

Analog and Digital Communications  
Final Exam  
December 14, 2004

I pledge that I have neither given nor received any assistance on this exam.

---

(signed)

---

Name (print)

---

Student Number

1. (20 points) Short Answer

a. (5 points) Power Efficiency

Rank the following modulation schemes in terms of power efficiency from most efficient (1) to least efficient (4)

16-FSK	
16-QAM	
16-PSK	
BPSK	

(b) (5 points) Matched Filtering

Assuming that BPSK modulation and matched filtering are used, which of the following pulse shapes performs the *worst* in terms of BER performance?

square pulse  
sinc pulse  
square root raised cosine roll-off factor 0.5  
square root raised cosine roll-off factor 0

(c) (5 points) Spectral Efficiency

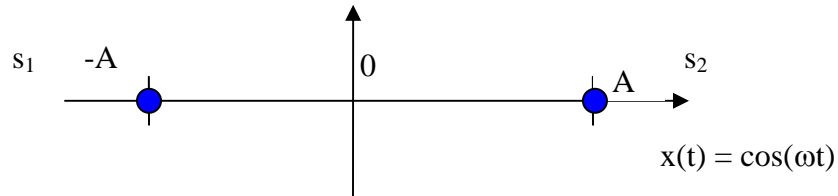
Rank the following modulation schemes in terms of bandwidth efficiency from most efficient (1) to least efficient (4)

16-QAM with square root raised cosine pulse (roll-off factor 0.25)	
16-PSK with square root raised cosine pulse (roll-off factor 0.5)	
BPSK with square root raised cosine pulse (roll-off factor 0)	
QPSK with square root raised cosine pulse (roll-off factor 0.75)	

(d) (5 points) Why are square pulse shapes impractical?

## 2. (20 points) Modulation Performance

Consider the following constellation diagram



The received signal after matched filtering and sampling is

$$r = s + n$$

where  $n$  is a random noise sample with the following distribution

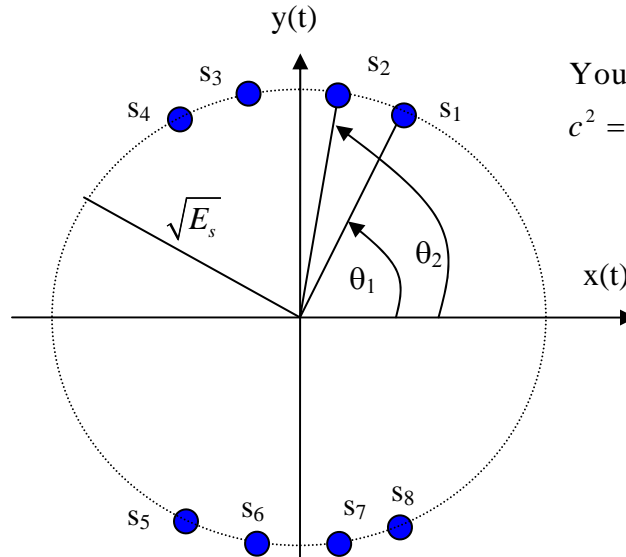
$$n = \begin{cases} 0 & p = 0.5 \\ B = \frac{\sigma_n}{2} & p = 0.5 \end{cases}$$

If the symbols  $\{s_1, s_2\}$  are equally likely, determine the probability of error in terms of the signal power  $S = E\{s^2\}$  and noise variance  $\sigma_n^2$ . (Assume that 0 is used as the threshold and that  $B > 0$ ).



3. (20 points) *M*-ary Modulation

Consider the following *M*-ary modulation scheme where  $\theta_1 = \pi/3$  and  $\theta_2 = 4\pi/9$ . Note that all four quadrants are symmetrical and Gray coding is used.



You may find the Law of Cosines Useful  
 $c^2 = a^2 + b^2 - 2ab \cos C$

a. (5 points) What is the spectral efficiency of this modulation scheme ?

b. (10 points) Determine the probability of bit error in an AWGN channel when matched filtering is used in terms of  $E_b/N_o$  (assume that all symbols are equally likely).

c. (5 points) If we wanted to minimize BER, what would be the optimal values of  $\theta_1$  and  $\theta_2$  ?

## 4. (20 points) Link Budgets and System Design

Consider a 10kbps mobile wireless system which has a mobile transceiver with a maximum transmit power of 200mW and a noise temperature of 600K. The base station transceiver has a maximum transmit power per channel of 2W and a noise temperature of 300K. The system uses BPSK and requires a BER of  $10^{-3}$  in the presence of Rayleigh fading. The base station transmit and receive antennas have identical gains of 15dBi and the mobile transmit and receive antenna gains are 1.5dBi. The center frequency is 2GHz.

a. (5 points) Which link (link from the base station to the mobile station or the link from the mobile station to the base station) limits the range of the system?

b. (5 points) What is the maximum range of the system if a 6dB margin is required to combat shadowing and the path loss exponent is 3?

c. (5 points) If coding can provide a 5dB gain in the link (i.e., 5dB less  $E_b/N_0$  is required) what is the maximum range obtainable?

d. (5 points) If the mobile receiver noise temperature could be reduced to 200K, what would be the range of the system?

## 5. (20 points) Communication System Design

Consider a satellite link that has a total bandwidth of 10MHz. The owners of the satellite would like to lease it for high speed data transfer with an error rate of  $10^{-6}$ . The transmitter has a maximum power of 10W, the transmit antenna has a gain of 35dBi while the receiver has a noise temperature of 400K and an antenna gain of 25dBi. The link is at 28GHz and the range is 42,000km. The following modulation schemes are available: BPSK, QPSK, 8-PSK, 16-QAM, 64-QAM, and BFSK. Square root raised cosine pulses can be used with a roll-off factor between 0.25 and 1.0. (Assume that near infinitely long pulses are possible.) What is the maximum data rate achievable ?

### The $Q$ -function

