Homework 2 solutions

1. Solution to problem 5.5

(a) The highest frequency component of $g(t) = \text{sinc}(200t)$ is 100 Hz. Hence, the Nyquist rate is 200 Hz and the Nyquist interval is 5 ms. (*2 points*)

(b) The highest frequency component of $g(t) = \text{sinc}^2(200t)$ is twice of that in part (a). Refer to HW1 for the convolution of two square waveforms in frequency domain. Thus, the Nyquist rate is 400 Hz and the Nyquist interval is 2.5 ms. (*2 points*)

(c) The highest frequency component of $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$ is determined by the one with higher frequency component, i.e., $\text{sinc}^2(200t)$. Therefore, the Nyquist rate is 400 Hz and the Nyquist interval is 2.5 ms. (*2 points*)

2. Solution to problem 5.6

Use Equation 5.2: $G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) = \sum_{n=-\infty}^{\infty} g(nT_s)\exp(-j2\pi nT_s f)$

(a) $G_\delta(f) = \sum_{n=-\infty}^{\infty} \cos\left(\frac{n\pi}{4}\right) e^{-j\frac{\pi}{2}nf}$ or $G_\delta(f) = 2\sum_{m=-\infty}^{\infty}\left[\delta\left(f - \tfrac{1}{2} - 4m\right) + \delta\left(f + \tfrac{1}{2} - 4m\right)\right]$

(b) $G_\delta(f) = \sum_{n=-\infty}^{\infty} \cos(n\pi) e^{-j2\pi nf}$ or $G_\delta(f) = \frac{1}{2}\sum_{m=-\infty}^{\infty}\left[\delta\left(f - \tfrac{1}{2} - m\right) + \delta\left(f + \tfrac{1}{2} - m\right)\right]$

(c) $G_\delta(f) = \sum_{n=-\infty}^{\infty} \cos\left(\frac{3n\pi}{2}\right) e^{-j3\pi nf}$ or $G_\delta(f) = \frac{1}{3}\sum_{m=-\infty}^{\infty}\left[\delta\left(f - \tfrac{1}{2} - \tfrac{2}{3}m\right) + \delta\left(f + \tfrac{1}{2} - \tfrac{2}{3}m\right)\right]$

3.

$X(f) = \frac{1}{5000}\left[\text{sinc}\left(\frac{\text{f -10,000}}{2{,}500}\right) = \text{sinc}\left(\frac{\text{f +10,000}}{2{,}500}\right)\right]$ This is plotted in Figure 3.1

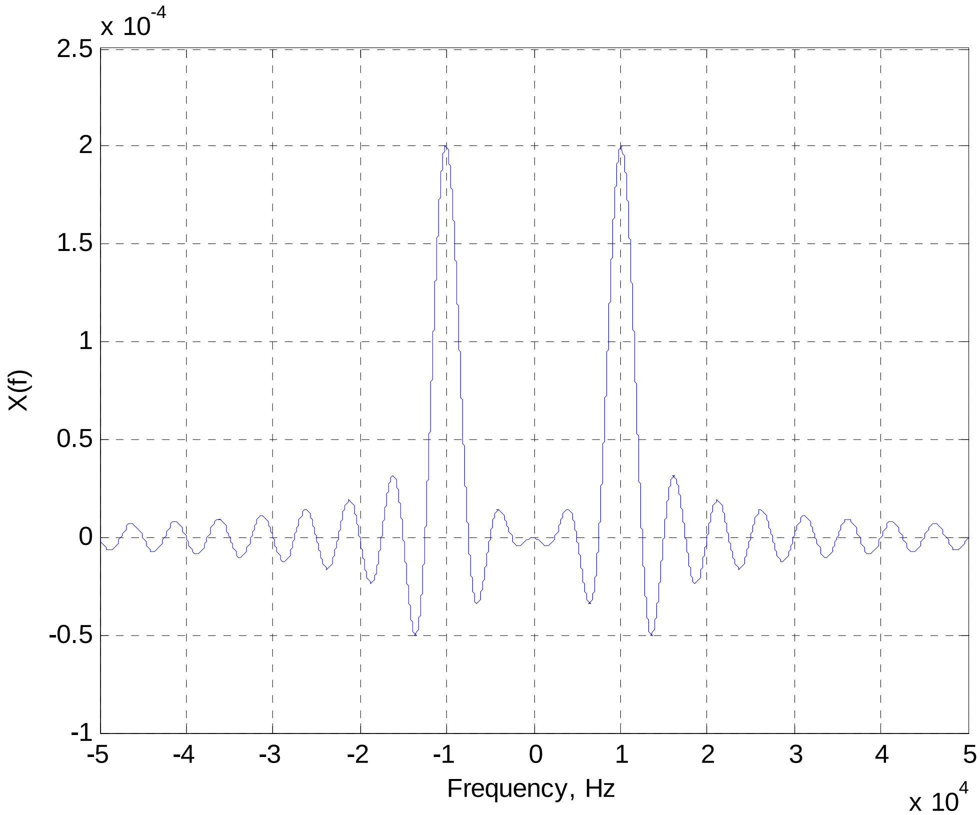


Figure 3.1 Amplitude Spectrum *X(f)*

(a) Technically the Nyquist rate is ***infinity*** because the sinc function never truly dies to zero and therefore has infinite frequency content.

NOTE: Several students used the first null-to-null bandwidth of 5000Hz, making the Nyquist sampling frequency ***10ksps.*** We accepted this answer since there was no other finite bandwidth value to use. However, this is technically wrong and would produce *severe* aliasing. Please be sure that you understand why this answer is incorrect.

(b) The filter transfer function is plotted with the original spectrum in Figure 3.2:

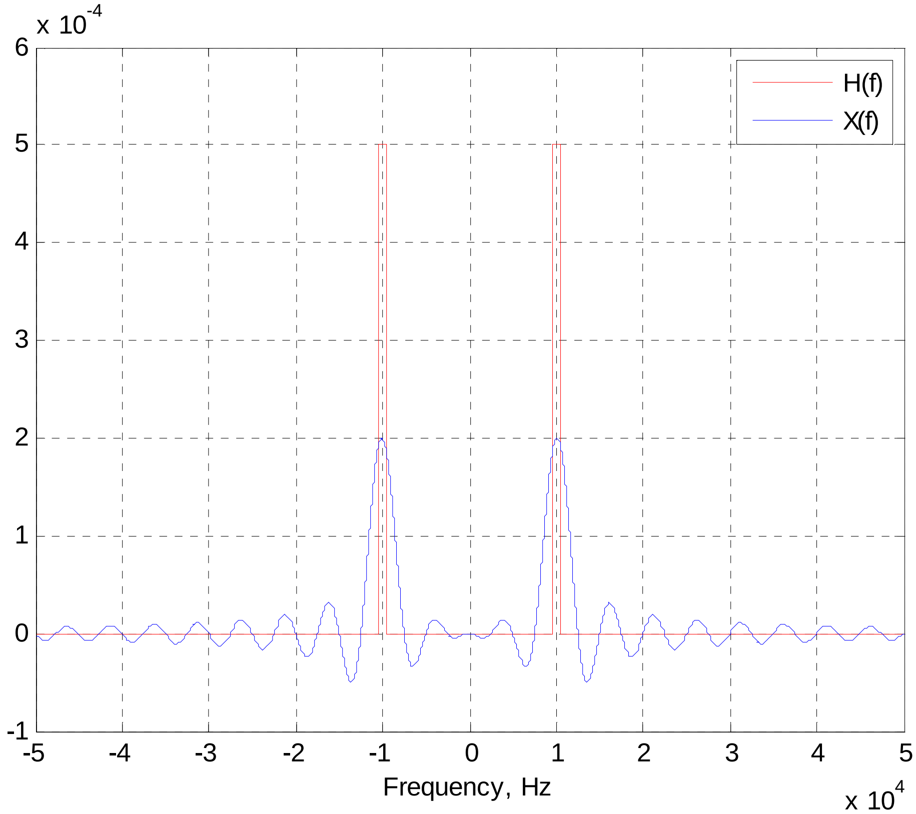


Figure 3.2 Amplitude Spectrum *X(f)* and Transfer Function *H(f)*

The spectrum is limited by the filter to a bandwidth of 1000Hz, with a maximum frequency of 10,500Hz.

Baseband: We must sample at twice the highest frequency. $f_s \geq 2 \cdot (10{,}000 + 500) = 21ksps$

Bandpass: We must sample at twice the bandwidth. $f_s \geq 2 \cdot (1000) = 2ksps$

4. (a) The bandwidth is technically infinite and thus, the Nyquist rate is infinity. (b) The 40dB bandwidth is 250kHz as shown in Figure 4.1. Thus, the Nyquist rate is 500kHz. Note that this is the same for both bandpass and baseband sampling theorems.

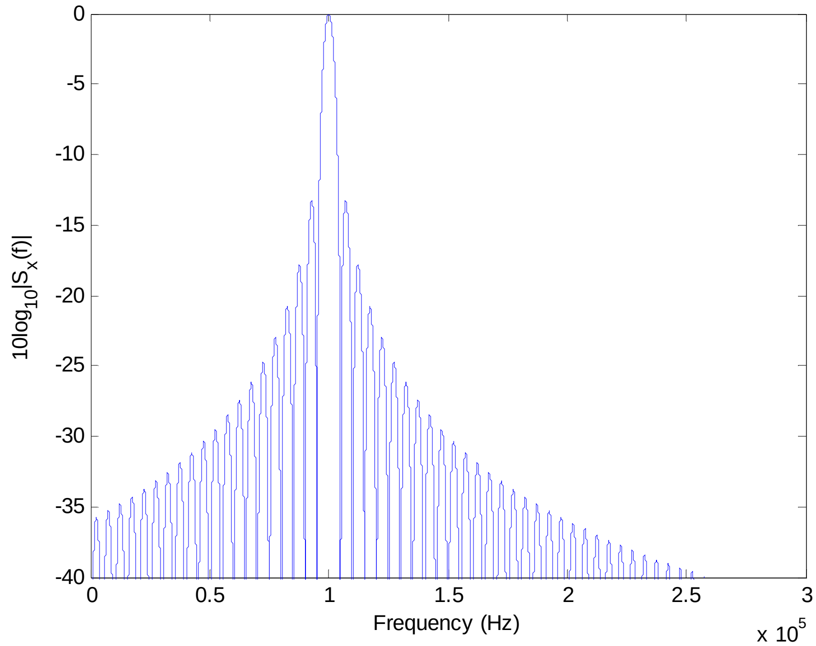


Figure 4.1 The 40dB Bandwidth is 250kHz

(c) The bandwidth is limited by the filter to 1000Hz, therefore the Nyquist rate is ***2ksps***.

Solution to problem 5.12

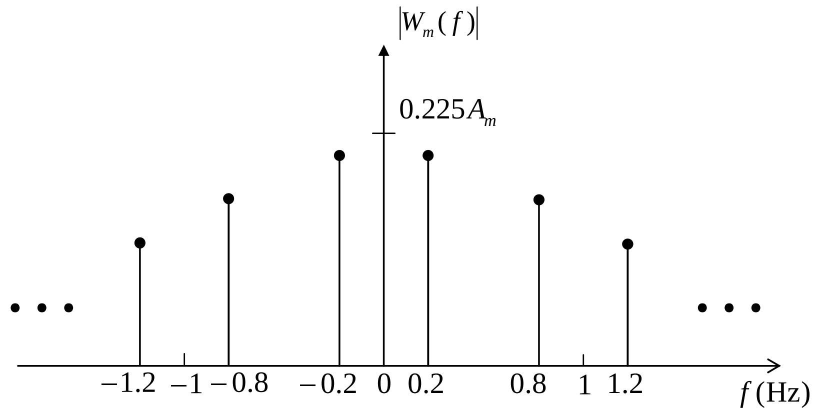
(a) Assume we use flat-top sampling (or you can use natural sampling), we can obtain the spectrum of the PAM signal as, ($T = 0.45$ s, $T_s = 1$ s, $f_m = 0.2$ Hz) (*2 points*)

$$W_m(f) = \frac{T}{T_s}\text{sinc}(Tf)\sum_{n=-\infty}^{+\infty} M(f - nf_s)$$

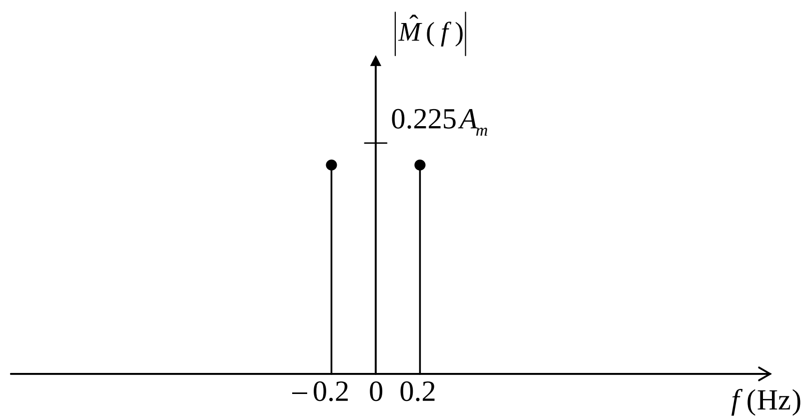
$$= \frac{T}{T_s}\text{sinc}(Tf)\sum_{n=-\infty}^{+\infty} \frac{A_m}{2}\left[\delta(f - f_m - nf_s) + \delta(f + f_m - nf_s)\right]$$

$$= 0.225A_m\text{sinc}(0.45f)\sum_{n=-\infty}^{+\infty}\left[\delta(f - 0.2 - n) + \delta(f + 0.2 - n)\right]$$

The power spectrum is shown as (*2 points for plot*)



(b) (*2 points*) an ideal unit-gain low-pass filter can be used to extract the two spectrum lines around the origin, as long as its cut-off frequency is between 0.2 and 0.8 Hz. The amplitude of the reconstructed signal is then given by, $0.225A_m \text{sinc}(0.9) = 0.222A_m$, as compared to amplitude $A_m/2$ corresponding to the case when there is no aperture effect. The power spectrum of the filtered output is shown below.



Solution to Problem 6

(a) No aliasing, filter leaves only the original signal.

$$y(t) = \cos(2\pi f_o t)$$

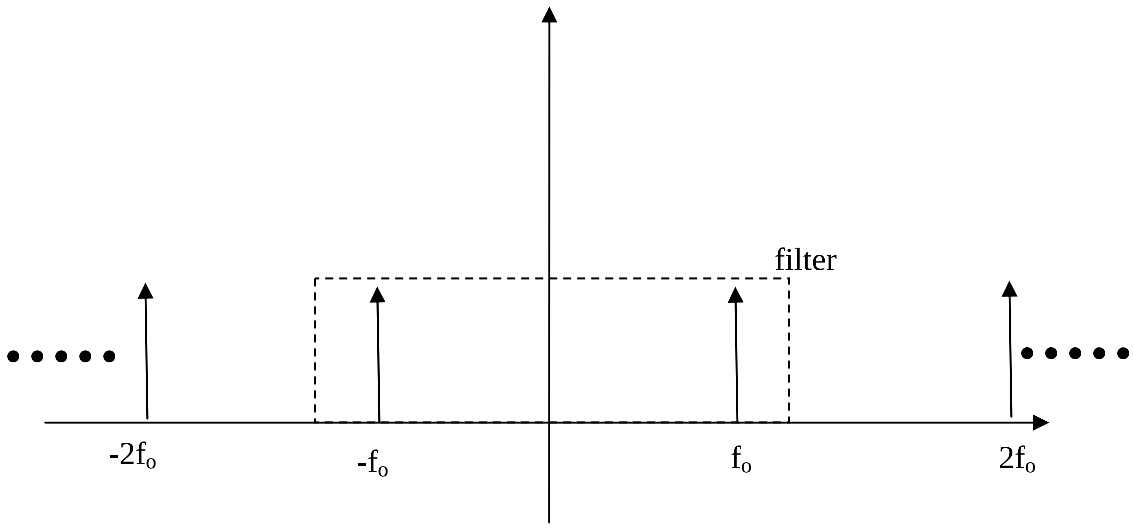


Figure 6.1 – $f_s = 3f_o$

(b) Filter occurs right on top of one of the sampled signal's frequency deltas at 1.25fo. Accepted either:

$y(t) = \cos(2\pi f_o t)$ or $y(t) = \cos(2\pi f_o t) + \frac{1}{2}\cos(2.5\pi f_o t)$

Figure 6.2 – $f_s = 2.2f_o$

(c) When fs= 1.5fo we can see that aliasing occurs at fo/2 as shown in Figure 6.3.

$y(t) = \cos(2\pi f_o t) + \cos(\pi f_o t)$

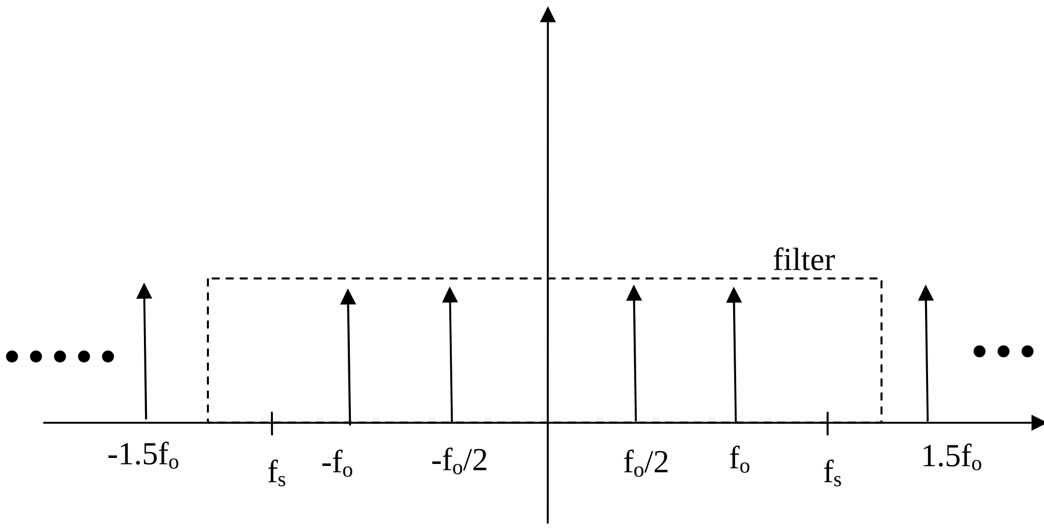


Figure 6.3 – f_s = 1.5f_o