

Digital Communications
Homework #5
Due 10/26/2007

- 1: (a) Draw the signal constellation diagram for a 16-QAM signal with maximum amplitude A . (b) What is the average energy per symbol? (c) What is the distance between neighboring signal points?
2. (a) Draw the signal constellation diagram for a 16-PSK signal with maximum amplitude A . (b) What is the average energy per symbol? (c) What is the distance between neighboring signal points? (c) Change the amplitude so that the average energy is the same as in 16-QAM presented in problem 1. How do the minimum distance between signal points compare?
3. In class we said that for differential encoding and decoding of M -ary PSK, we can obtain the proper decision statistics using

$$R_1 = R_1^i R_1^{i-1} + R_2^i R_2^{i-1}$$
$$R_2 = R_2^i R_1^{i-1} - R_1^i R_2^{i-1}$$

Show that this eliminates the unknown phase. (*Hint: use the definitions for R_1^i and R_2^i .*)

4. Explain the difference between coherent reception, non-coherent reception and differentially coherent reception.
5. (a) Explain the trade-off between energy efficiency and bandwidth efficiency for MPSK and MASK. (b) Does this trade-off differ for MFSK?
6. Assume that a QPSK signal is used to send data at a rate of 30Mbps over a satellite transponder. The transponder has a bandwidth of 24MHz. (a) If the satellite signal is to have an equivalent raised cosine filter characteristic after matched filtering, what is the roll-off factor r that is required? (b) Could a rolloff factor r be found so that a 50-Mbps data rate could be supported?
7. (a) Using Matlab generate 10,000 Gaussian random variables with mean 1 and variance $\frac{1}{2}$. Use the “**randn**” function. Create and plot a histogram. Hint: Use the following commands

```
range = -4:0.01:6;
```

```
h = hist(g, range);  
plot( range, h);
```

where g is the vector of Gaussian random variables. (b) Superimpose a plot of the Gaussian density distribution on the histogram. (The command **hold on;** will allow you to put multiple plots on one graph.) Hint: To superimpose the plots they must have the same scale. This can be done by ensuring that both sum to unity.

8 (a) Using Matlab create 10,000 instances of the random variable $Y = \sqrt{X_1^2 + X_2^2}$ where X_1 and X_2 are Gaussian random variables with mean 0 and variance = $\frac{1}{2}$. Plot the histogram. (b) What type of random variable is Y ? (c) Superimpose a plot of the probability density function.

9. Using Matlab create 10,000 sample functions of the random process $X(t) = 2 \cos(2\pi t + \theta)$ over the time period $0 < t < 10$ where θ is a uniform random variable distributed on $(0, 2\pi)$. Find the histogram of $X(t=1)$.

10. A random variable x has a PDF

$$f(x) = \begin{cases} \frac{3}{32}(-x^2 + 8x - 12) & 2 < x < 6 \\ 0 & \text{else} \end{cases}$$

- (a) Demonstrate that it is a valid PDF.
- (b) Find the mean
- (c) Find the second moment.
- (d) Find the variance.