

ECE4634 Digital Communications Fall 2007

Instructor: R. Michael Buehrer
Lecture #15: Bandpass
Modulation – BASK



1

Motivation



- Baseband signals $w(t)$ may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t) \cos(\omega_c t + \theta) \Leftrightarrow \frac{1}{2} \left[e^{j\theta} W(f - f_c) + e^{-j\theta} W(f + f_c) \right]$$

- Most transmitted signals are modulated onto a carrier because
 - Modulated signals propagate well through the atmosphere
 - Modulation allows many signals with different carrier frequencies to share the spectrum
- We investigate a very simple form of sinusoidal modulation that directly uses the Fourier Modulation property: Binary Amplitude Shift Keying (BASK)
- What to read – Section 7.2

RM Buehrer
Virginia Tech
Fall 2007

2

Lecture Objective



- The objective of today's lecture is to describe the most simple form of digital bandpass modulation, Binary Amplitude Shift Keying (BASK)
- Specifically we will describe
 - Bandpass representations for BASK
 - Transmitter and Receiver design
 - Spectral characteristics

RM Buehrer
Virginia Tech
Fall 2007

3

Three Ways of Representing Bandpass Signals



- We will use the following analytical tools to handle bandpass signals

- Magnitude and Phase

$$v(t) = R(t)\cos[\omega_c t + \theta(t)]$$

- In Phase and Quadrature

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \text{Re}\left[g(t)e^{j\omega_c t}\right]$$

RM Buehrer
Virginia Tech
Fall 2007

4

Bandpass Modulation - ASK



- Amplitude Shift Keying

- Basic Idea:

- Send a sinusoid for 1, nothing for a 0

- Let T_s be the duration of one data symbol (also bit)

- Then we transmit the signal $v(t)$:

$$b_k = 1 \Rightarrow v(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \Big|_0^{T_s}$$

$$b_k = 0 \Rightarrow v(t) = 0 \Big|_0^{T_s}$$

$$v(t) = \sqrt{\frac{2E_b}{T_b}} \left(\sum_k b_k \text{rect}\left(\frac{t-kT_s}{T_s}\right) \right) \cos(\omega_c t)$$

- Analogous to Unipolar NRZ signaling

- Also called On-Off Keying (OOK)

For binary modulation
 $T_b = T_s$

RM Buehrer
Virginia Tech
Fall 2007

5

ASK - Magnitude and Phase Representation



- $v(t) = R(t)\cos[\omega_c t + \theta(t)]$ where

$$1 \Rightarrow R(t) = \sqrt{\frac{2E_b}{T_b}} \Big|_0^{T_s}$$

$$0 \Rightarrow R(t) = 0 \Big|_0^{T_s}$$

$$\theta(t) = 0 \Big|_0^{T_s}$$

$$R(t) = x(t) = \sqrt{\frac{2E_b}{T_b}} \sum_k b_k \text{rect}\left(\frac{t-kT_s}{T_s}\right)$$

unipolar NRZ linecode

- All of the information is in the amplitude (i.e., no information in the phase)

- This modulation is nice since it can be detected non-coherently (i.e., we do not need a phase reference)

RM Buehrer
Virginia Tech
Fall 2007

6

ASK - I & Q Representation



- $v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$ where
 - $y(t) = R(t)\sin(\theta(t)) = 0 \Big|_0^{T_s}$
 - No Q component
- $1 \Rightarrow x(t) = R(t)\cos(\theta(t)) = \sqrt{\frac{2E_b}{T_b}} \Big|_0^{T_s} \Rightarrow x(t) = \sqrt{\frac{2E_b}{T_b}} \sum_k b_k \text{rect}\left(\frac{t-kT_s}{T_s}\right)$
- $0 \Rightarrow x(t) = R(t)\cos(\theta(t)) = 0 \Big|_0^{T_s}$
- I component is just a unipolar NRZ signal
- If a second ASK signal is transmitted as the Q-component, then we can transmit two bits
- ASK can also be used with noncoherent reception

RM Buehrer
Virginia Tech
Fall 2007

7

ASK - Complex Envelope Representation

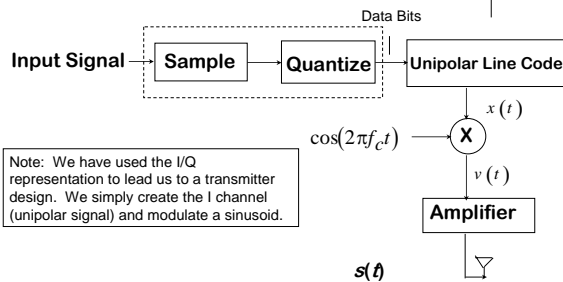


- $v(t) = \text{Re}\left[g(t)e^{j\omega_c t}\right]$
 - $g(t) = x(t)$
 - $1 \Rightarrow g(t) = \sqrt{\frac{2E_b}{T_b}} \Big|_0^{T_s} \Rightarrow g(t) = \sqrt{\frac{2E_b}{T_b}} \sum_k b_k \text{rect}\left(\frac{t-kT_s}{T_s}\right)$
 - $0 \Rightarrow g(t) = 0 \Big|_0^{T_s}$
- Complex envelope is entirely real
- Complex envelope is equivalent to unipolar NRZ signaling
- BPSK is more energy efficient
 - Note ASK has energy in impulse at f_c

RM Buehrer
Virginia Tech
Fall 2007

8

Transmitter for ASK

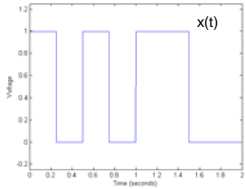


RM Buehrer
Virginia Tech
Fall 2007

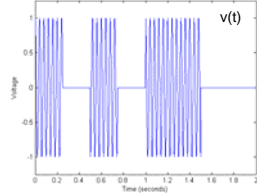
9

Example

- $b = [1, 0, 1, 0, 1, 1, 0, 0]$



First step is to create a baseband pulse modulated signal



Second step is to create a bandpass signal by linearly modulating the carrier by the¹⁰ baseband signal

RM Buehrer
Virginia Tech
Fall 2007



Receivers for BASK

- A digital receiver can be broken down into two general functions
 - Converting the bandpass signal into a baseband signal (often termed *demodulation*)
 - Determining the data bits from the baseband signal (often termed *detection*)
- Receiver structures for BASK can be found by recognizing the similarity between BASK and AM
 - Envelope detection
 - Product detector
 - Non-coherent product detector
- Sampling vs. Filtering
 - Determining the data bits can be done by simply sampling the bandpass signal and making a decision
 - However in the presence of noise better performance can be obtained by filtering or integrating the signal first and then sampling

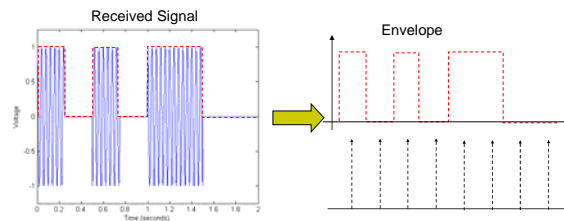
RM Buehrer
Virginia Tech
Fall 2007

11



Envelope Detector

- The simplest receiver is an envelope detector



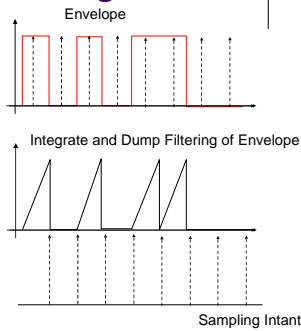
RM Buehrer
Virginia Tech
Fall 2007

Sampling instances ¹²



Sampling vs. Filtering

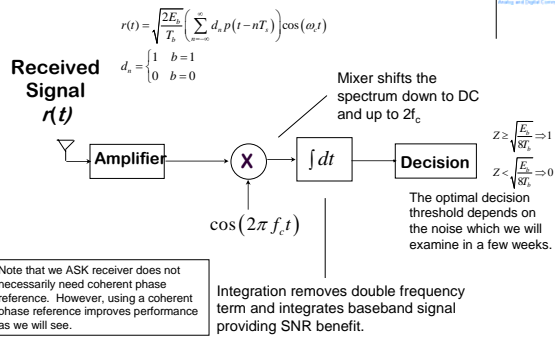
- Integration improves SNR (as we will see later in the course) but may require more accurate timing



RM Buehrer
Virginia Tech
Fall 2007



Coherent Product Detector



RM Buehrer
Virginia Tech
Fall 2007



Receiver for ASK

- Decision Variable Z (assume that $b=1$):

$$\begin{aligned} Z &= \frac{1}{T_b} \int_0^{T_b} r(t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_b} \int_0^{T_b} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_b} \int_0^{T_b} x(t) \frac{1}{2} [1 + \cos(4\pi f_c t)] dt \quad \text{Integrates to } \approx 0 \\ &\approx \sqrt{\frac{E_b}{2T_b}} \end{aligned}$$

$$x(t) = \sqrt{\frac{2E_b}{T_b}} p(t-nT_b) \cos(\omega_c t)$$

- Decision Variable Z (assume that $b=0$):

$$\begin{aligned} Z &= \frac{1}{T_b} \int_0^{T_b} r(t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_b} \int_0^{T_b} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) dt \\ &\approx 0 \end{aligned}$$

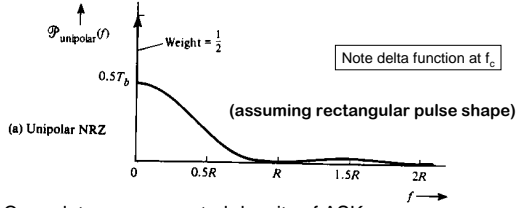
RM Buehrer
Virginia Tech
Fall 2007



Power Spectral Density of ASK



- Complex envelope $g(t)$ of ASK is unipolar NRZ



- Can relate power spectral density of ASK:

$$P_v(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

RM Buehrer
Virginia Tech
Fall 2007

19

Power Spectral Density for ASK



We have shown for unipolar NRZ signaling:

$$P_g(f) = \frac{A^2 T_b}{4} \left[\frac{\sin(\pi f T_b)}{\pi f T_b} \right]^2 \left[1 + \frac{1}{T_b} \delta(f) \right]$$

due to pulse shape due to DC component

Thus,

$$\begin{aligned} P_v(f) &= \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)] \\ &= \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c) T_b)}{\pi(f - f_c) T_b} \right)^2 + \left(\frac{\sin(\pi(-f - f_c) T_b)}{\pi(-f - f_c) T_b} \right)^2 \right] + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(-f - f_c) \\ &= \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c) T_b)}{\pi(f - f_c) T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c) T_b)}{\pi(f + f_c) T_b} \right)^2 \right] + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(f + f_c) \\ &= \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c) T_b)}{\pi(f - f_c) T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c) T_b)}{\pi(f + f_c) T_b} \right)^2 \right] + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(f + f_c) \end{aligned}$$

$$A = \sqrt{\frac{2E_b}{T_b}}$$

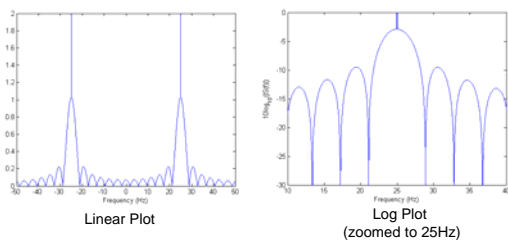
RM Buehrer
Virginia Tech
Fall 2007

20

Example - Theory



- $f_c = 25\text{Hz}$, $R_b = 4\text{bps}$



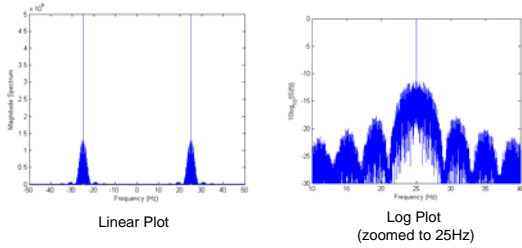
RM Buehrer
Virginia Tech
Fall 2007

21

Example – Measured Signal



- $f_c = 25\text{Hz}$, $R_b = 4\text{bps}$



RM Buehrer
Virginia Tech
Fall 2007

22

Pulse Shaping



- Recall that the power spectral density of BASK is dominated by the square pulse from the NRZ line code:

$$P_s(f) = \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f-f_c)T_b)}{\pi(f-f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(f+f_c)T_b)}{\pi(f+f_c)T_b} \right)^2 + \frac{1}{T_b} \delta(f-f_c) + \frac{1}{T_b} \delta(f+f_c) \right]$$

- Thus, we can improve the spectral characteristics of BASK by changing the pulse shape $p(t)$ associated with the NRZ line code:

$$x(t) = \sqrt{\frac{2E_b}{T_b}} \sum_k p\left(\frac{t-kT_b}{T_b}\right)$$

Pulse Shape
 $p(t) \iff P(f)$

- And the new PSD is

$$P_s(f) = \frac{A^2 T_b}{16} \left[P^2(f) + P^2(f) + \frac{1}{T_b} \delta(f-f_c) + \frac{1}{T_b} \delta(f+f_c) \right]$$

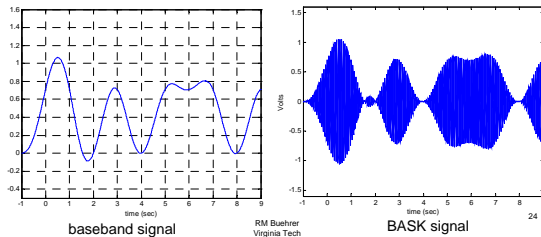
RM Buehrer
Virginia Tech
Fall 2007

23

Example – Sinc Functions



- Truncation length – 20 symbols
- Carrier frequency – 15Hz
- Data rate – 1sps



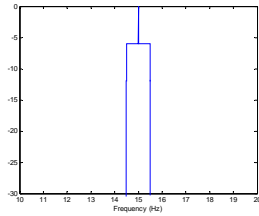
RM Buehrer
Virginia Tech
Fall 2007

24

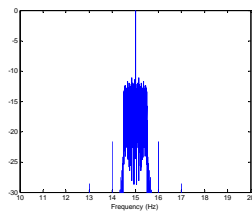
Example – PSD.



Theory



Measured



RM Buehrer
Virginia Tech
Fall 2007

25

In class Drill



- If time permits...

RM Buehrer
Virginia Tech
Fall 2007

26

Summary



- We have begun our investigation of binary digital bandpass modulation schemes with perhaps the simplest scheme Binary Amplitude Shift Keying (BASK)
- BASK can be viewed as a simple binary NRZ line code linearly modulating a sinusoid
- The transmitter and receiver are similar to those for analog Amplitude Modulation
- The spectral properties of BASK can be easily modified through changing the pulse shape in the NRZ line code

RM Buehrer
Virginia Tech
Fall 2007

27
