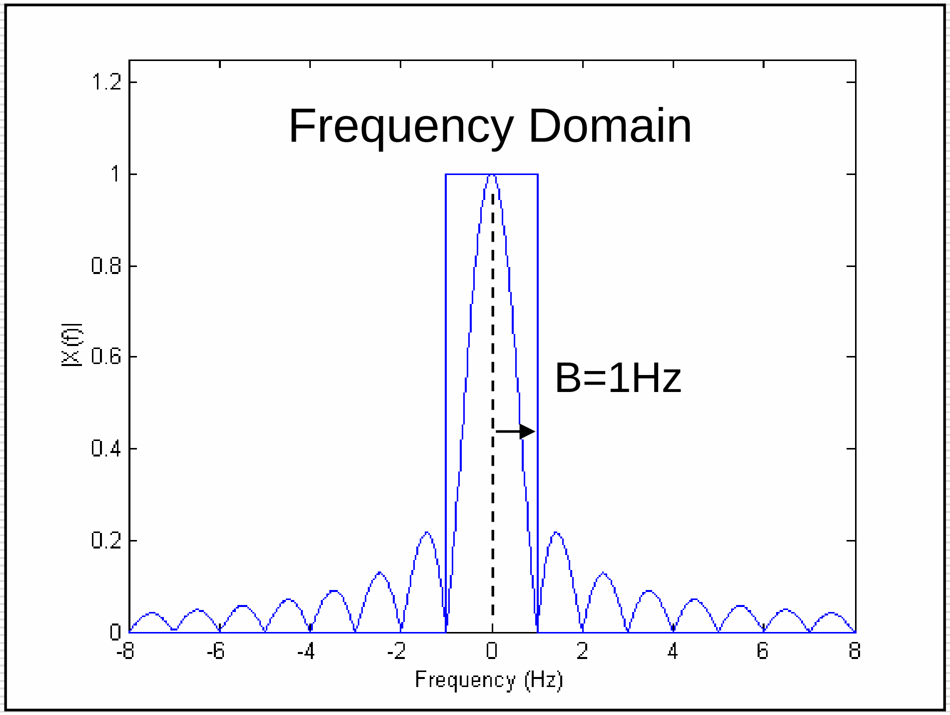
- Now consider a a filter with a bandwidth of 2Hz – BT = 2
- The bandwidth restriction now begins to cause a more substantial change in the pulse shape
- Sharp transitions cannot occur

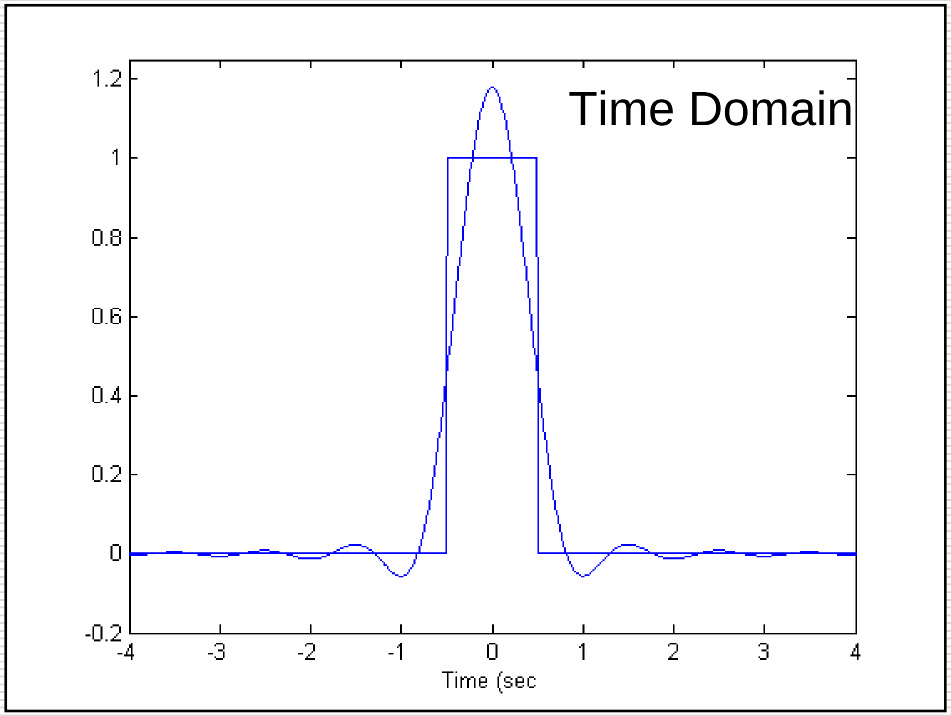




Example 7.1 – cont.

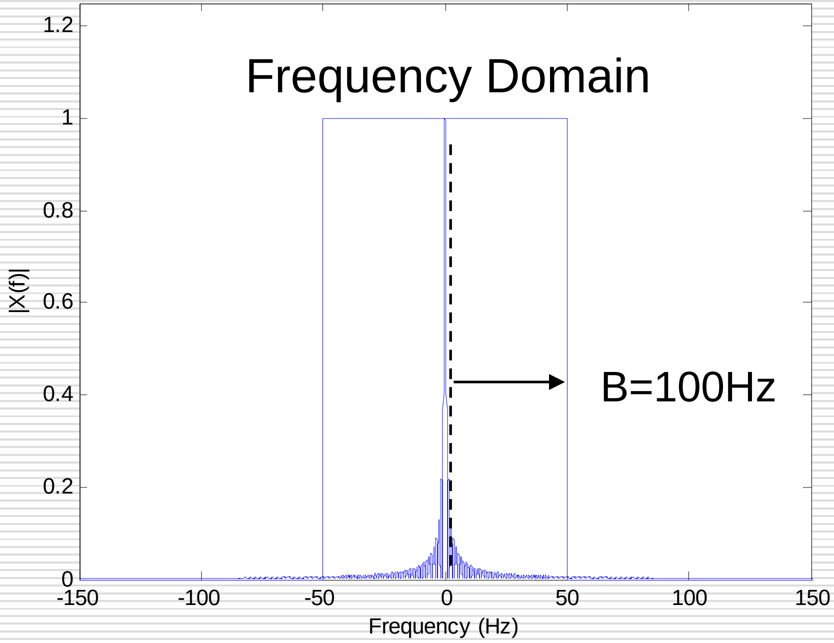
- Now consider a a filter with a bandwidth of 1Hz (BT =1)
- The bandwidth restriction now changes the shape considerably

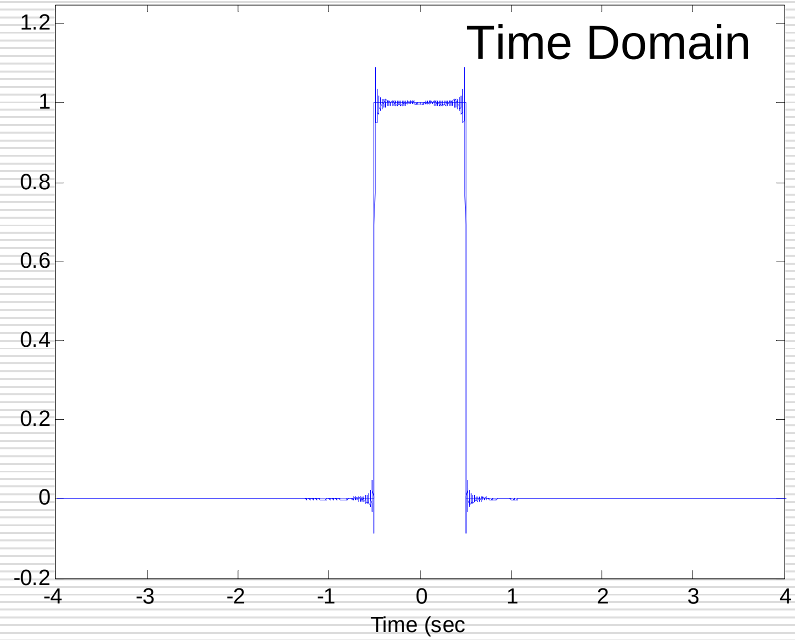




Example 7.1 – final

- Finally, consider a a filter with a bandwidth of 100Hz (BT =100)
- The pulse shape is nearly unchanged with the exception of Gibbs phenomenon.





Ideal Bandpass Filter

- The ideal bandpass filter can be written as

$$H(f) = \begin{cases} Ae^{-j2\pi f t_o} & f_L \le |f| \le f_H \\ 0 & else \end{cases}$$

$$= \underbrace{A\left[rect\left(\frac{f-f_o}{\Delta f}\right) + rect\left(\frac{f+f_o}{\Delta f}\right)\right]}_{\substack{magnitude \\ response}} \underbrace{e^{-j2\pi f t_o}}_{\substack{time \\ delay}}$$

$$h(t) = F^{-1}\{H(f)\}$$

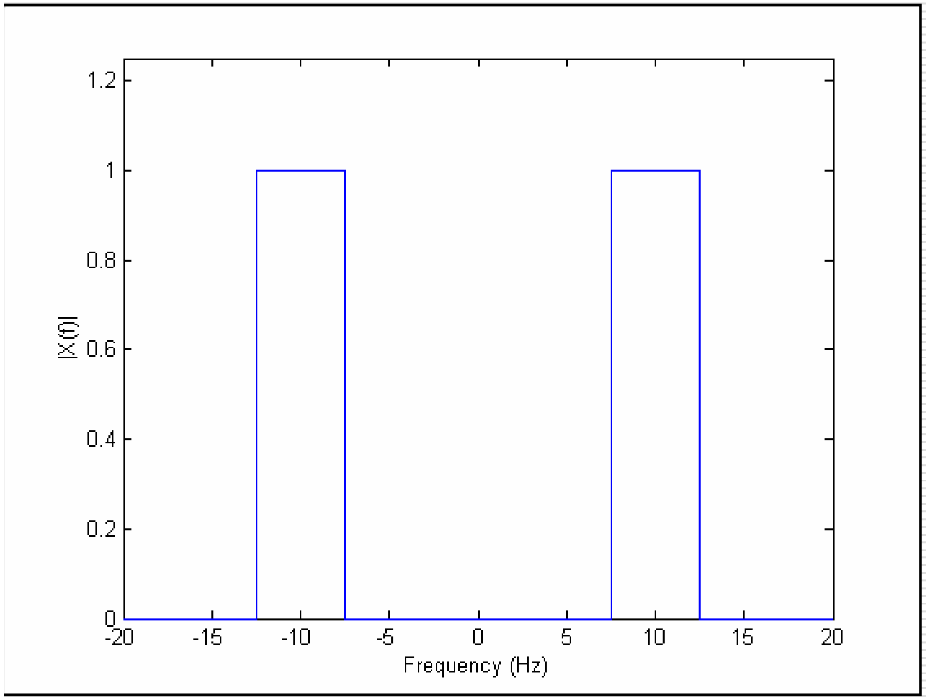
$$= F^{-1}\left\{A\left[rect\left(\frac{f-f_o}{\Delta f}\right) + rect\left(\frac{f+f_o}{\Delta f}\right)\right]e^{-j2\pi f t_o}\right\}$$

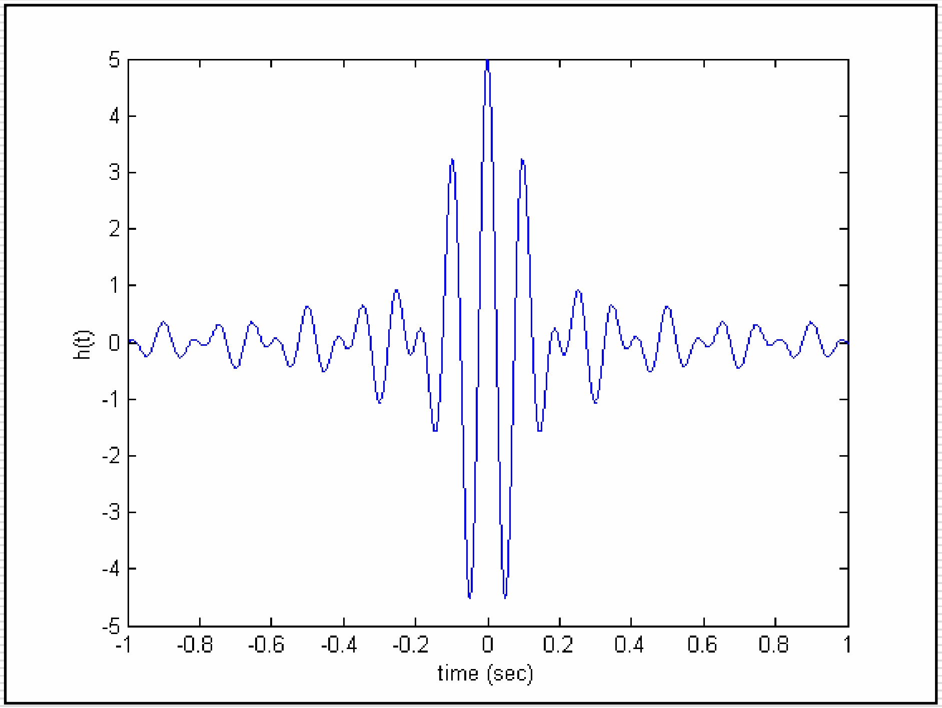
$$= 2A\Delta f\,\mathrm{sinc}\left(\Delta f_m\left(t-t_o\right)\right)\cos\left(2\pi f_o\left(t-t_o\right)\right)$$

f_L = lower frequency limit
f_H = upper frequency limit
$\Delta f = f_H - f_L$
$f_o = (f_H – f_L)/2$

Ideal Bandpass Filter

- Ideal BPF is also non-causal
- Realistic bandpass filters will be causal and not have a perfect frequency response



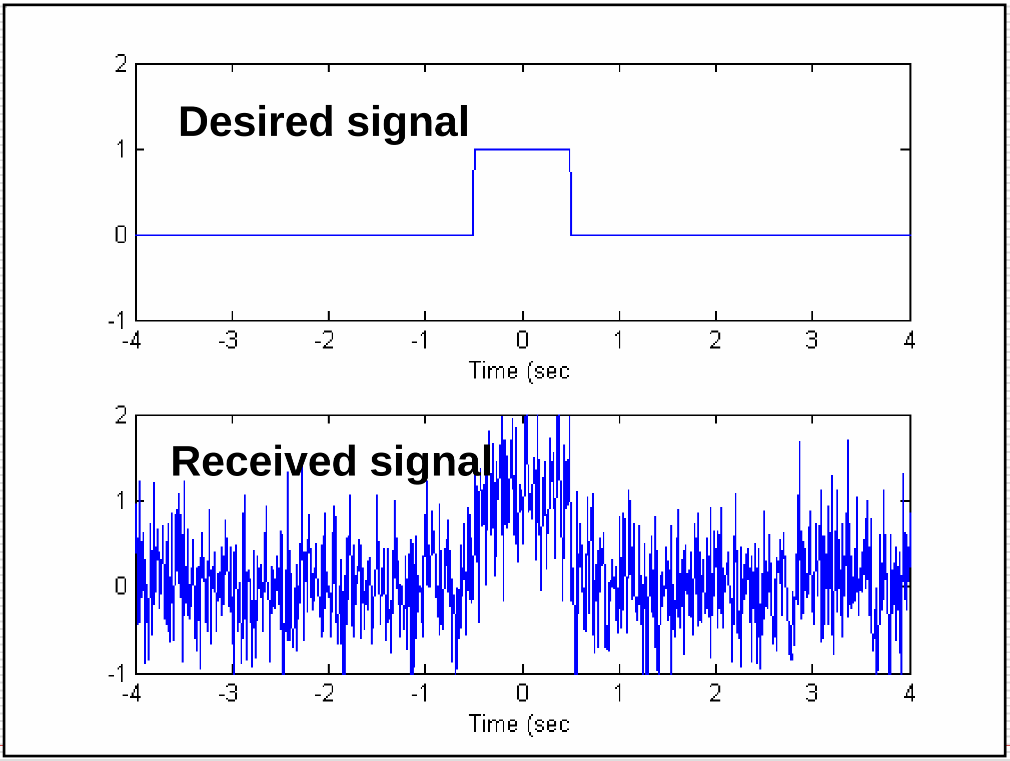


Example 7.2 - Noise

- A major use of filters is the elimination of noise.
- Noise typically has a much larger bandwidth than the signal of interest.
  - Filtering the received signal with a bandpass or lowpass filter can reduce the amount of noise
- We typically consider *additive noise* where the received signal $r(t)$ is equal to the desired signal $x(t)$ plus noise $n(t)$
  - $r(t) = x(t) + n(t)$

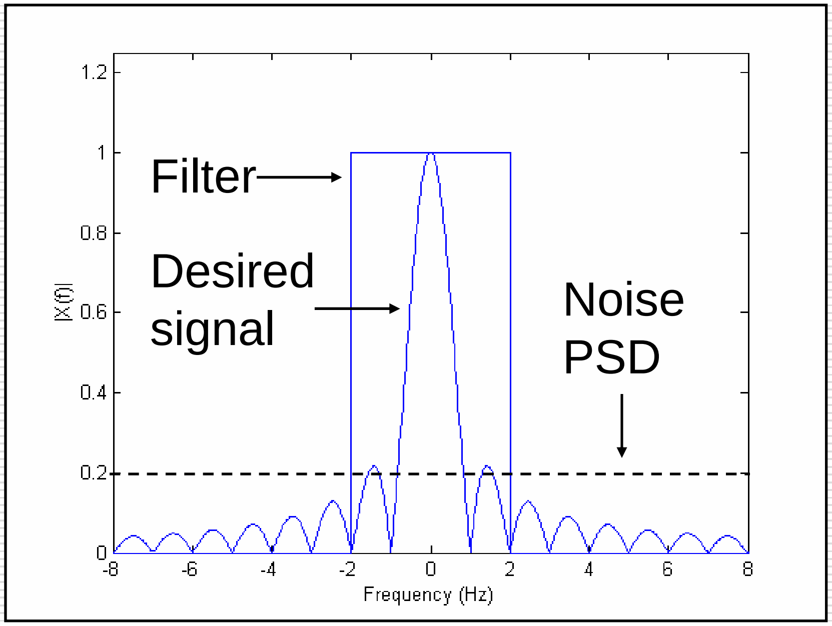
Example 7.2 (cont.)

- Consider a square pulse with duration one second that is received with the addition of noise
- The ratio of the received desired signal power to the noise power, signal-to-noise ratio or SNR, is 2

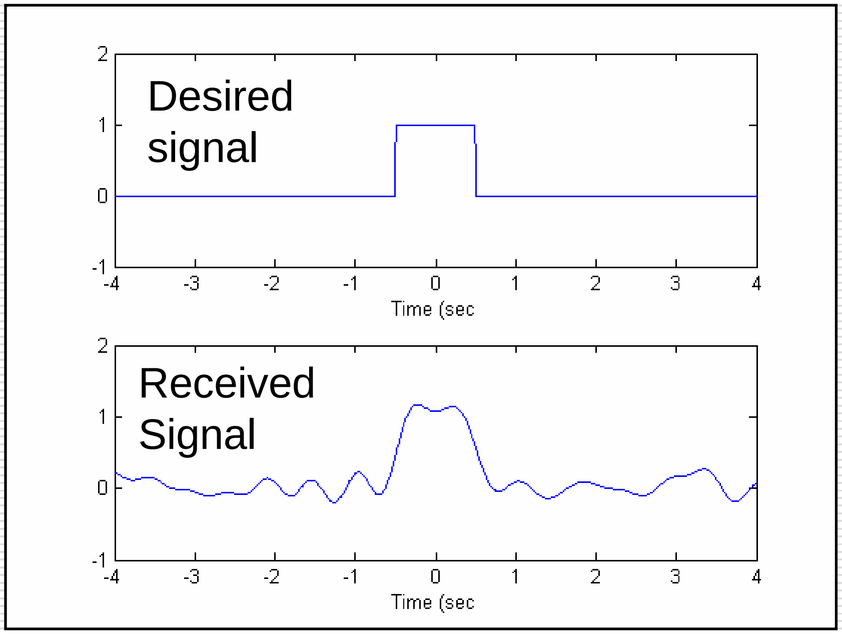


Example 7.2 (cont.)

- If we filter the signal with an ideal LPF with bandwidth B = 2Hz, we know that we will introduce some distortion to the desired signal, but we can also eliminate much of the noise



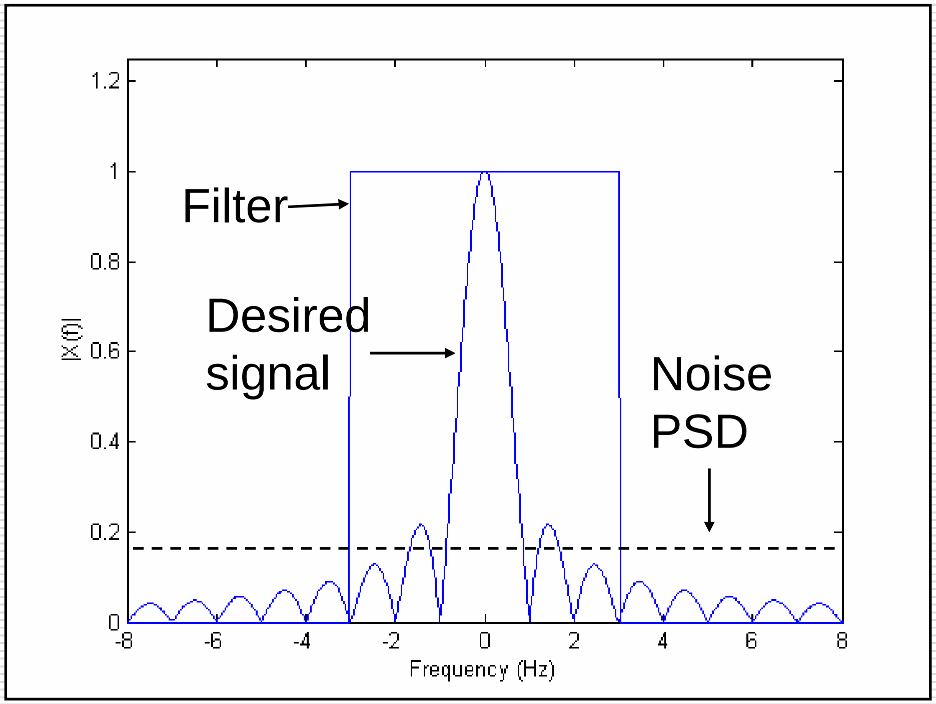
Frequency Domain



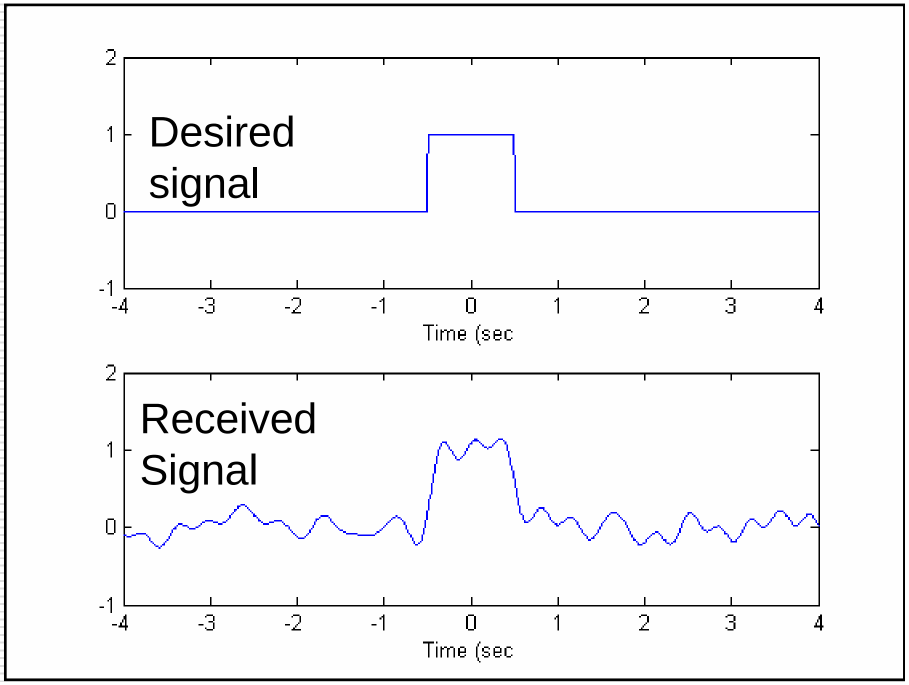
Time Domain

Example 7.2 (cont.)

- We know that increasing the bandwidth to B = 3Hz will reduce the amount of distortion to the original signal
- However, it also lets more noise in



Frequency Domain



Time Domain

Summary

- An important function in communication systems is to filter the input signal
  - Filtering helps eliminate noise in the system but can also distort the desired signal
- Increasing the value of the time-bandwidth product *BT* tends to reduce the rise time and decay time of the filter pulse response and helps to preserve the pulse shape.
- Increasing the bandwidth of the filter also allows more noise into the system.
- Ideal filters are non-causal. Causal filters can approximate the ideal filter using delay. More delay can allow better approximation.
- Filters are built in the analog domain using resistors, capacitors and inductors. They are often implemented digitally using simple tapped-delay lines in DSP.