3614: Introduction to Communication Systems

Midterm Exam I

September 20, 2007

SOLUTION

I pledge that I have neither given nor received any assistance on this exam.

_____

(signed)

Name (print) _____ Student Number _____

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1. (20 points) Multiple Choice – Choose the answer which best completes the sentence or answers the question.

1.1 [5 points] Which of the following is causal filter?

(a) H(f) = rect(f/1000)
(b) h(t) = exp(a (t+2)) u(t)
(c) sinc(10000(t-10)) u(t)
(d) None of the above

either - Both (b) and (c) are causal. Either answer accepted.

1.2 [5 points] Increasing a filter's bandwidth

(a) increases the time duration of its impulse response
(b) speeds up the impulse response (in time)
(c) has no impact on the impulse response
(d) None of the above

1.3 [5 points] The bandwidth of a signal $m(t)$ is

(a) less than the bandwidth of $m(t)\cos(2\pi ft)$
(b) greater than the bandwidth of $m(t)\cos(2\pi ft)$
(c) the same as the bandwidth of $m(t)\cos(2\pi ft)$
(d) unrelated to the bandwidth of $m(t)\cos(2\pi ft)$
(e) None of the above

1.4 [5 points] The Fourier Series applies to

(a) periodic signals
(b) aperiodic signals
(c) Energy signals
(d) None of the above

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2. (30 points) Fourier Transform

(a) [15 points] Determine the inverse Fourier Transform of the signal

$$X(f)=\frac{200e^{-j2\pi(f-1000)/6000}}{10000+4\pi^2(f-1000)^2}$$

$$\begin{cases} \exp(-a|t|) \rightleftharpoons \frac{2a}{a^2+(2\pi f)^2} & 3 \\ x(t-t_0) \rightleftharpoons X(f)e^{-j2\pi f t_0} & 3 \\ e^{+j2\pi f_0 t}x(t) \rightleftharpoons X(f-f_0) & 3 \end{cases}$$

$$\exp(-100|t|) \rightleftharpoons \frac{200}{10{,}000+4\pi^2 f^2}$$

$$\exp(-100|t-1/6000|) \rightleftharpoons \frac{200\,e^{-j2\pi f/6000}}{10{,}000+4\pi^2 f^2}$$

$$\exp(-100|t-1/6000|)\exp(j2000\pi t)$$

$$\Updownarrow$$

$$\frac{200\,e^{-j2\pi(f-1000)/6000}}{10{,}000+4\pi^2(f-1000)^2}$$

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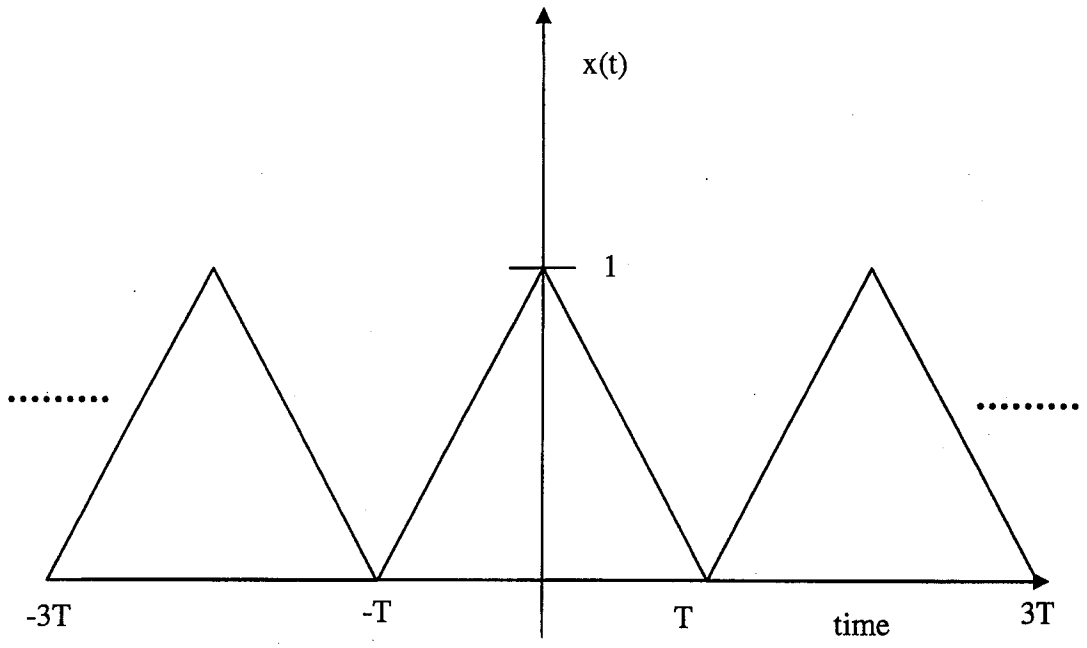
$$\exp(-100|t-1/6000|)\exp(j2000\pi t)$$

(b) [15 points] Determine the Fourier Transform of the function shown in the plot below.



$$X(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0) \quad 5$$

$$f_0 = 1/2T$$

For $x(t) = \sum_{n=-\infty}^{\infty} tri\left(\frac{t-nT}{T}\right) \overset{FS}{\rightleftharpoons} c_n = T/\tau \, sinc^2(T/\tau n)$ 5

$$\tau = 2T$$

$$x(t) = \sum_{n=-\infty}^{\infty} tri\left(\frac{t-n2T}{T}\right) \rightleftharpoons c_n = \frac{1}{2} sinc^2(n/2)$$

$$\boxed{X(f) = \sum_{n=-\infty}^{\infty} \frac{1}{2} sinc^2(n/2)\, \delta(f - n/2T)} \quad 5$$

Alternatively $x(t) = tri(t/T) * \sum_{n=-\infty}^{\infty} \delta(t - n \cdot 2T)$ 5

or $x(t) = T sinc^2(fT) \cdot \frac{1}{2T} \sum \delta(f - nf_0)$ 5

$= \sum_{n=-\infty}^{\infty} \frac{1}{2} sinc^2(nf_0T)\, \delta(f - nf_0) = \sum_{n=-\infty}^{\infty} \frac{1}{2} sinc^2(n/2) \cdot \delta(f - n/2T)$ 5

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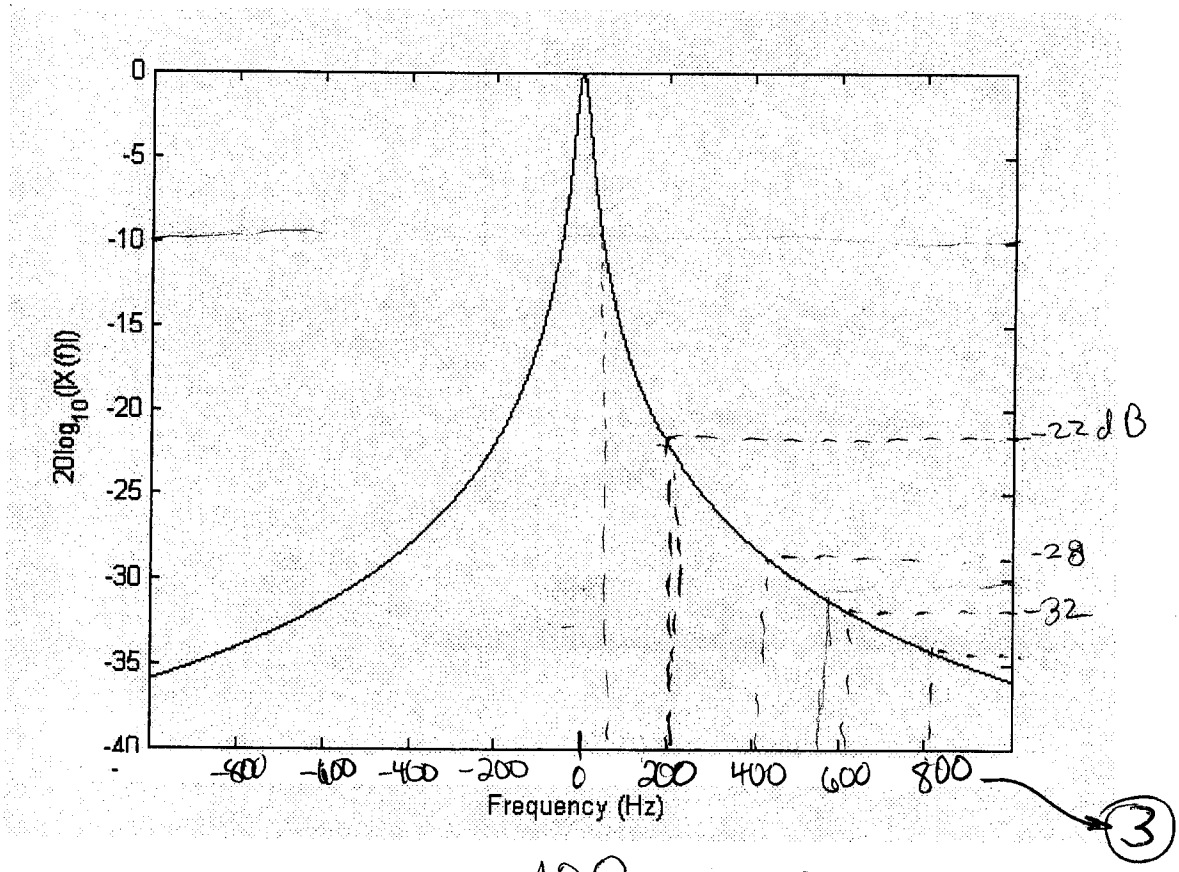
3. (25 points) Filters

Consider a filter with impulse response h(t) = 100exp(-100t) u(t) .

(a) [10 points] Fill in the x-axes values for the the Transfer Function plots below.



$$+|(f) = \frac{100}{100 + j2\pi f} \quad (3)$$

$$|H(f)| = \frac{100}{\sqrt{100^2 + 4\pi^2 f^2}} \quad (2)$$

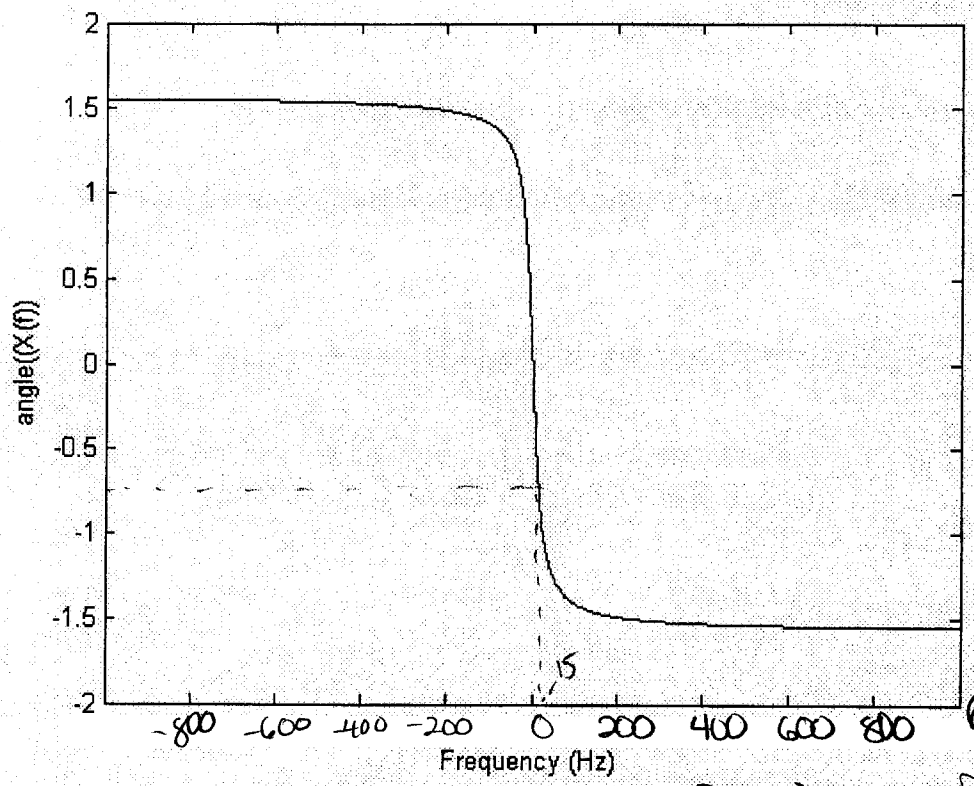
Y	F
-22dB	200Hz
-29dB	400
-32	620
30	503Hz
-10	47Hz
-20	158Hz

$$20\log(|H(f)|) = \underbrace{20\log_{10}(100)}_{40} - 10\log_{10}\left(100^2 + 4\pi^2 f^2\right)$$

$f = 0 \rightarrow 0dB$

$Y = 40 - 10\log_{10}(\ )$

$100^2 + 4\pi^2 f^2 = {}_{10}^{(40-Y)/10} \rightarrow f = \sqrt{\frac{1}{4\pi^2}\left(10^{(40-Y)/10} - 100^2\right)}$



(2)

$\arg(H(f)) = 0 - \tan^{-1}\left(2\pi f/100\right)$

$f = 100, -1.41$

$f = 200, -1.49$

Approaches $\pi/2$ very fast

$-\frac{\pi}{4} \rightarrow f = \frac{100}{2\pi} = 15\text{Hz}$

(b) [5 points] What is the 30dB bandwidth of the filter?

$Y = -30$

$f \cong 500\text{Hz}$

(c) [5 points] Is the filter causal?

Yes $h(t) = 0, t < 0$

(d) [5 points] If we changed the value 100 to 1000 in the impulse response would the 30dB bandwidth increase or decrease?

Increase

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4. (25 points) Power Spectral Density and Energy Spectral Density

(a) [15 points] If $x(t) = exp(-at)\, u(t)$ what is $R_x(\tau)$?

$$R_x(\tau) = \mathcal{F}^{-1}\{\Psi_x(f)\} \quad (3)$$

$$\Psi_x(f) = |X(f)|^2 \quad (3)$$

$$X(f) = \frac{1}{a + j2\pi f} \quad (3)$$

$$|X(f)|^2 = \frac{1}{a^2 + 4\pi^2 f^2} \quad (3)$$

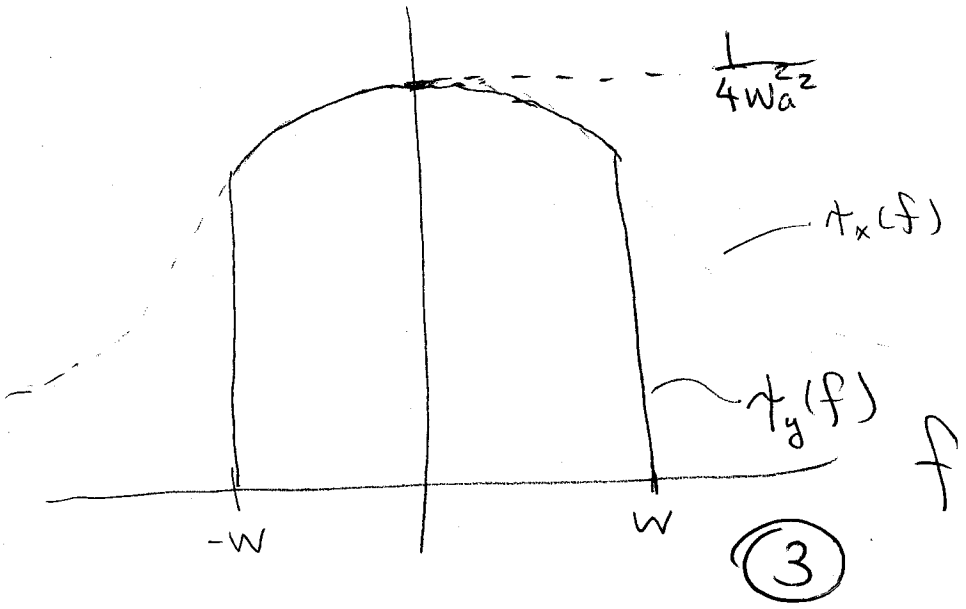
$$\boxed{R_x(\tau) = \frac{1}{2a} \exp(-a|t|)} \quad (3)$$

(b) [10 points] If x(t) is passed through a system with impulse response h(t) = sinc(2Wt) sketch the energy spectral density of y(t). ***Label all axes***.

$$\mathcal{Y}_y(f) = |H(f)|^2 \mathcal{Y}_x(f) \quad (3)$$

$$H(f) = \frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right) \quad (2)$$

$$\mathcal{Y}_y(f) = \frac{1}{4W^2} \text{rect}\left(\frac{f}{2W}\right) \cdot \frac{1}{a^2 + 4\pi^2 f^2} \quad (2)$$



(3)

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