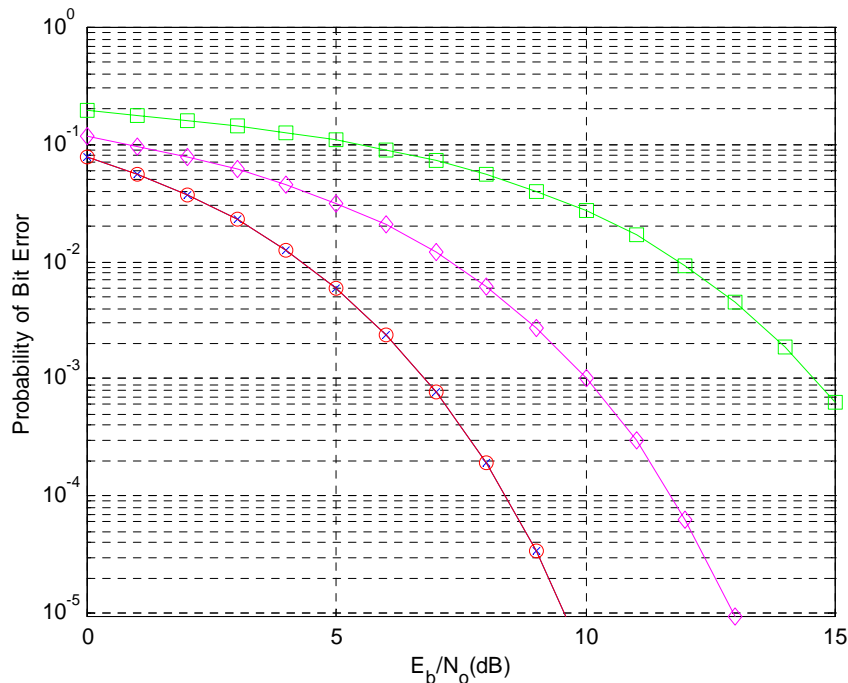


Digital Communications
Homework #7
SOLUTION

1. Plot the probability of bit error error vs. E_b/N_0 (in dB) for M -PSK for $M = 2,4,8, 16$. How much loss (in dB) is there when for QPSK, 8-PSK and 16-PSK compared to BPSK at a bit error rate of 10^{-3} .



From the figure we can see that there is no loss in energy efficiency for QPSK, a loss of approximately 3dB for 8-PSK and approximately a loss of 7.5dB for 16-PSK at an error rate of 10^{-3} .

2. A receiver with a system noise figure of $F=5dB$ is being used to receive a microwave signal ($f=2GHz$). The transmit power is 10mW and the transmit and receive antennas are identical with 2dB of gain. If the modulation scheme is 8-PSK, what is the maximum data rate that can be achieved at a distance of 10km if a target BER of 10^{-5} is required? Note that matched filtering is assumed. Assume free space propagation. This is a point-to-point link, so ignore fading considerations. Don't consider bandwidth.

A link budget created in an Excel spread sheet is shown below. Note that the EIRP is equal to $10\log(0.01) + 2dB = -18dBW$. The path loss is $20\log(4\pi d/\lambda) = 118.5dB$. With a receive antenna gain of 2dB the received power is $-18dBW - 118dB + 2dB = -135dBW$. The receiver noise temperature is calculated as $290K(NF-1)$ where $NF = 10^{5/10}$. The resulting noise temperature is $T_R = 627K$ or 27.97dBK. The noise power spectral density is $-228.6dB/Hz/K$ (Boltzman's Constant) + $T_R + T_A$ where T_A is antenna noise*

temperature. Since I didn't specify this value, you could assume that it is zero (normally it should be assumed to be approximately 290K). Thus, the noise PSD is -201dBW/Hz. From Figure 1, (the plot of the probability of error versus E_b/N_o for 8-PSK) we can see that an E_b/N_o of 13.2dB is required. The received E_b/N_o is then 13.2dB = -135dBW - $10\log(R_b) + 201\text{dBW/Hz}$. Solving for R_b results in

$$\begin{aligned}
 10\log(R_b) &= -13.2\text{dB} - 135\text{dBW} + 201\text{dBW/Hz} \\
 &= 52.9\text{dB b/s} \\
 R_b &= 195\text{ kbps}
 \end{aligned}$$

Transmit Power	0.01W	-20	dBW		
Transmit Antenna Gain		2	dB		
EIRP		-18	dBW	distance	10000 m
Path Loss	7.113E+11	118.5	dB	frequency	2 GHz
				wavelength	0.149 m
Receive Antenna Gain		2	dB		
Received Power		-135	dBW	PL exponent	2
Noise Figure		5	dB		
Noise Temp	627.06052	27.97	dBK		
Noise PSD		-201	dBW/Hz	-228.6	dBW/Hz/K
Bit Rate	195000	52.9	dB b/s		
Energy Per Bit		-187	dBJ		
Received Eb/No		13.2	dB		
Required Margin					
Required Eb/No		13.2	dB		

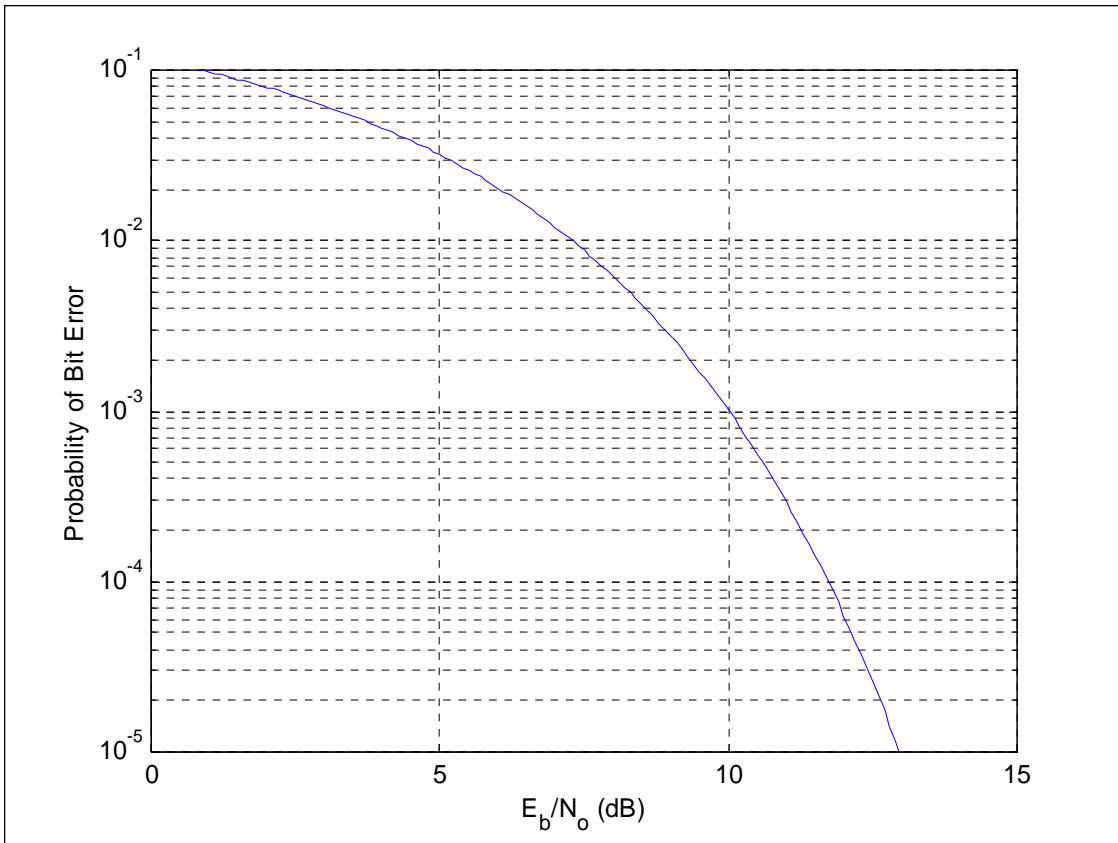


Figure 1: Probability of Error for 8-PSK

3 An engineer in your group has found an error correction coding chip that will provide your system with 3dB of gain at the target error rate. Using the same system as in (2) determine the data rate that is now supported.

With a 3dB coding gain the required E_b/N_o is reduced to 10.2dB. Thus, the bit rate can be increased to 390kbps.

Transmit Power	0.01	-20	dBW		
Transmit Antenna Gain		2	dB		
EIRP		-18	dBW	distance	10000 m
Path Loss	7.11E+11	118.5205	dB	frequency	2 GHz
				wavelength	0.149 m
Receive Antenna Gain		2	dB		
Received Power		-134.52	dBW	PL exponent	2
Noise Figure		5	dB		
Noise Temp	627.0605	27.97309	dBK		
Noise PSD		-200.627	dBW/Hz	-228.6	dBW/Hz/K
Bit Rate	390000	55.91065	dB b/s		
Energy Per Bit		-190.431	dBJ		
Received Eb/No		10.19579	dB		
Required Margin					
Required Eb/No		13.2	dB		
Coding Gain		3	dB		
Required Eb/No with Coding		10.2	dB		

4. It turns out that this system will be used to transmit to systems throughout an entire county. Since there is log-normal shadowing due to the hilly terrain, it is determined that 10dB of fade margin is required to obtain 95% coverage. Assuming the same system as in (3) what data rate can be supported?

Due to the 10dB fading margin and the 3dB coding gain, we have a required E_b/N_o of 20.2dB. In order to achieve this higher E_b/N_o value we must reduce the data rate to 39kbps as seen below.

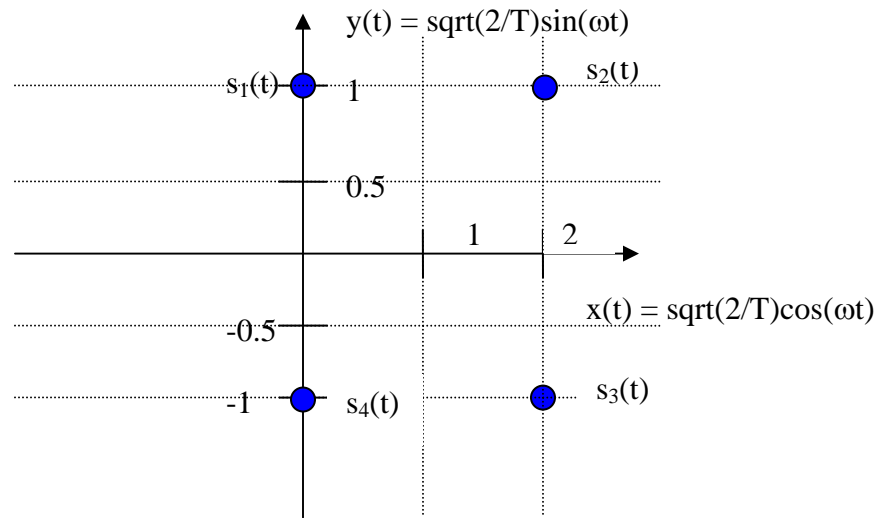
Transmit Power	0.01	-20	dBW		
Transmit Antenna Gain		2	dB		
EIRP		-18	dBW	distance	1000 m
Path Loss	7.1E+1	118.520		frequency	2 GHz
	1	5	dB	wavelength	0.149 m
Receive Antenna Gain		2	dB	PL exponent	2
Received Power		-134.52	dBW		
Noise Figure		5	dB		
Noise Temp	627.06	27.9730			
	1	9	dBK		
Noise PSD		-	dBW/H		
		200.627	z	228.6	dBW/Hz/K
Bit Rate		45.9106			
	39000	5	dB b/s		
Energy Per Bit		-			
		180.431	dBJ		
Received E_b/N_o		20.1957			
		9	dB		
Required Margin		10	dB		
Required E_b/N_o		13.2	dB		
Coding Gain		3	dB		
Required E_b/N_o with Coding		20.2	dB		

5. Which of the two systems listed below requires higher transmit power?

	<u>System A</u>	<u>System B</u>
Tx Antenna Gain	0 dB	0 dB
Tx Power	W	W
Path Loss	110 dB	110 dB
Data Rate	10 kbps	1 Mbps
Rx Antenna Gain	2.3 dB	2.3 dB
No	-195 dB W/Hz	-195 dB W/Hz
Modulation	8-PSK	QPSK
Error Rate Required	0.001	0.01
Fading Margin	10 dB	10 dB

We know that the performance of System A (8-PSK) is $P_e = \frac{2}{3}Q\left(\sqrt{\frac{0.88E_b}{N_o}}\right)$. From the Q-function table we find that $Q(3.1)=0.001$. Solving for E_b/N_o results in a 10dB requirement. For System B, since we are using QPSK, $P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$. Further, since the target is 0.01, our required E_b/N_o falls to 4.3dB. Thus, System A requires 5.7dB more E_b/N_o . Now, all terms in the link budget are identical except data rate. Since System B has a data rate which is 100 times that of System A, it will require 100 times more received power to achieve the same E_b/N_o . Thus, while System B requires a smaller E_b/N_o value, it has a 20dB higher data rate. Since all other things are equal, System B will require $20\text{dB}-5.7\text{dB} = 14.3\text{dB}$ more transmit power than System A.

6. Consider the following constellation diagram



(a) Derive the probability of bit error in terms of E_b/N_o assuming that all symbols are equally likely, that symbol errors are dominated by nearest neighbors, and that Gray coding is used.

The probability of symbol error can be written as

$$\begin{aligned}
 P_s &= \sum_{i=1}^4 \Pr\{s = i\} \Pr\{e|s = i\} \\
 &= \frac{1}{4} \sum_{i=1}^4 \Pr\{e|s = i\}
 \end{aligned}$$

Since all of the symbols have the same number of nearest neighbors which are the same distance away, we can simplify the expression as

$$\begin{aligned}
 P_s &= \frac{1}{4} \sum_{i=1}^4 \Pr\{e|s = i\} \\
 &= 2Q\left(\frac{d}{\sqrt{2N_o}}\right)
 \end{aligned}$$

where d is the distance between adjacent symbols. Now, substituting $d = 2$,

$$\begin{aligned}
P_s &= 2Q \left(\frac{d}{\sqrt{2N_o}} \right) \\
&= 2Q \left(\frac{2}{\sqrt{2N_o}} \right) \\
&= 2Q \left(\sqrt{\frac{2}{N_o}} \right)
\end{aligned}$$

In order to put this in terms of E_b/N_o , we must find the average energy per symbol (assuming that $T=1$):

$$\begin{aligned}
E_s &= \frac{T}{4} \sum_{i=1}^4 P_i \\
&= \frac{1}{4} \{1+1+5+5\} \\
&= 3
\end{aligned}$$

Thus,

$$\begin{aligned}
P_s &= 2Q \left(\sqrt{\frac{2}{N_o}} \right) \\
&= 2Q \left(\sqrt{\frac{2E_s}{3N_o}} \right) \\
&= 2Q \left(\sqrt{\frac{4E_b}{3N_o}} \right)
\end{aligned}$$

*Now, since Gray coding is used, $P_b = 1/2 * P_s$. Thus,*

$$\begin{aligned}
P_b &= \frac{P_s}{2} \\
&= Q \left(\sqrt{\frac{4E_b}{3N_o}} \right)
\end{aligned}$$

(b) (10 points) An analog message signal with uniform distribution and bandwidth 10kHz is to be digitized assuming Nyquist sampling and transmitted using the modulation scheme above. The receiver noise power spectral density is found to be $N_o = -200\text{dBW/Hz}$ and due to range considerations the maximum receive power is limited to $P_r = -135\text{dBW}$ (note that $E_b = P_r T_b$ where T_b is the bit duration in seconds). Using the expression derived above, determine the maximum quantization SNR when the probability of error is required to be equal to or below 10^{-3} .

The required E_b/N_o is found as

$$P_b = Q\left(\sqrt{\frac{4E_b}{3N_o}}\right)$$

$$0.001 = Q\left(\sqrt{\frac{4E_b}{3N_o}}\right)$$

$$3.1 = \sqrt{\frac{4E_b}{3N_o}}$$

$$\frac{E_b}{N_o} = 7.2$$

or 8.6dB. Now, E_b/N_o is found from

$$\frac{E_b}{N_o}(\text{dB}) = P_r(\text{dB}) - R_b(\text{dB}) - N_o(\text{dB})$$

$$R_b(\text{dB}) = -\frac{E_b}{N_o}(\text{dB}) + P_r(\text{dB}) - N_o(\text{dB})$$

$$= 56.4\text{dB}$$

which corresponds to 439kbps. Assuming Nyquist sampling, $f_s = 20\text{kHz}$. Thus,

$$n \leq \frac{R_b}{f_s}$$

$$\leq \frac{439\text{kbps}}{20\text{kHz}}$$

$$\leq 21.9$$

Since the quantization improves 6dB per quantization bit and we have 21 bits, the quantization SNR is 126dB.

7. Link Budgets - Consider a system which has a transmit power of 40mW, a transmit antenna gain of 5dB, a receive antenna gain of 8dB, a path loss of 90dB, a required fading margin of 5dB and the noise power spectral density is measured as -195dBW/Hz. The required error rate is 0.1%. Further, assume raised cosine pulse shaping with a roll-off factor of $\alpha = 0.25$ and a matched filter receiver. The system bandwidth is limited to 500kHz. What is the maximum achievable data rate with this system? (You need only consider BPSK, QPSK, and 8-PSK modulation schemes).

There are two limits in this problem, energy and bandwidth. The bandwidth limits are easier, so we can tackle them first. With raised cosine pulse shaping, the maximum supportable data rate is

$$\begin{aligned} R_b &= r * R_s \\ &= r * \frac{B}{1 + \alpha} \\ &= r * 400\text{ksp/s} \end{aligned}$$

Thus, due to bandwidth considerations we are limited to 400kbps, 800kbps, and 1.2Mbps when using BPSK, QPSK, and 8-PSK respectively.

Now concerning energy limitations:

$$\begin{aligned} \frac{E_b}{N_o} &= P_t(\text{dB}) + G_t(\text{dB}) - L_p(\text{dB}) + G_r(\text{dB}) - R_b(\text{dB}) - N_o(\text{dB}) - M(\text{dB}) \\ &= -14\text{dBW} + 5\text{dB} - 90\text{dB} + 8\text{dB} - R_b(\text{dB}) + 195\text{dBW / Hz} - 5\text{dB} \\ &= 99\text{dB} - R_b(\text{dB}) \end{aligned}$$

The 0.001 error rate means that we require an E_b/N_o of 6.75dB for BPSK or QPSK and an E_b/N_o of 10dB for 8-PSK. Using these values we can find that the data rates are limited to 1.6Gbps for BPSK and QPSK and 780Mbps for 8-PSK due to energy concerns. Combining the two sets of limitations, we can see that the best data rate achievable is 1.2Mbps with 8-PSK.