3614: Introduction to Communication Systems

Midterm Exam II

November 6, 2007

SOLUTION

I pledge that I have neither given nor received any assistance on this exam.

(signed)

Name (print)

Student Number

ECE 3614 Midterm II – Test A

1. (20 points) Short answer and multiple choice

1.1 [5 points] Consider a message signal with a peak voltage of 1V and a bandwidth of 25kHz. Rank the following modulation schemes in terms of required bandwidth (least bandwidth = 1, most bandwidth = 4)

2 DSBSC
1 Single Sideband AM
4 FM with $k_f = 5000$
3 FM with $k_f = 500$

1.2 [5 points] The Fourier Transform of a periodic signal

(a) is a series of impulses
(b) is discrete in frequency
(c) is the multiplication of an impulse train with the Fourier Transform of one period
(d) None of the above
(e) All of the above

1.3 [5 points] Vestigial Sideband modulation has a bandwidth which is

(a) greater than DSBSC
(b) the same as LC-AM
(c) greater than SSB
(d) All of the above
(e) None of the above

1.4 [5 points] Single Sideband Modulation

(a) reduces bandwidth by eliminating the unmodulated carrier from LC-AM.
(b) reduces bandwidth by also transmitting a phase-shifted version of the message in order to eliminate the lower or upper sideband.
(c) reduces bandwidth by eliminating the need for a quadrature channel.
(d) None of the above

ECE 3614 Midterm II – Test A

2. (30 points) Amplitude Modulation

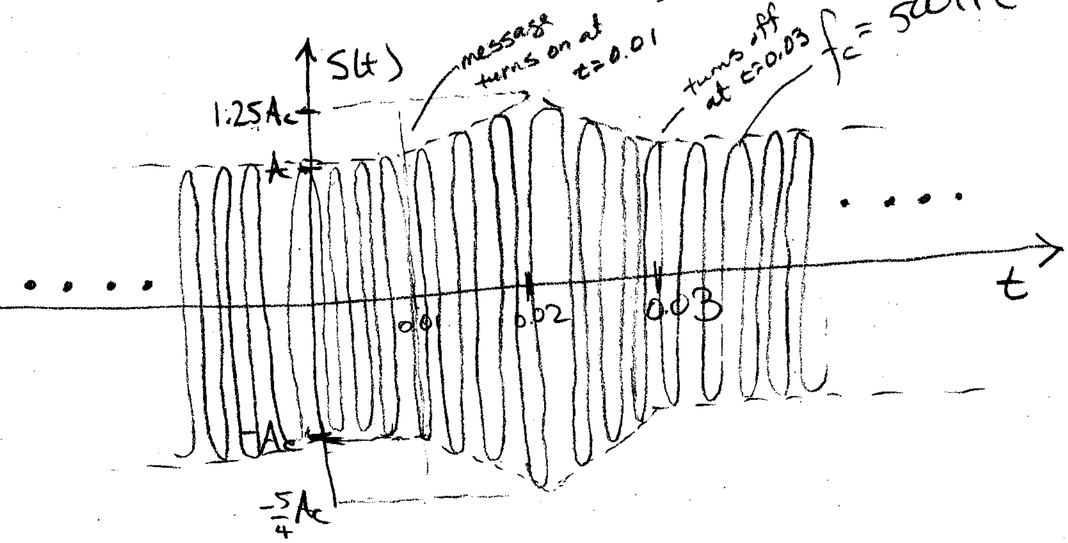
Consider a message signal

$$m(t) = tri\left(\frac{t-0.02}{0.01}\right)$$

(a) [8 points] Sketch the time domain signal of a large carrier AM modulated signal if k_a = 0.25 and f_c = 500Hz. Clearly label all axes.

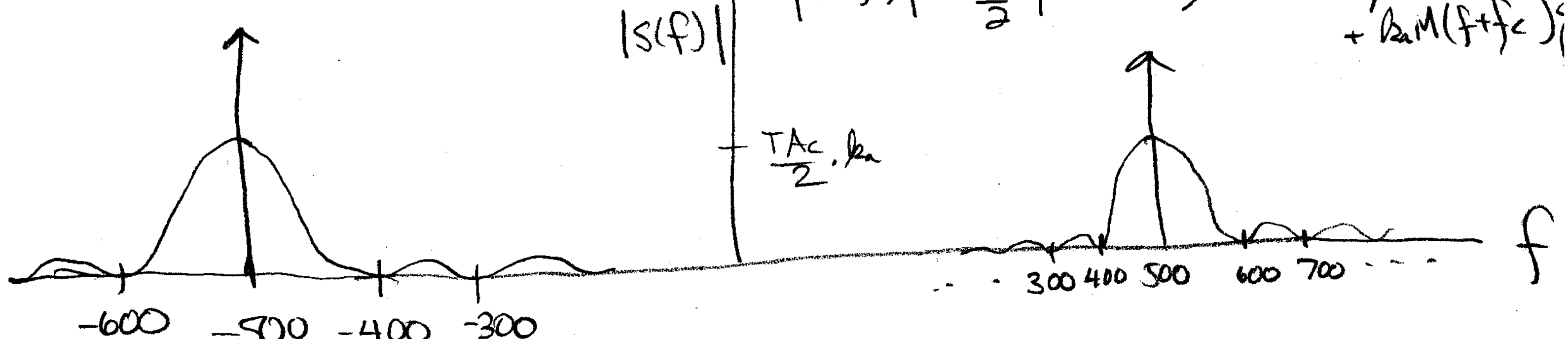
$$s(t) = A_c\left(1 + k_a m(t)\right)\cos(2\pi f_c t)$$
$$= A_c\left(1 + \tfrac{1}{4}\, tri\left(\tfrac{t-0.02}{0.01}\right)\right)\cos(2\pi f_c t)$$

labeling 1
tri-function - 3
correct values - 2
sinusoid - 2



(b) [7 points] Sketch the frequency domain of the transmit signal described in part (a).

$$|S(f)| = \frac{A_c}{2}\left\{\delta(f-f_c) + \delta(f+f_c) + k_a M(f-f_c) + k_a M(f+f_c)\right\}$$



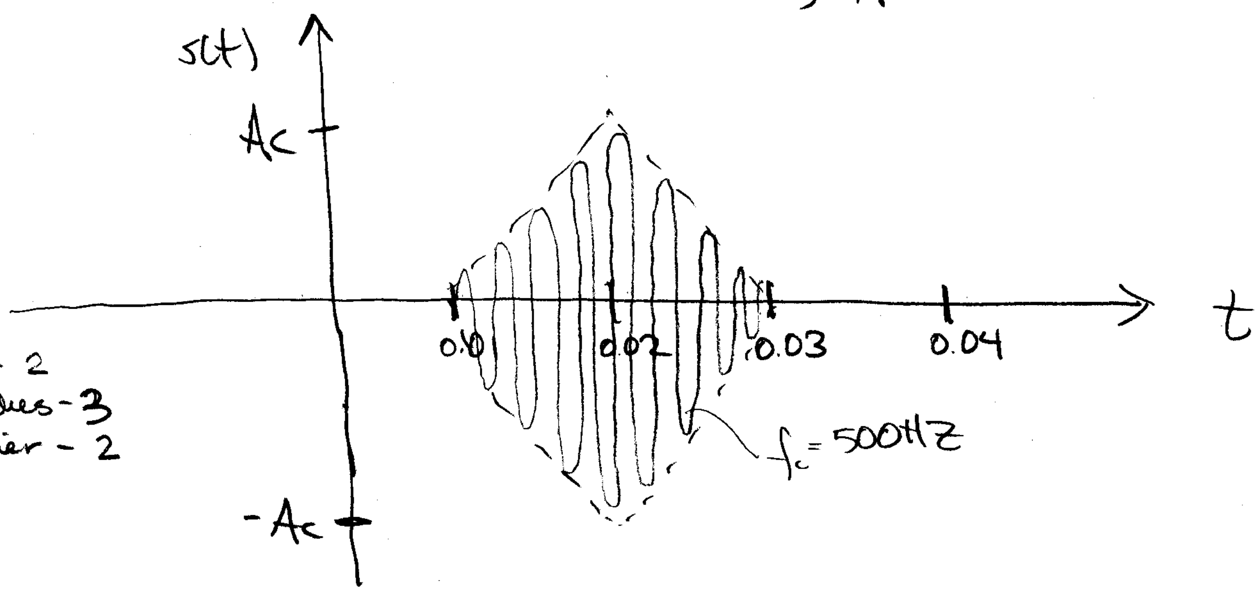
$$M(f) = T\,\text{sinc}^2(fT)\,e^{-j2\pi f t_0}$$

Impulses - 2
labeling - 1
center freq - 1
$\text{sinc}^2(f)$ - 3

$T = 0.01$
$t_0 = 0.02$

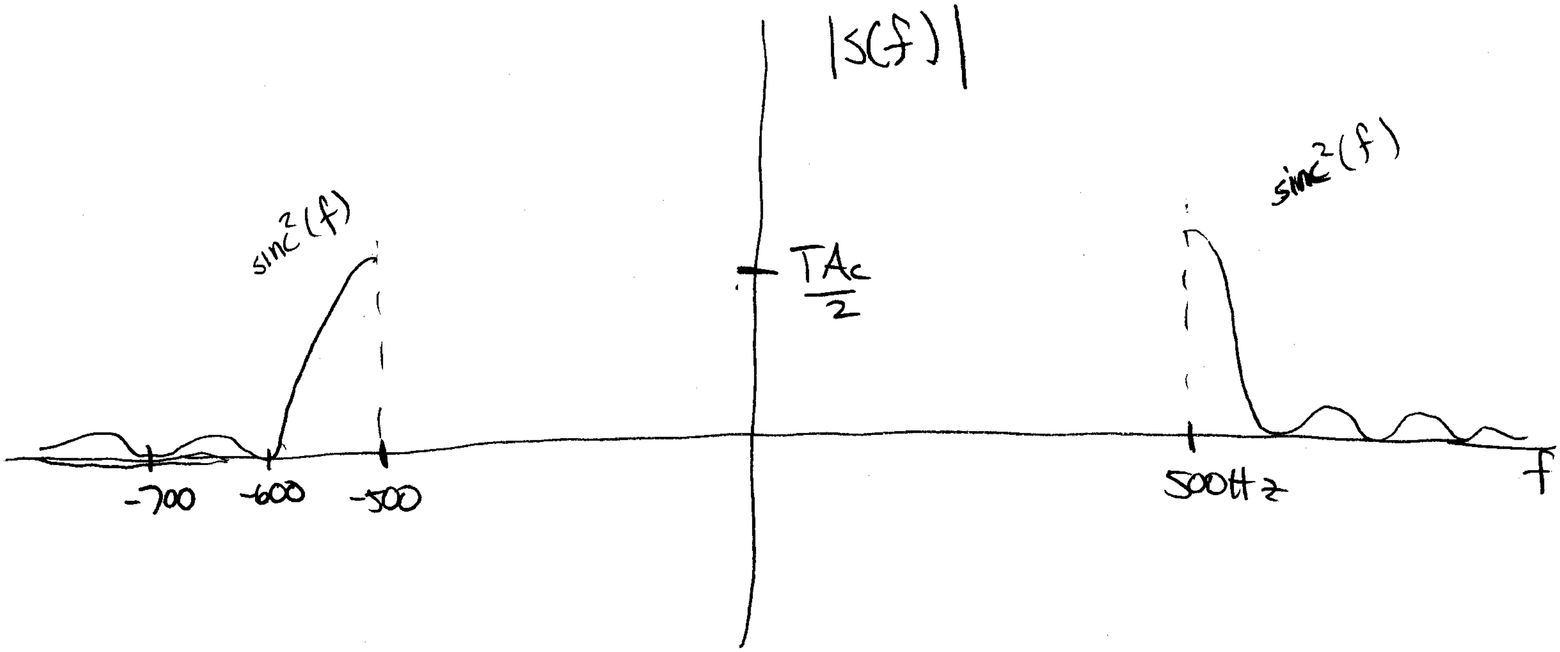
(c) [8 points] Sketch the time domain of a DSB-SC signal when f_c = 500Hz. Clearly label all axes and points.

$s(t) = A_c m(t) \cos(2\pi f_c t)$



turn-on/off - 2
tri/ peak/time values - 3
on/off carrier - 2
label - 1

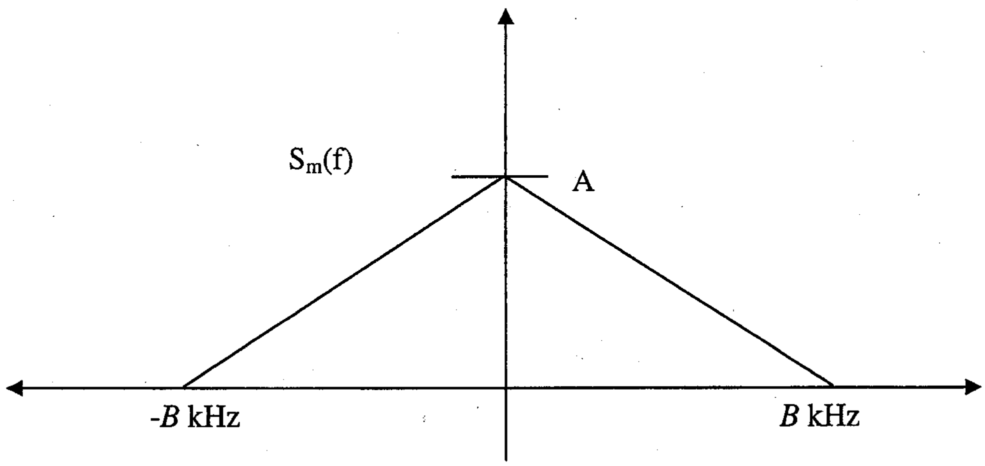
(d) [7 points] Sketch the frequency domain of a U-SSB signal when f_c = 500Hz.



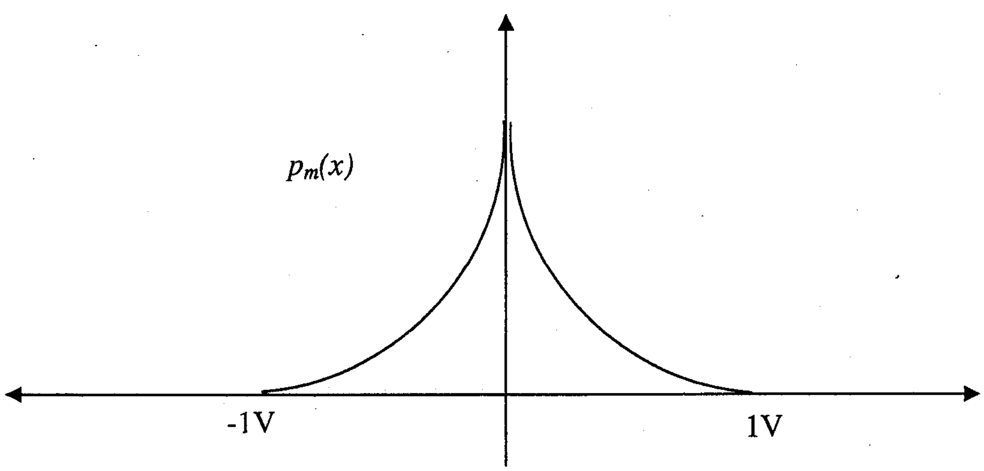
No impulse - 1
labeling - 1
Upper - 1
sinc² shape - 3
fc - 1

3. (30 points) Frequency Modulation

Consider a message signal with power spectral density



and probability density function $f_m(x) = ke^{-|x|}$ where $k = \frac{1}{2\left(1+e^{-2}\right)}$ which is plotted below:

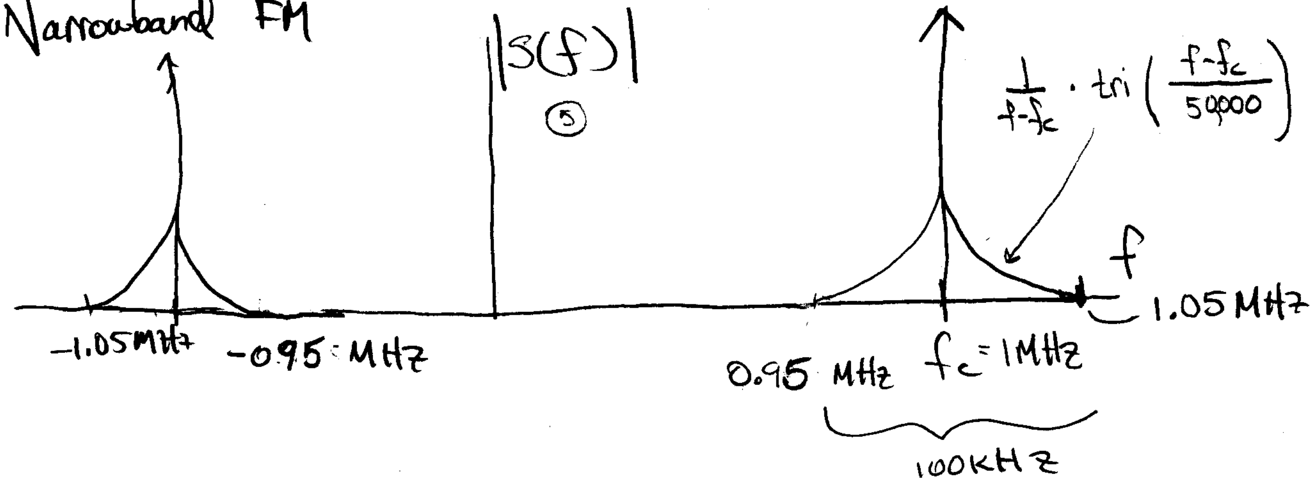


ECE 3614 Midterm II – Test A

(a) [10 points] Plot the Power spectral density (or spectrum) of an FM signal if f_c = 1MHz and k_f = 200 (Hz/V), and $B = 50$[redacted]

③ – $D = \frac{\Delta f}{W} = \frac{k_f \cdot V_p}{W} = \frac{200 \cdot 1}{50{,}000} = 4e^{-3}$ N.B. approx.

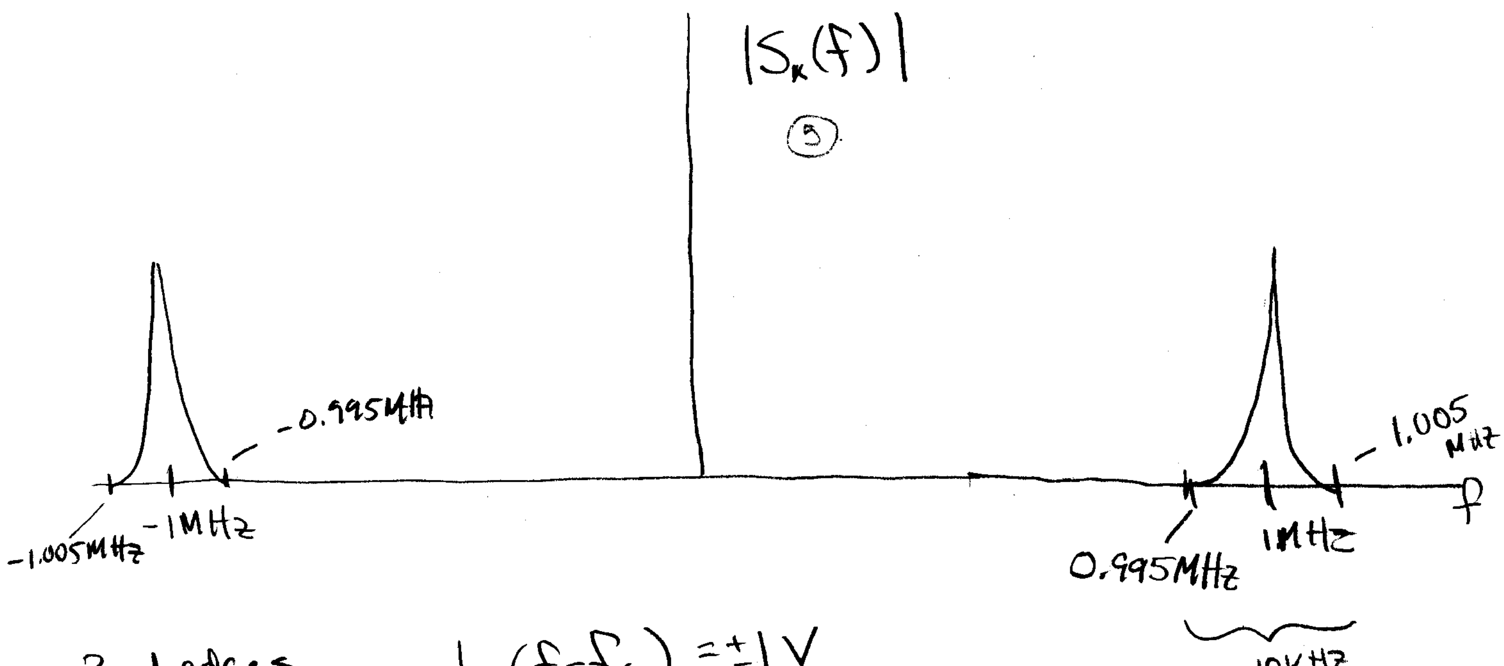
② – Narrowband FM



(b) [10 points] Plot the Power spectral density of an FM signal if f_c = 1MHz and k_f = 5[redacted]000 and $B = 1$.

③ $D = \frac{5000 \cdot 1}{1000} = 5$

Wideband approx. ②



Band edges

$\frac{1}{k_f}(f-f_c) = \pm 1V$

$f - f_c = \pm 1V \cdot 5000\,Hz/V$

$f = \pm 5KHz + 1MHz$

(c) [5 points] How does the bandwidth of the signal plotted in part (a) compare to Carson's Rule?

Carson's Rule → $B = 2(\Delta f + W)$ (2)

$= 2(200 + 50{,}000)$

$= 100.4\,\text{KHz}$ (1)

Figure $2 \times 50\,\text{KHz} = 100\,\text{KHz}$ (2)

0.4% difference

(d) [5 points] How does the bandwidth of the signal plotted in part (b) compare to Carson's Rule?

Carson's Rule $B = 2(\Delta f + W)$

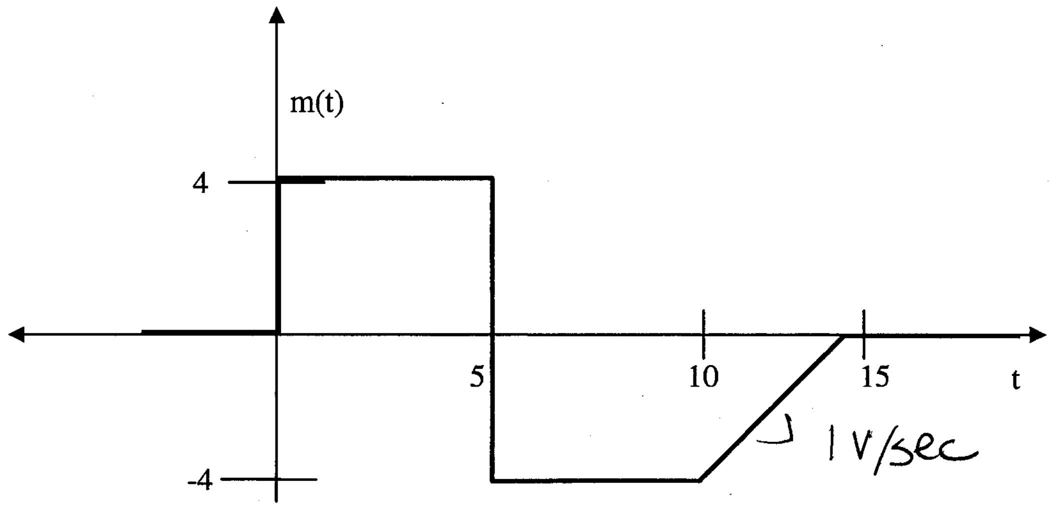
$= 2(5000 + 1000)$

$= 12\,\text{KHz}$

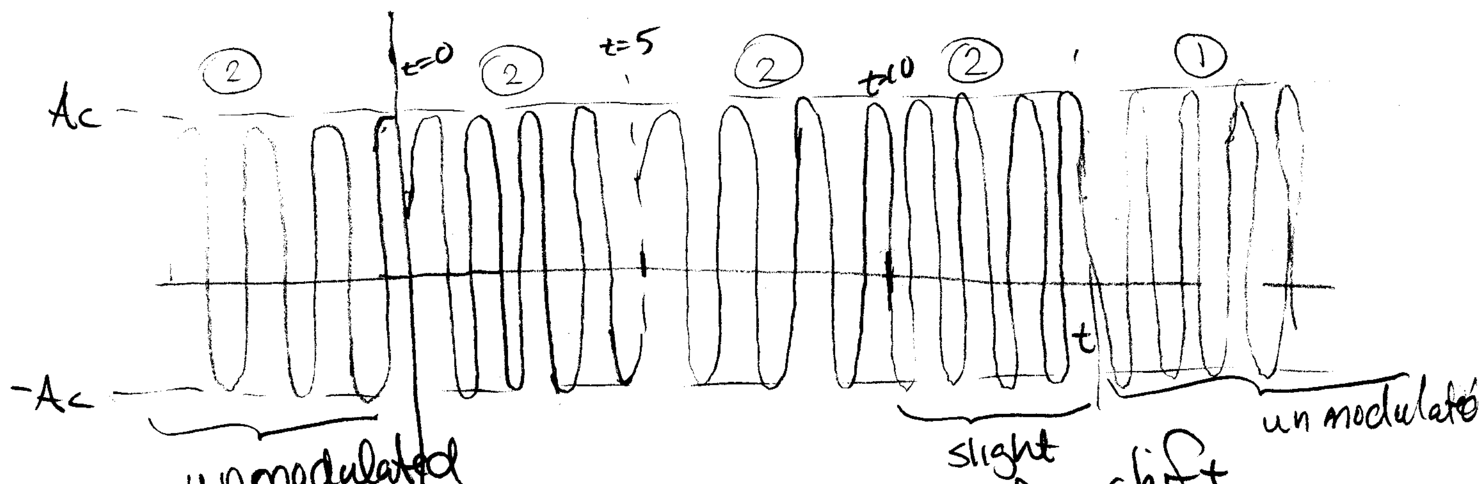
Figure $2 \times 5\,\text{KHz} = 10\,\text{KHz}$

20% difference

4. (20 points) Consider the following message signal



(a) [10 points] ~~Write an expression for the time domain signal and~~ plot the time domain signal for a phase modulated signal when fc = 1Hz and $k_p = \pi/10$ rad/V



Labels ①

$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$

$t < 0$ $m(t) = 0$ $s(t) = A_c \cos(2\pi f_c t)$

$0 < t \leq 5$ $m(t) = 4$ $s(t) = A_c \cos(2\pi f_c t + 4\pi/10)$

$5 < t \leq 10$ $m(t) = -4$ $s(t) = A_c \cos(2\pi f_c t - 4\pi/10)$

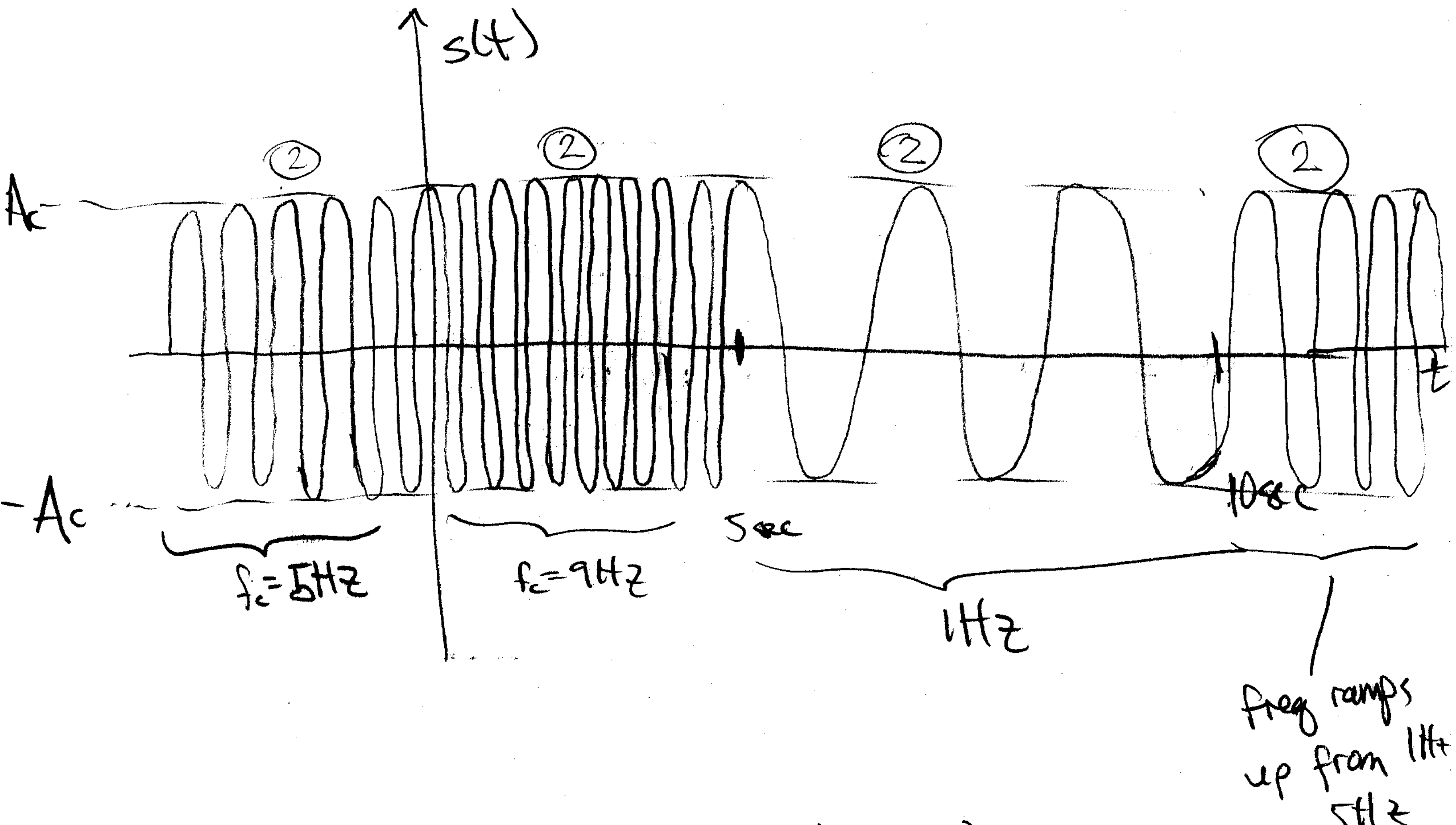
$10 \leq t \leq 14$ $m(t) = t - 14$ $s(t) = A_c \cos(2\pi f_c t + (\pi/10)\cdot(t-14))$

$= A_c \cos(2\pi (f_c + 1/20)t - 14\pi/10)$

$t > 14$ $m(t) = 0$ $s(t) = A_c \cos(2\pi f_c t)$

(b) [10 points] ~~Write an expression for the time domain signal and~~ Plot the time domain signal for a frequency modulated signal when f_c = 5Hz and k_f = 1Hz/V

$1\,Hz/V(\pm 4V) = \pm 4\,Hz$



$t < 0$ $m(t) = 0$ $s(t) = A_c \cos(2\pi \cdot 5t)$

$0 \le t \le 5$ $m(t) = 4$ $s(t) = A_c \cos(2\pi \cdot 9t)$

$5 < t \le 10$ $m(t) = -4$ $s(t) = A_c \cos(2\pi t)$

$10 < t \le 14$ $m(t) = t-14$ $s(t) = A_c \cos(2\pi \cdot 5t + 2\pi(-4+t)t)$

$= A_c \cos(2\pi(1+t)t)$

$t > 14$ $m(t) = 0$ $s(t) = A_c \cos(2\pi \cdot 5t)$