ECE4634
Digital Communications
Fall 2007

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Lecture #14: Representation of Bandpass Signals

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Review

- We have been examining the transmission of analog (or digital) information signals in a *digital* communications system
- To date we have examined
  - Conversion of an analog signal to a string of bits
    - Sampling
    - Quantization
  - Mapping those bits onto a *baseband* waveform
    - Line codes
    - Pulse trains
- Starting with the last lecture we began discussing the mapping of bits onto a *bandpass* waveform (sinusoidal modulation)
- What to read – Section 3.8 in the text

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Lecture Objectives

- The objective of this lecture is to show that there are three main ways of representing a bandpass signal using only baseband signals.
- We also wish to show that the three bandpass representations are equivalent.

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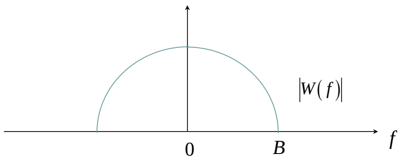
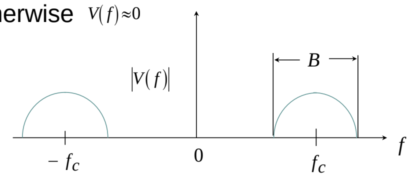
Baseband Signals

- A <u>baseband</u> signal *w(t)* with bandwidth *B* is a signal for which $W(f)$ is non-negligible for $|f| \le B$ and for which $W(f) \approx 0$ for $|f| > B$



Bandpass Signals

- A <u>bandpass</u> signal *v(t)* with bandwidth *B* is a signal for which *V(f)* is non-negligible some region about $\pm f_c$ and for which otherwise $V(f) \approx 0$



Modulation

- So far, we have dealt primarily with baseband signals
- Baseband signals *w*(*t*) may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t)\cos(\omega_c t + \theta) \Leftrightarrow \frac{1}{2}\left[e^{j\theta} W(f - f_c) + e^{-j\theta} W(f + f_c)\right]$$

- Most transmitted signals are modulated onto a carrier because
  - Modulated signals propagate well through the atmosphere
  - Modulation allows many signals with different carrier frequencies to share the spectrum

Three Ways of Representing Bandpass Signals

- We will need some additional analytical tools to handle bandpass signals *v(t)*
- Magnitude and Phase

  $$v(t) = R(t)\cos[\omega_c t + \theta(t)]$$

- In Phase and Quadrature

  $$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Complex Envelope

  $$v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$$

Magnitude and Phase Representation

- Any bandpass signal can be represented as:
  - $v(t) = R(t)\cos[\omega_c t + \theta(t)]$ where
  - $R(t) \geq 0$ is a real valued baseband signal representing the magnitude of the signal
  - $\theta(t)$ is a real valued baseband signal representing the phase of the signal
- The bandpass signal is modeled as a sinusoid whose amplitude and phase are time-varying.
- The representation is easy to interpret physically, but often is not mathematically convenient

Transmission of Information using Modulated Signals

- **Modulated signals can represent digital information through changing three parameters of the signal:**
- Amplitude: $R(t)$
  - "Amplitude Shift Keying" (ASK)
    - Or in analog systems we call it Amplitude Modulation (AM)
- Phase: $\theta(t)$
  - "Phase Shift Keying" (PSK)
- Frequency: $d\theta(t)/dt$
  - "Frequency Shift Keying" (FSK)
    - Or in analog systems we call it Frequency Modulation (FM)

In-phase and Quadrature (I&Q) Representation

- **Any bandpass signal *v(t)* can also be represented as**
  - $v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$ where
  - $x(t)$ is a real-valued baseband signal called the In-phase (I) component
  - $y(t)$ is a real-valued baseband signal called the Quadrature (Q) component
  - Note: the two components are *orthogonal*
- **This is often a convenient form which**:
  - Emphasizes the fact that two signals may transmitted within the same bandwidth
  - Closely parallels the physical implementation of the transmitter and receiver

Relationship Between Magnitude/Phase and I/Q Forms:

• Beginning with the amplitude/phase representation:

$$\begin{aligned} v(t) &= R(t)\cos(\omega_c t + \theta(t)) \\ &= R(t)\left[\cos(\omega_c t)\cos(\theta(t)) - \sin(\omega_c t)\sin(\theta(t))\right] \\ &= \underbrace{R(t)\cos(\theta(t))}_{x(t)\ \text{In-phase}}\cos(\omega_c t) - \underbrace{R(t)\sin(\theta(t))}_{y(t)\ \text{Quadrature}}\sin(\omega_c t) \end{aligned}$$

Relationship Between Magnitude/Phase and I/Q Forms:

- To transform from Magnitude/Phase to I/Q

$$x(t) = R(t)\cos\theta(t)$$

$$y(t) = R(t)\sin\theta(t)$$

- To transform from I/Q to Magnitude/Phase
  - $R(t) = \sqrt{x^2(t) + y^2(t)}$

$$\theta(t) = \tan^{-1}\left[\frac{y(t)}{x(t)}\right]$$

I and Q Portions of the Signal are Orthogonal

- Look at the correlation between
  - I portion: $x(t)\cos(\omega_c t)$
  - Q portion: $-y(t)\sin(\omega_c t)$

$$\int_0^{T_b} x(t)\cos(\omega_c t)\cdot y(t)\sin(\omega_c t)dt$$

$$=\int_0^{T_b} x(t)y(t)\frac{1}{2}\left[\sin(\omega_c t-\omega_c t)+\sin(\omega_c t+\omega_c t)\right]dt$$

$$=\int_0^{T_b} x(t)y(t)\frac{1}{2}\left[\sin(0)+\sin(2\omega_c t)\right]dt \approx 0$$

Note that x(t) and y(t) change slowly compared to $\sin(2\omega_c t)$

Complex Envelope (or Baseband) Representation

- Any bandpass signal can also be represented as
  - $v(t)=\mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$ where
  - $g(t)$ is a complex-valued baseband signal called the complex envelope
- This form is convenient many instances for analysis because it is
  - Compact
  - Easy to manipulate complex exponentials without recourse to trigonometric identities

Relationship Between Complex Envelope and I/Q Forms:

- We would like to deal with only the *baseband* portion of the *bandpass* signal

$$v(t)=\underbrace{x(t)}_{baseband}\cos(\omega_c t)-\underbrace{y(t)}_{baseband}\sin(\omega_c t)$$

- Since the I and Q portions of the signal are orthogonal, we can represent the baseband parts with an *orthogonal representation*. One such representation is complex notation.

$$g(t)=\underbrace{x(t)}_{baseband}+j\underbrace{y(t)}_{baseband}$$

- Thus the term, *complex baseband* notation

Relationship Between Complex Envelope and I/Q Forms:

- Proof:
  - Let *g(t) = a(t) + j b(t)*

$$v(t) = \mathrm{Re}\left\{g(t)e^{j\omega_c t}\right\}$$
$$= \mathrm{Re}\left\{\left[a(t) + jb(t)\right]\left[\cos(\omega_c t) + j\sin(\omega_c t)\right]\right\}$$
$$= \mathrm{Re}\left\{\left[a(t)\cos(\omega_c t) - b(t)\sin(\omega_c t)\right] + j\left[a(t)\sin(\omega_c t) + b(t)\cos(\omega_c t)\right]\right\}$$
$$= a(t)\cos(\omega_c t) - b(t)\sin(\omega_c t)$$

  - Thus, $g(t) = \underbrace{x(t)}_{baseband} + j\underbrace{y(t)}_{baseband}$

Relationship Between Complex Envelope and I/Q Forms:

- To transform from Complex Envelope to I/Q:

$$x(t) = \mathrm{Re}\left[g(t)\right]$$
$$y(t) = \mathrm{Im}\left[g(t)\right]$$

- To transform from I/Q to complex envelope:

$$g(t) = x(t) + jy(t)$$
$$v(t) = \mathrm{Re}\left[g(t)e^{j\omega_c t}\right]$$
$$= \mathrm{Re}\left[\left(x(t) + jy(t)\right)\cdot\left(\cos\omega_c t + j\sin\omega_c t\right)\right]$$
$$= x(t)\cos\omega_c t - y(t)\sin\omega_c t$$

Relationship Between Complex Envelope and Magnitude/Phase Forms:

- To transform from Complex Envelope to Magnitude/Phase:

$$R(t) = \sqrt{x^2(t) + y^2(t)}$$
$$= \left|g(t)\right|$$
$$\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$
$$= \angle g(t)$$

- To transform from Magnitude/Phase to Complex Envelope: $g(t) = \left|g(t)\right|e^{j\angle g(t)} = R(t)e^{j\theta(t)}$

Relationship Between Spectral Representations

- Assume:

$$v(t) = \text{Re}\left\{ g(t) e^{j\omega_c t} \right\}$$

- Fourier Transform (Deterministic Signals):

$$V(f) = \frac{1}{2}\left[G(f - f_c) + G^*(-f - f_c) \right]$$

- Power Spectral Density (Random Signals):

$$P_V(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c) \right]$$

Spectrum of Bandpass Signals

$$v(t) = \text{Re}\left\{ g(t) e^{j\omega_c t} \right\}$$

$$= \frac{1}{2} g(t) e^{j\omega_c t} + \frac{1}{2} g^*(t) e^{-j\omega_c t}$$

$$V(f) = F\left\{ \frac{1}{2} g(t) e^{j\omega_c t} + \frac{1}{2} g^*(t) e^{-j\omega_c t} \right\}$$

$$= \frac{1}{2} F\left\{ g(t) e^{j\omega_c t} \right\} + \frac{1}{2} F\left\{ g^*(t) e^{-j\omega_c t} \right\}$$

$$= \frac{1}{2} G(f - f_c) + \frac{1}{2} G^*(-(f + f_c))$$

$$= \frac{1}{2} G(f - f_c) + \frac{1}{2} G^*(-f - f_c)$$

Relationship Between Power of Bandpass and Complex Envelope Representations

- Power of bandpass signal is one half of power in complex envelope:

$$P_V = R_V(0) = \frac{1}{2}\left\langle |g(t)|^2 \right\rangle = \frac{1}{2} R_g(0) = \frac{1}{2} P_g$$

- This is because:

$$R_v(\tau) = \frac{1}{2} \text{Re}\left\{ R_g(\tau) e^{j\omega_c \tau} \right\}$$

Proof:

$$
\begin{aligned}
R_v(\tau) &= \left\langle v(t)v(t+\tau) \right\rangle \\
&= \left\langle \mathrm{Re}\left\{ g(t)e^{j\omega_c t} \right\} \mathrm{Re}\left\{ g(t+\tau)e^{j\omega_c (t+\tau)} \right\} \right\rangle \\
&= \frac{1}{2}\left\langle \mathrm{Re}\left\{ g^*(t)g(t+\tau)e^{-j\omega_c t}e^{j\omega_c (t+\tau)} \right\} \right\rangle \\
&\quad + \frac{1}{2}\left\langle \mathrm{Re}\left\{ g(t)g(t+\tau)e^{j\omega_c t}e^{j\omega_c (t+\tau)} \right\} \right\rangle \\
&= \frac{1}{2}\mathrm{Re}\left\{ \left\langle g^*(t)g(t+\tau) \right\rangle e^{j\omega_c \tau} \right\} + \frac{1}{2}\mathrm{Re}\left\{ \underbrace{\left\langle g(t)g(t+\tau)e^{j2\omega_c t} \right\rangle}_{negligible} e^{j\omega_c \tau} \right\} \\
&= \frac{1}{2}\mathrm{Re}\left\{ R_g(\tau)e^{j\omega_c \tau} \right\}
\end{aligned}
$$

$$\boxed{\mathrm{Re}\{a\}\,\mathrm{Re}\{b\} = \frac{1}{2}\mathrm{Re}\{ab^*\} + \frac{1}{2}\mathrm{Re}\{ab\}}$$

In-class drill

Summary

- Today we described three ways of representing bandpass signals
  - Magnitude/Phase representation
  - In-phase/Quadrature representation
  - Complex baseband representation
- Each representation has its own usefulness
- We will heavily rely on the I/Q and the complex baseband representations throughout the course