ECE3614 Introduction to Communications Systems Fall 2007

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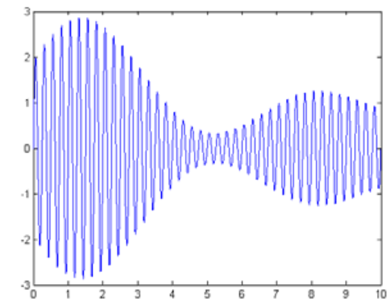
Lecture #9: Introduction to Amplitude Modulation



Overview

- Today we will begin our discussion of analog communication systems by discussing sinusoidal modulation techniques
- Specifically we will discuss amplitude modulation or AM
  - The next 3-4 lectures will discuss this technique which is a fundamental broadcast radio technique
- Reading
  - 3.1 - 3.2

Modulation

- Modulation is defined as a process where an *information-bearing signal* causes changes to some characteristic of a carrier signal.
- The carrier signal can be either a pulse stream (a series of pulses in time) as in baseband communications or a sinusoid as in bandpass communications
- We will examine sinusoidal carrier modulation first

Types of Sinusoidal Modulation

- In general time-varying modulation of a sinusoid can be written as

$$s(t) = A(t)\cos\left(2\pi f_c t + \theta(t)\right)$$

  - f_c – the nominal carrier frequency
  - $A(t)$ – time varying amplitude
  - $\theta(t)$ – time varying angle
- The information-bearing signal can be used to modulate (change) the amplitude or the angle of the sinusoid
  - The receiver can then recover the original information by examining the amplitude or angle of the received sinusoid

Types of Sinusoidal Modulation

- Amplitude Modulation
  - The amplitude of the carrier is varied according to the message signal
  - Let $F_{AM}(m(t))$ be the function or mapping of the message to the amplitude:

$$s(t) = F_{AM}\left(m(t)\right)\cos\left(2\pi f_c t\right)$$

- Angle Modulation
  - The angle of the carrier is varied according to the message of the signal
  - Frequency modulation – message directly affects the carrier frequency

$$s(t) = A_c \cos\left(2\pi\left[f_c + F_{FM}\left(m(t)\right)\right]t\right)$$

  - Phase modulation – message directly affects the carrier phase

$$s(t) = A_c \cos\left(2\pi f_c t + F_{PM}\left(m(t)\right)\right)$$

Analog Modulation

- Later in the course we will consider *digital modulation* where the message signal is digital.
- For now we consider *analog* modulation of a sinusoidal carrier
  - The transmitted (carrier) signal varies continuously with message signal
- Types of analog modulation
  - Amplitude modulation (AM) - we consider first
  - Phase modulation (PM) - not widely used
  - Frequency modulation (FM) - we will consider this in a couple of weeks

Amplitude Modulation

- $m(t)$ – analog message signal
- Transmitted signal:

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos\left(2\pi f_c t\right)$$

  - Message modulates the amplitude
  - First term can be thought of as adding a pilot tone to spectrum
  - Also called "Large carrier" AM or "Double Sideband Large Carrier (DSB-LC) AM"
  - Note that we require $|k_a m(t)| < 1$. When this is violated we call this "over-modulation".
    - A coherent (product) detector is required to recover the message signal in this case.
    - A simple envelope detector cannot be used
  - Modulation % =

$$\frac{A_{\max} - A_{\min}}{2A_c} \times 100 \xrightarrow{\text{Also equal to}} \max\left\{\left|k_a m(t)\right|\right\} \times 100$$

Definitions

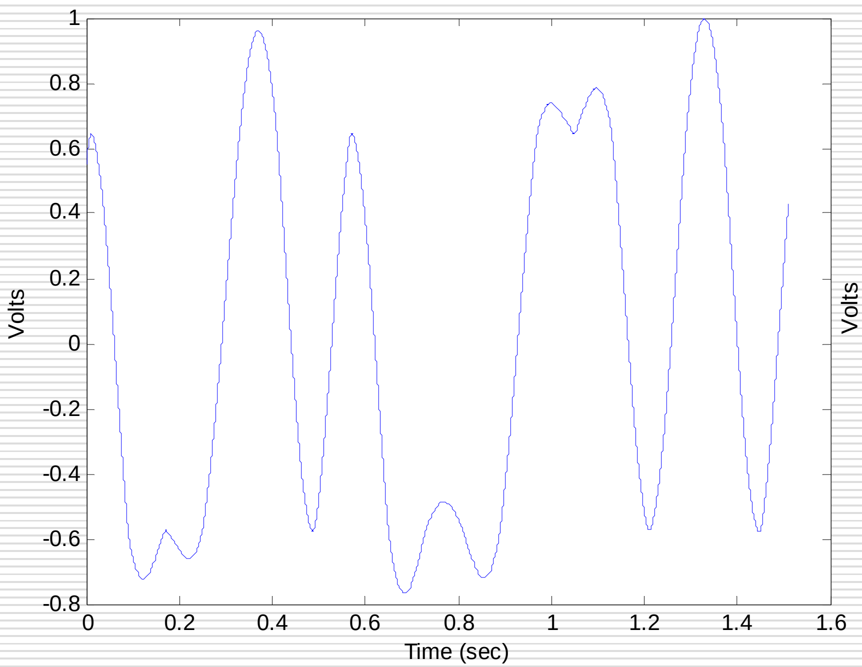
- Message signal – information-bearing signal that is to be recovered at the receiver [*m(t)*]
- Carrier – the sinusoid with frequency *fc* that is used to “carry” the information signal
- Envelope – the time-varying magnitude of the sinusoidal signal

Envelope Variation

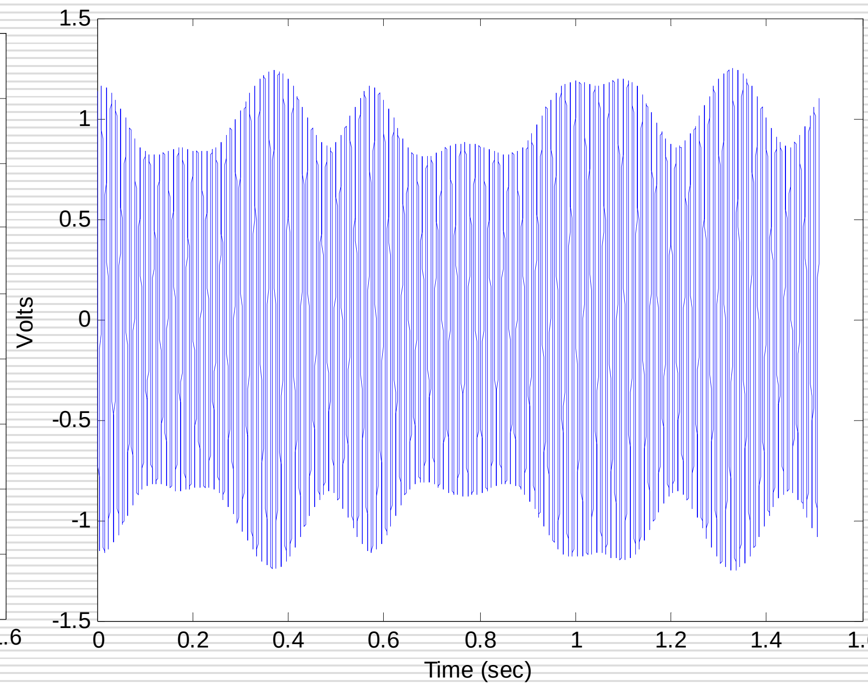
- The envelope of the transmit signal *s(t)* has the same shape as the message signal provided that
  1. Over-modulation doesn't occur. In other words as long as $|k_a m(t)| < 1$
     - This is the same as saying that $(1+k_a m(t))$ must be positive. Since this represents the amplitude of the carrier, we say that the amplitude cannot be "negative"
     - Negative amplitude corresponds to a phase reversal
  2. The carrier frequency is much greater than the message bandwidth ($f_c >> W$)
     - This is the same as saying that the carrier signal changes much more quickly than the message signal

Example – $k_a = 0.25$, $\max\{|m(t)|\} = 1$

Message signal

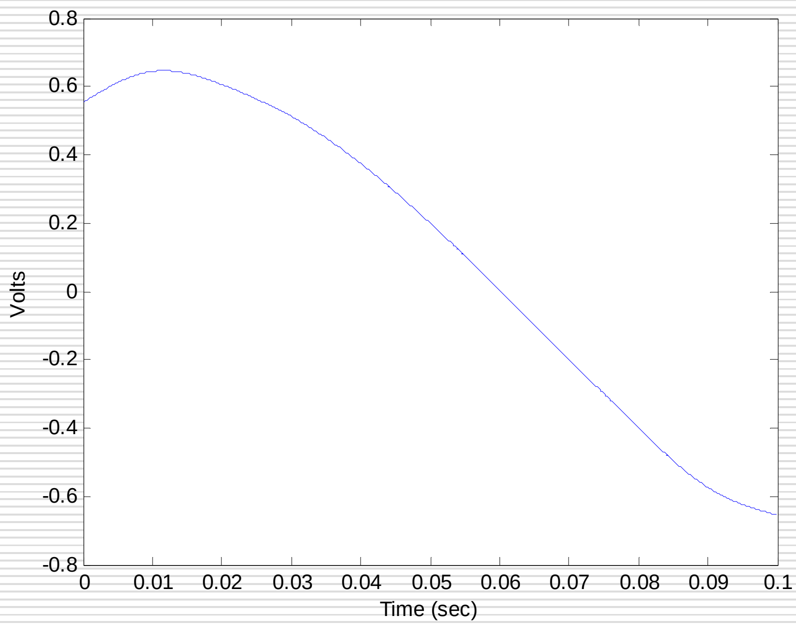


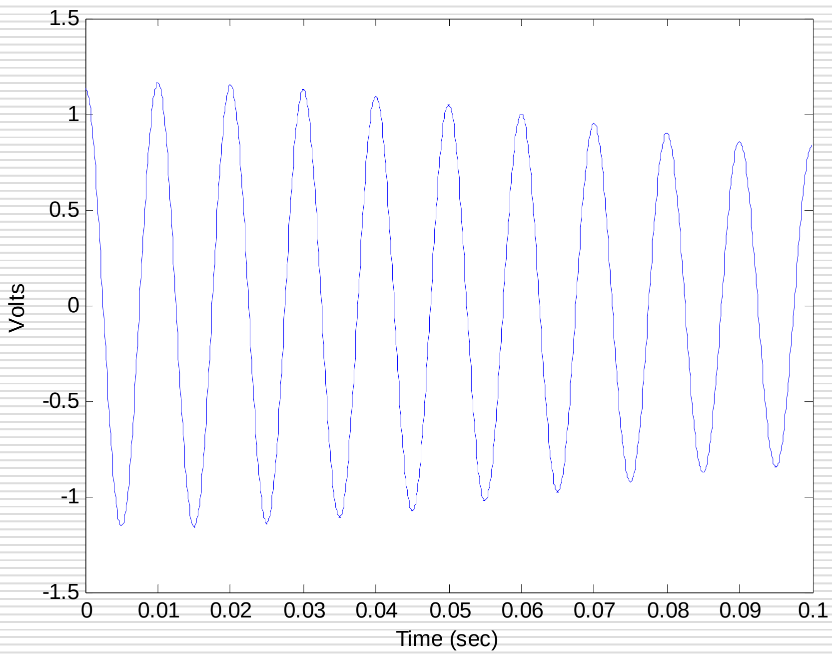
Modulated carrier



Example – k_a = 0.25, max{m(t)} = 1

- Close-up

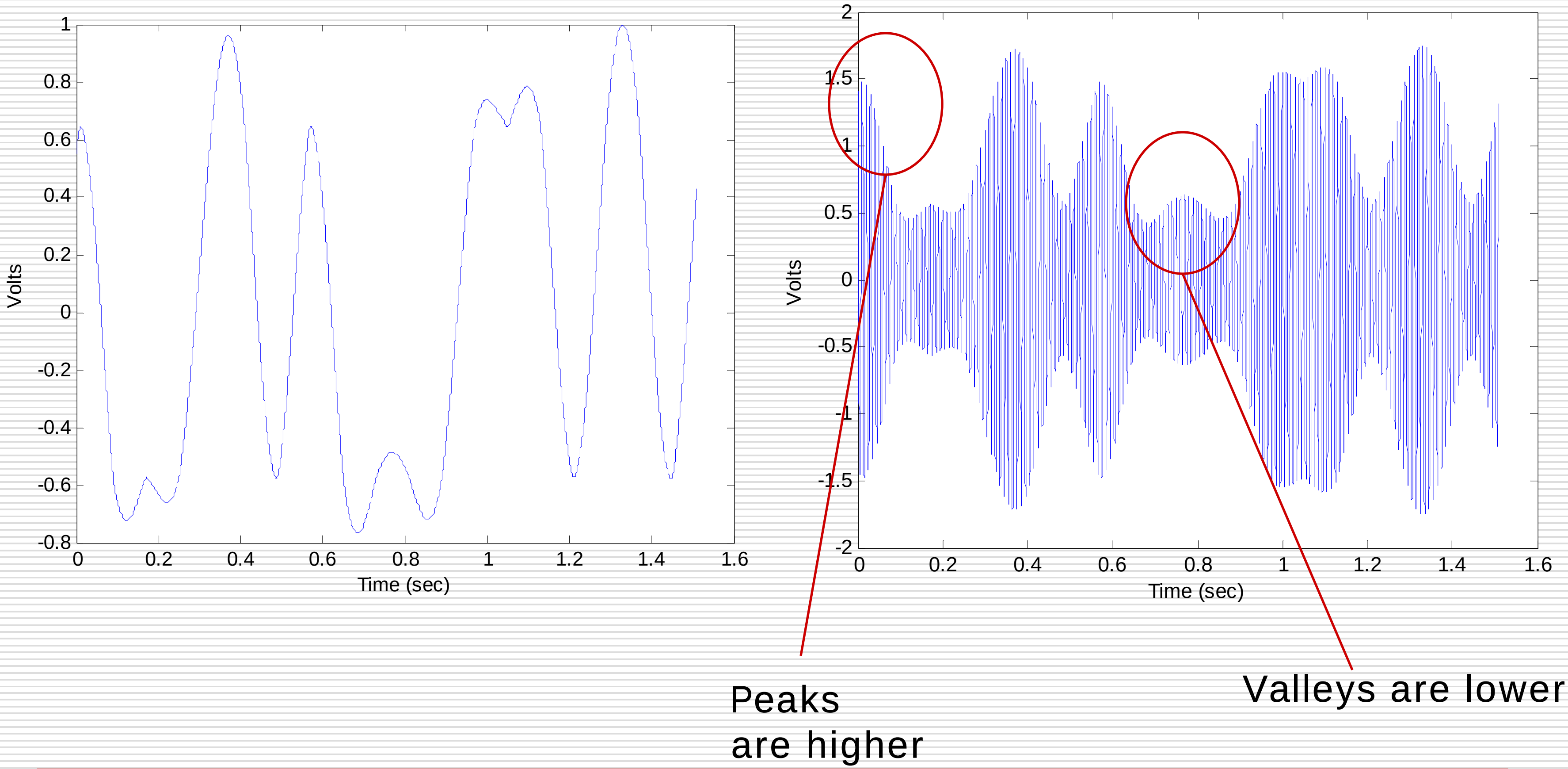




Example – k_a = 0.75, max{m(t)} = 1

Message signal

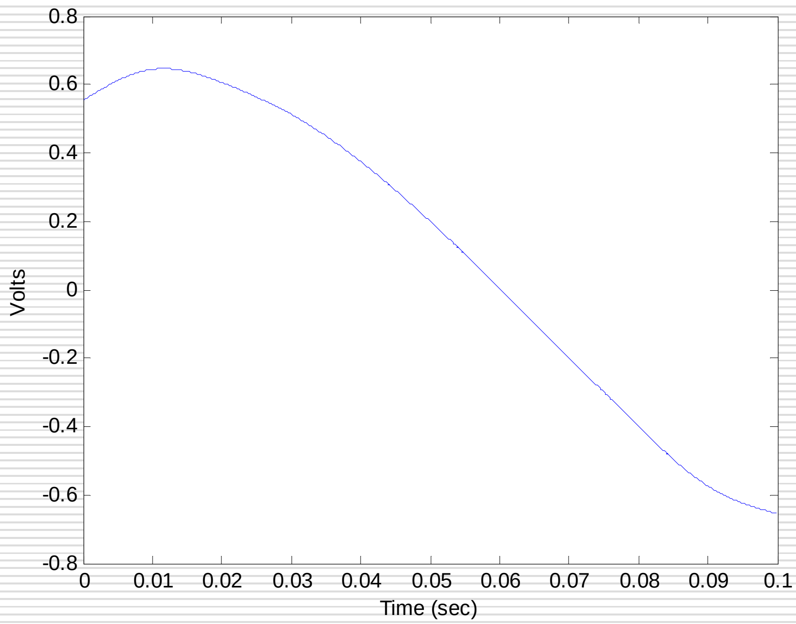
Modulated carrier



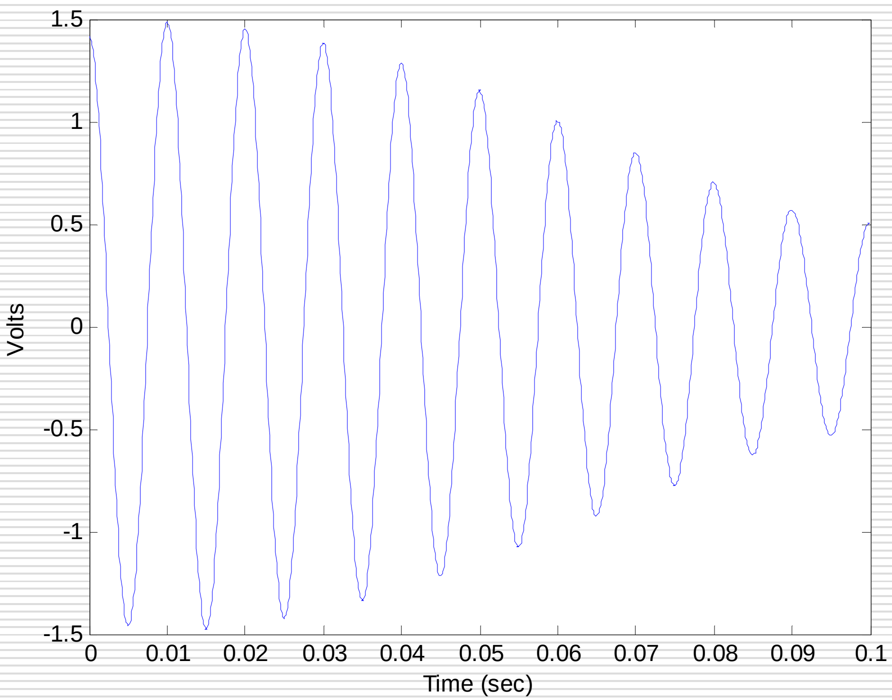
Example – k_a = 0.75, max{m(t)} = 1

- Close-up

Message signal

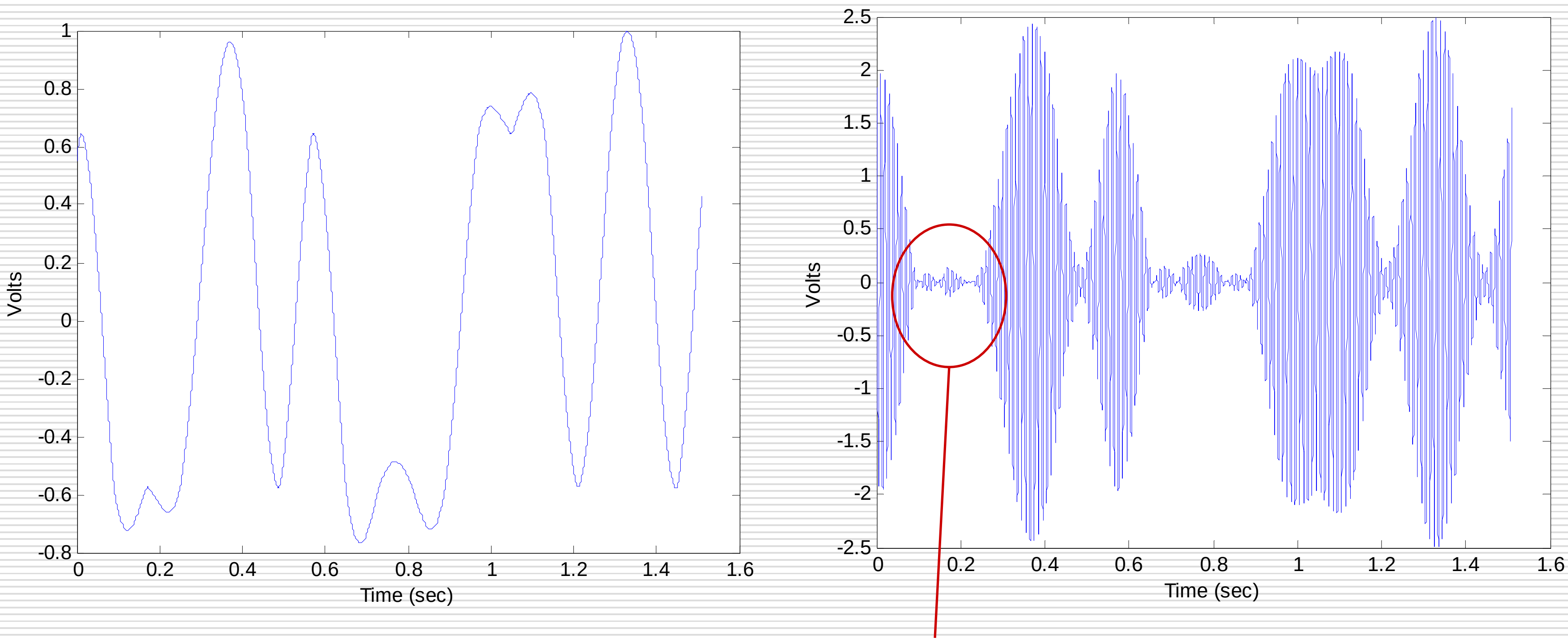


Modulated carrier



Example – $k_a = 1.5$, $\max\{m(t)\} = 1$

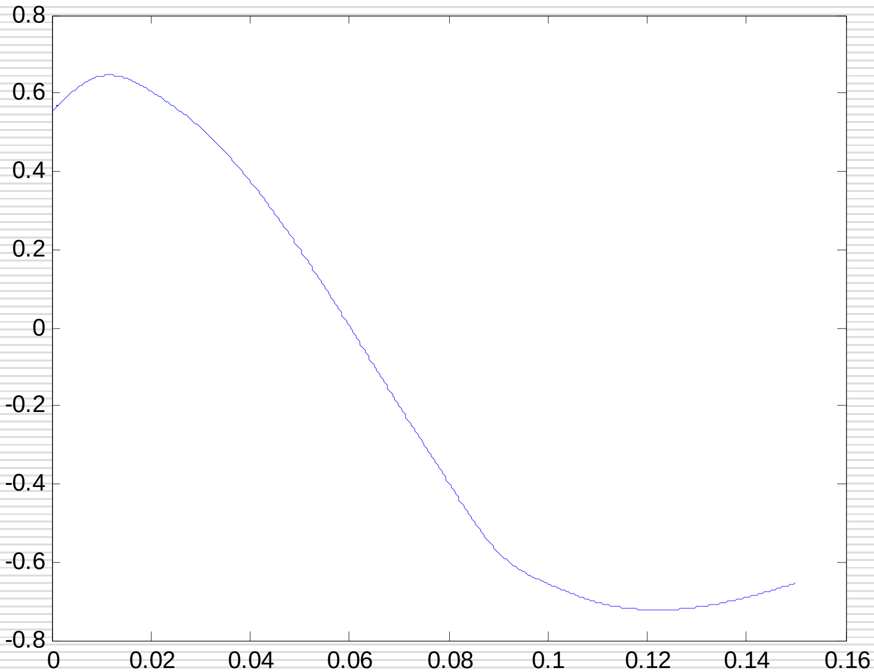
- Example of over-modulation.

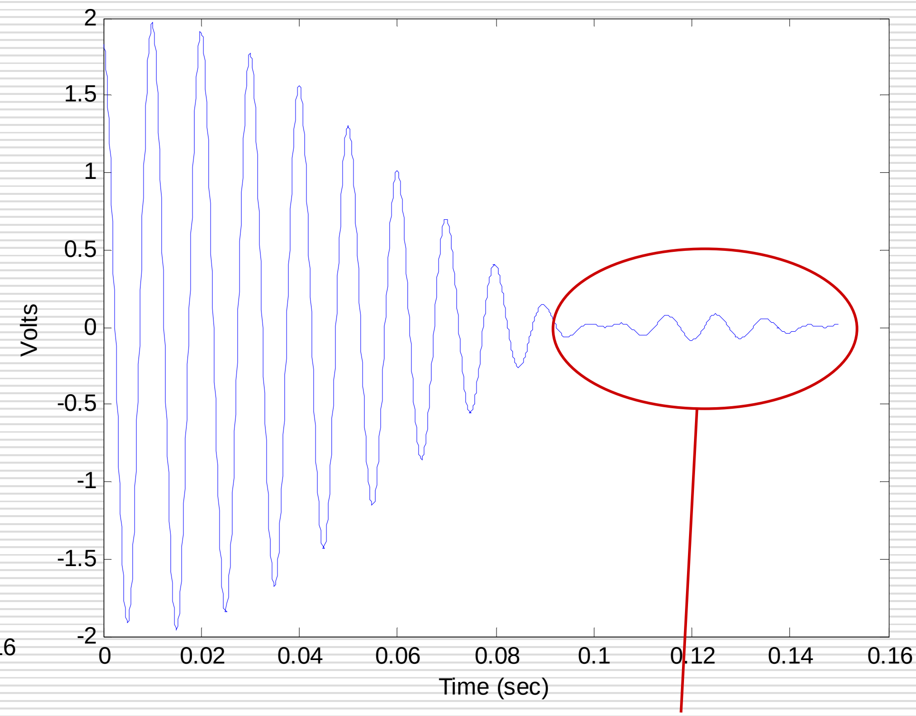


Note envelope distortion

Example – k_a = 1.5, max{m(t)} = 1

- Close-up





Note envelope distortion

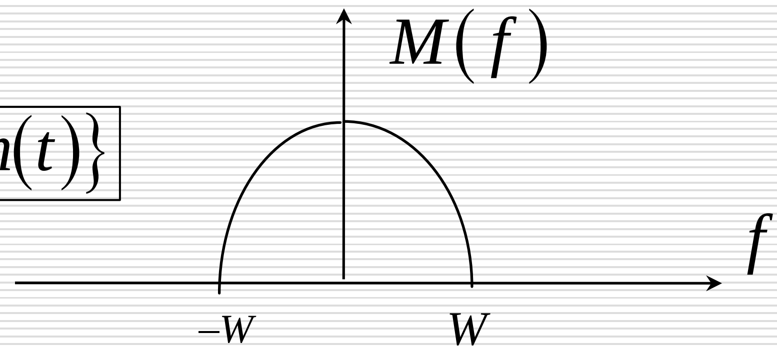
Frequency Domain

- The spectrum *S(f)* can be determined from Fourier Theory using the modulation property

$$
\begin{aligned}
S(f) &= F\{s(t)\} \\
&= F\{A_c[1+k_a m(t)]\cos(2\pi f_c t)\} \\
&= F\{A_c\cos(2\pi f_c t)\} + F\{A_c k_a m(t)\cos(2\pi f_c t)\} \\
&= \frac{A_c}{2}\{\delta(f-f_c)+\delta(f+f_c)\} + \frac{A_c k_a}{2}\{M(f-f_c)+M(f+f_c)\}
\end{aligned}
$$

Spectrum of DSB-LC AM

$$M(f) = F\{m(t)\}$$

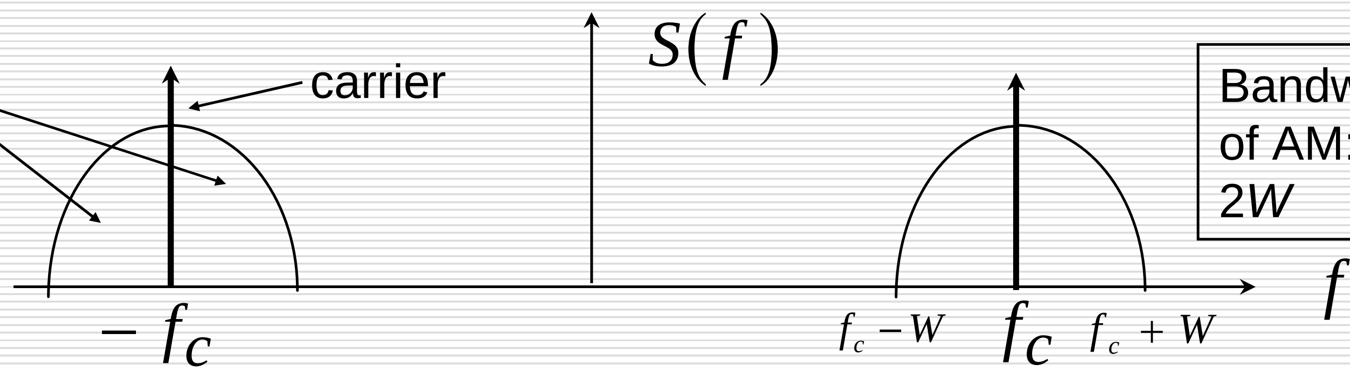


Bandwidth of message : *W*

Message

$$S(f) = \frac{A_c}{2}\left[\underbrace{k_a M(f - f_c) + k_a M(f + f_c)}_{F\{k_a m(t)\cos(2\pi f_c t)\}} + \underbrace{\delta(f - f_c) + \delta(f + f_c)}_{F\{\cos(2\pi f_c t)\}}\right]$$

Since both "sidebands" are present we call this *double* sideband



Bandwidth of AM: 2*W*

Example – Sinusoidal modulation

- Consider the message

$$m(t) = A_m \cos(2\pi f_m t)$$

- The AM signal is then

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$
$$= A_c [1 + k_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

- The modulation factor (modulation percentage divided by 100) is

$$\mu = \max\{|k_a m(t)|\} = k_a A_m$$

- From the equation for the AM signal we can determine the ratio of the maximum amplitude value to the minimum amplitude value:

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$

Example (cont.)

- Rearranging $\mu = \dfrac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$
- Further, the product of cosines can be expressed in terms of the sum and difference frequencies

$$s(t) = A_c\left[1 + k_a A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{1}{2}A_c\mu\cos\left(2\pi(f_c - f_m)t\right) + \frac{1}{2}A_c\mu\cos\left(2\pi(f_c + f_m)t\right)$$

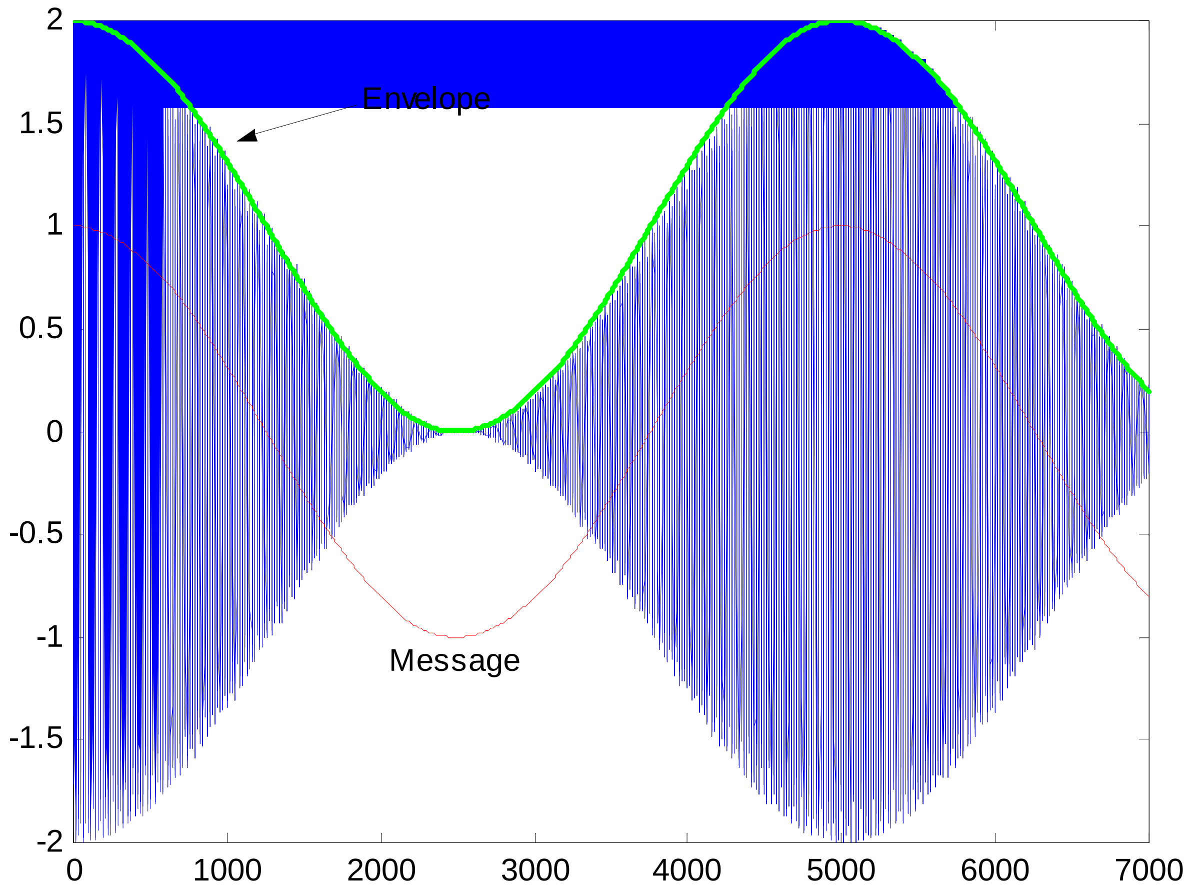
- Clearly, the Fourier Transform is

$$S(f) = \frac{A_c}{2}\left\{\delta(f - f_c) + \delta(f + f_c)\right\} + \frac{1}{4}A_c\mu\left\{\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right\}$$

$$+ \frac{1}{2}A_c\mu\left\{\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right\}$$

Amplitude Modulation: DSB-LC

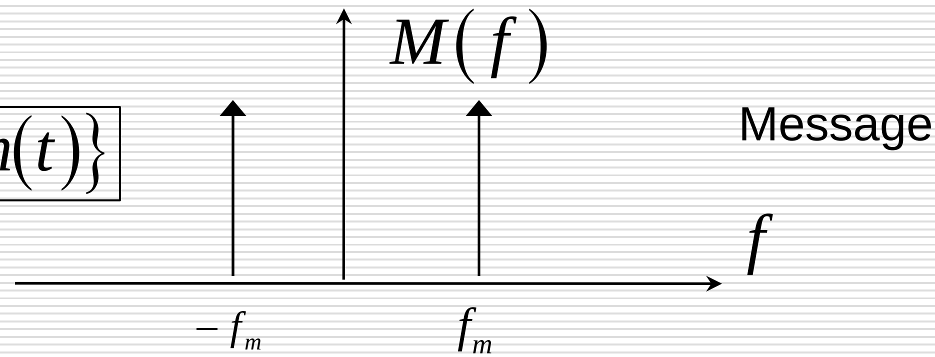
- Ex: AM wave modulated by a sinusoid. (100% Modulation)

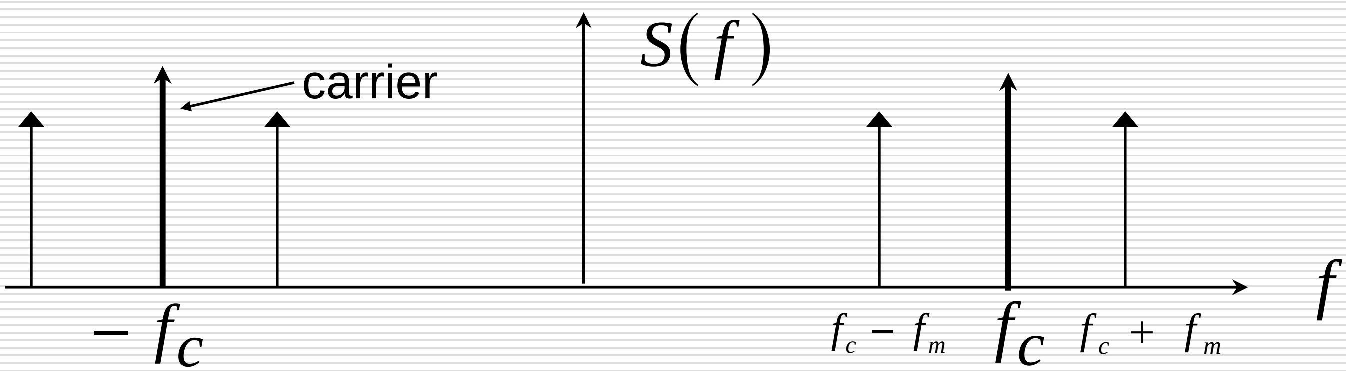


Note that the information is carried in the "envelope" $[1+m(t)]$ of the waveform

Example (spectrum)

$$M(f) = F\{m(t)\}$$





Power in the carrier: $P_c = \frac{1}{2} A_c^2$

Power in the message: $P_m = \frac{1}{4} \mu^2 A_c^2$

$$\frac{P_m}{P_c} = \frac{\frac{1}{4}\mu^2 A_c^2}{\frac{1}{2} A_c^2} = \frac{\mu^2}{2}$$

Even with 100% modulation, only 1/3 of the total power is dedicated to the message

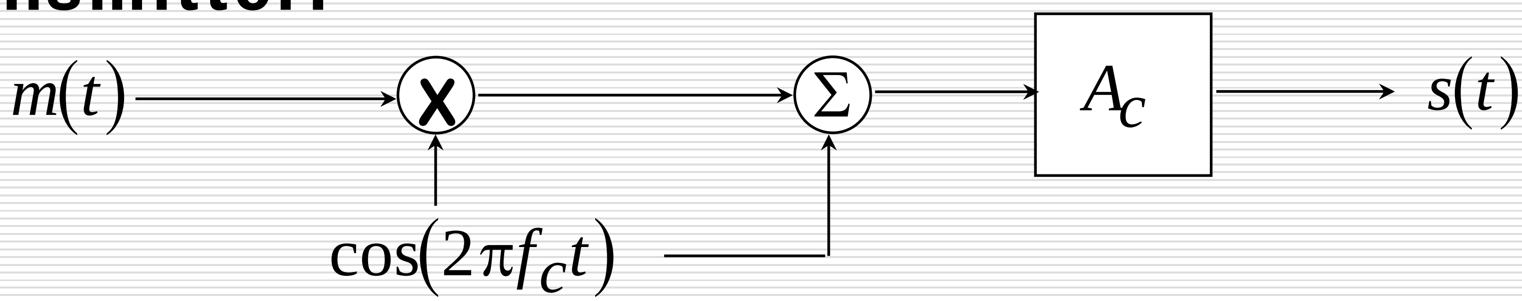
Notes

- fc >> *W* in order to allow the message signal to be recovered from the envelope
  - This can be seen from the spectrum plot
- Doubling of bandwidth – due to the fact that "negative" frequencies are moved into the positive frequency range we see that the bandwidth of the transmit signal doubles
  - Shows the usefulness of negative frequency
- Power is wasted in the carrier.
  - Since we want to avoid over-modulation we must transmit an unmodulated carrier which carries no information

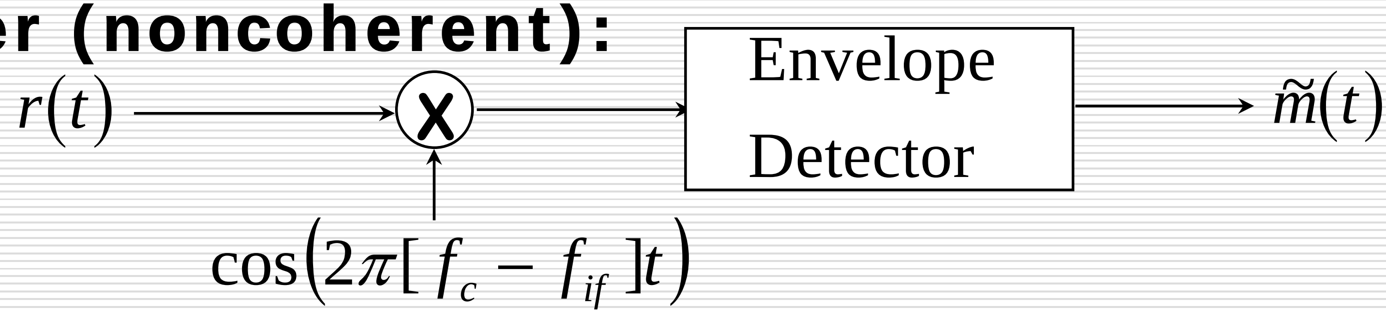
In-class Drill

Transmitter and Receiver for DSB-LC AM

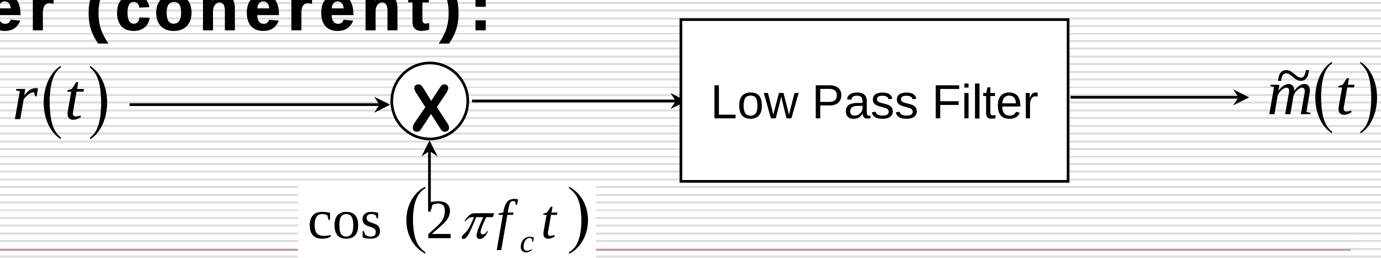
- **Transmitter:**



- **Receiver (noncoherent):**

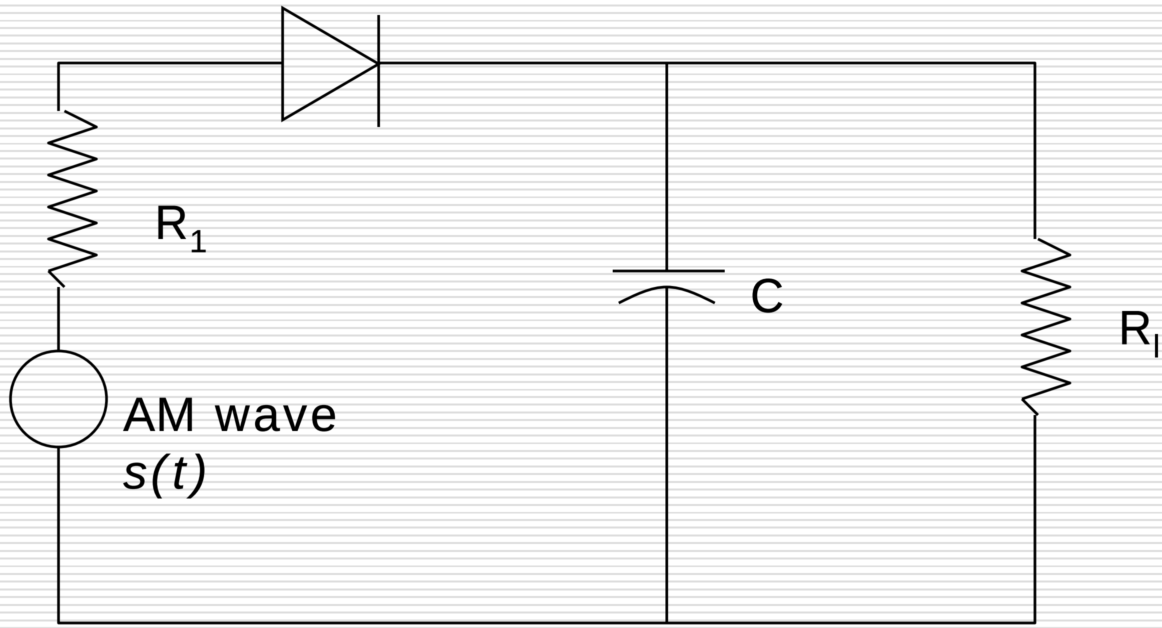


- **Receiver (coherent):**



Envelope Detector

- Consider the circuit



- This circuit can be used to detect an AM signal provided that (a) the carrier frequency is much greater than the bandwidth of the message and (b) the modulation is less than 100%

Envelope Detector

1. When the input signal is positive, the diode is forward-biased (with small resistance r_f) and the capacitor C charges rapidly to the peak value.
   - (This requires that $(r_f+R_1)C << 1/f_c$)
2. When the input signal is falls below the capacitor charged value, the diode is reversed-biased and the capacitor discharges slowly through the load resistor R_l.
   - This requires that $1/f_c << R_l C << 1/W$
3. Discharging continues until the next positive half-cycle.
4. When the input signal is greater than the capacitor voltage, the diode conducts again and allows the capacitor to charge.

Summary

- In this lecture we have introduced the concept of Amplitude Modulation
- This modulation scheme allows a sinusoidal carrier to be used to carry a message signal in its time-varying amplitude or envelope
- The simple modulation scheme
  - Doubles the signal bandwidth
  - Wastes power in the unmodulated carrier
  - Allows for an extremely simple receiver
  - Was first widely implemented in the 1920's with the emergence of broadcast AM radio
- We will examine techniques which are more complicated but are more bandwidth and energy efficient