

ECE4634

Digital Communications

Fall 2007

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Lecture #15: Bandpass
Modulation – BASK



Analog and Digital Communications

Motivation



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- Baseband signals $w(t)$ may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t)\cos(\omega_c t + \theta) \Leftrightarrow \frac{1}{2} \left[e^{j\theta} W(f - f_c) + e^{-j\theta} W(f + f_c) \right]$$

- Most transmitted signals are modulated onto a carrier because
 - Modulated signals propagate well through the atmosphere
 - Modulation allows many signals with different carrier frequencies to share the spectrum
- We investigate a very simple form of sinusoidal modulation that directly uses the Fourier Modulation property: Binary Amplitude Shift Keying (BASK)
- What to read – Section 7.2



Lecture Objective

- The objective of today's lecture is to describe the most simple form of digital bandpass modulation, Binary Amplitude Shift Keying (BASK)
- Specifically we will describe
 - Bandpass representations for BASK
 - Transmitter and Receiver design
 - Spectral characteristics

Three Ways of Representing Bandpass Signals



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- We will use the following analytical tools to handle bandpass signals

- Magnitude and Phase

$$v(t) = R(t) \cos[\omega_c t + \theta(t)]$$

- In Phase and Quadrature

$$v(t) = x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \operatorname{Re} \left[g(t) e^{j\omega_c t} \right]$$

Bandpass Modulation - ASK



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- Amplitude Shift Keying
- Basic Idea:
 - Send a sinusoid for 1, nothing for a 0
 - Let T_s be the duration of one data symbol (also bit)
 - Then we transmit the signal $v(t)$:

$$b_k = 1 \Rightarrow v(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \Big|_0^{T_s}$$
$$b_k = 0 \Rightarrow v(t) = 0 \Big|_0^{T_s}$$

$$v(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}} \left(\sum_k b_k \text{rect}\left(\frac{t-kT_s}{T_s}\right) \right)}_{x(t)} \cos(\omega_c t)$$

- Analogous to Unipolar NRZ signaling
- Also called On-Off Keying (OOK)

For binary modulation

$$T_b = T_s \quad 5$$

ASK - Magnitude and Phase Representation



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- $v(t) = R(t) \cos[\omega_c t + \theta(t)]$

where

- $1 \Rightarrow R(t) = \sqrt{\frac{2E_b}{T_b}} \Big|_0^{T_s}$

$$0 \Rightarrow R(t) = 0 \Big|_0^{T_s}$$

$$\theta(t) = 0 \Big|_0^{T_s}$$

$$R(t) = x(t)$$



$$= \underbrace{\sqrt{\frac{2E_b}{T_b}} \sum_k b_k \text{rect}\left(\frac{t - kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$$

- All of the information is in the amplitude (i.e., no information in the phase)
- This modulation is nice since it can be detected non-coherently (i.e., we do not need a phase reference)



ASK - I & Q Representation

- $v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$ where

- $y(t) = R(t)\sin(\theta(t))$

$$= 0 \Big|_0^{T_s}$$

- No Q component

- $1 \Rightarrow x(t) = R(t)\cos(\theta(t)) = \sqrt{\frac{2E_b}{T_b}} \Big|_0^{T_s} \Rightarrow x(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}} \sum_k b_k \text{rect}\left(\frac{t-kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$
- $0 \Rightarrow x(t) = R(t)\cos(\theta(t)) = 0 \Big|_0^{T_s}$

- I component is just a unipolar NRZ signal
- If a second ASK signal is transmitted as the Q-component, then we can transmit two bits
- ASK can also be used with noncoherent reception

ASK - Complex Envelope Representation



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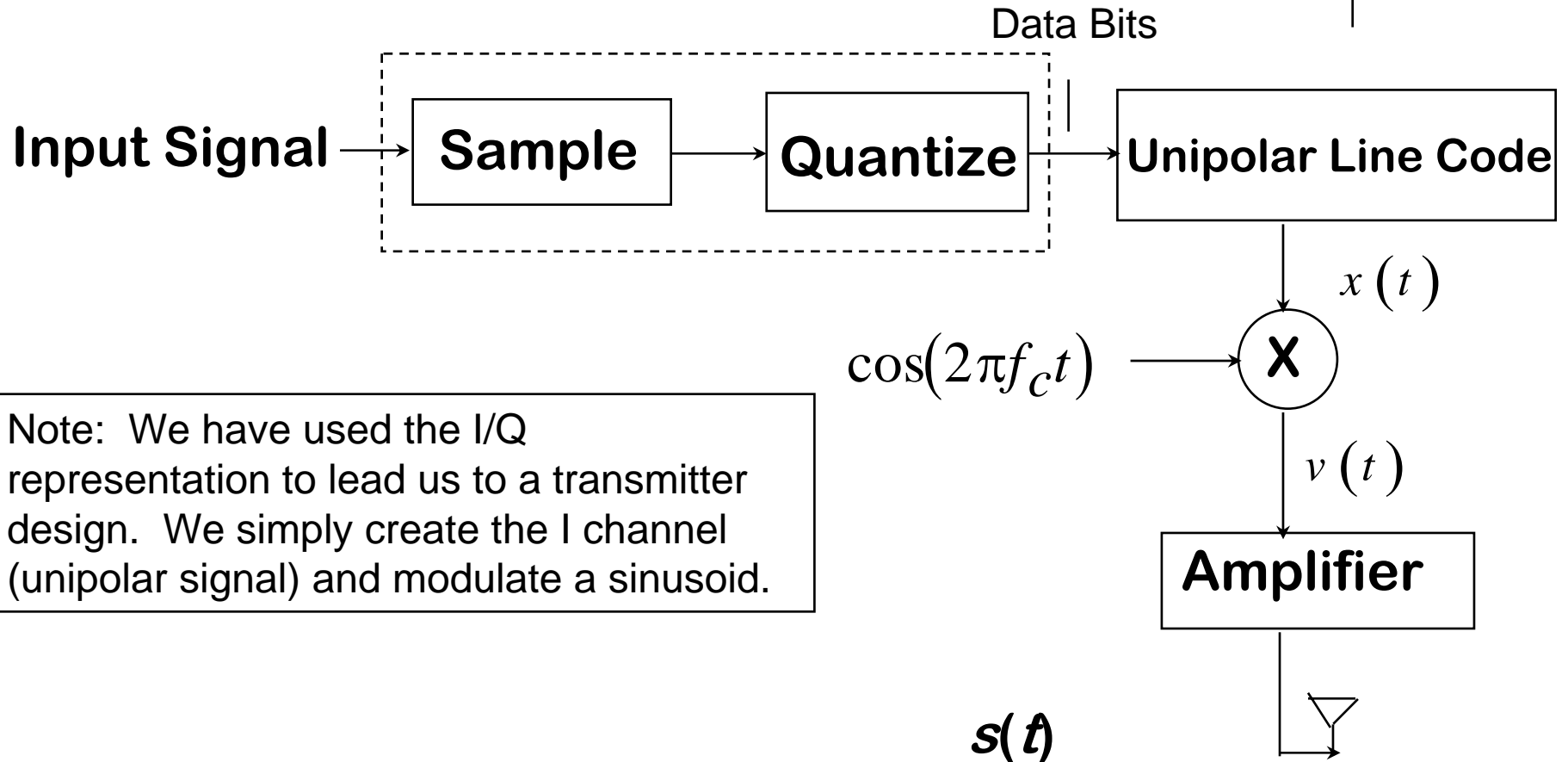
- $v(t) = \text{Re} \left[g(t) e^{j\omega_c t} \right]$
 $1 \Rightarrow g(t) = \sqrt{\frac{2E_b}{T_b}} \Big|_0^{T_s}$
 $0 \Rightarrow g(t) = 0 \Big|_0^{T_s}$
 $\Rightarrow g(t) = x(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}} \sum_k b_k \text{rect} \left(\frac{t - kT_s}{T_s} \right)}_{\text{unipolar NRZ linecode}}$

- Complex envelope is entirely real
- Complex envelope is equivalent to unipolar NRZ signaling
- BPSK is more energy efficient
 - Note ASK has energy in impulse at f_c

Transmitter for ASK



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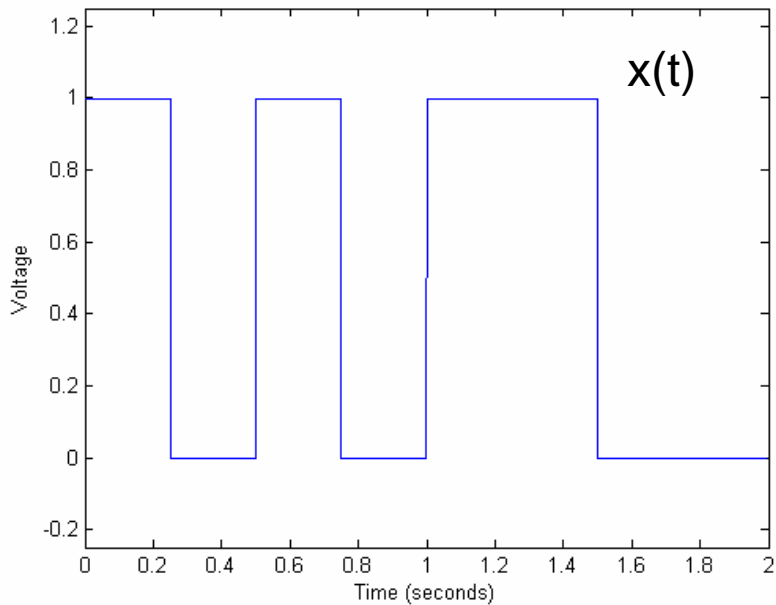


Note: We have used the I/Q representation to lead us to a transmitter design. We simply create the I channel (unipolar signal) and modulate a sinusoid.

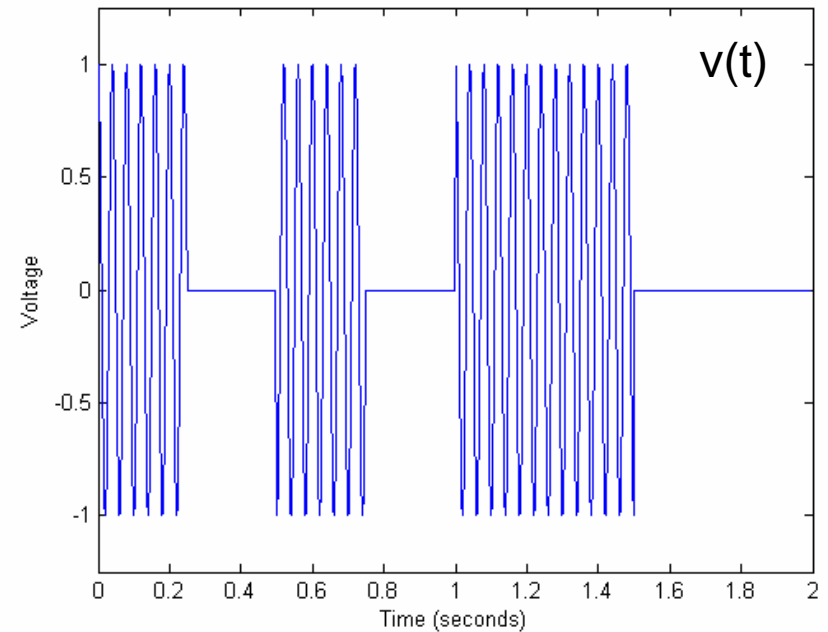


Example

- $b = [1, 0, 1, 0, 1, 1, 0, 0]$



First step is to create a baseband pulse modulated signal



Second step is to create a the bandpass signal by linearly modulating the carrier by the¹⁰ baseband signal

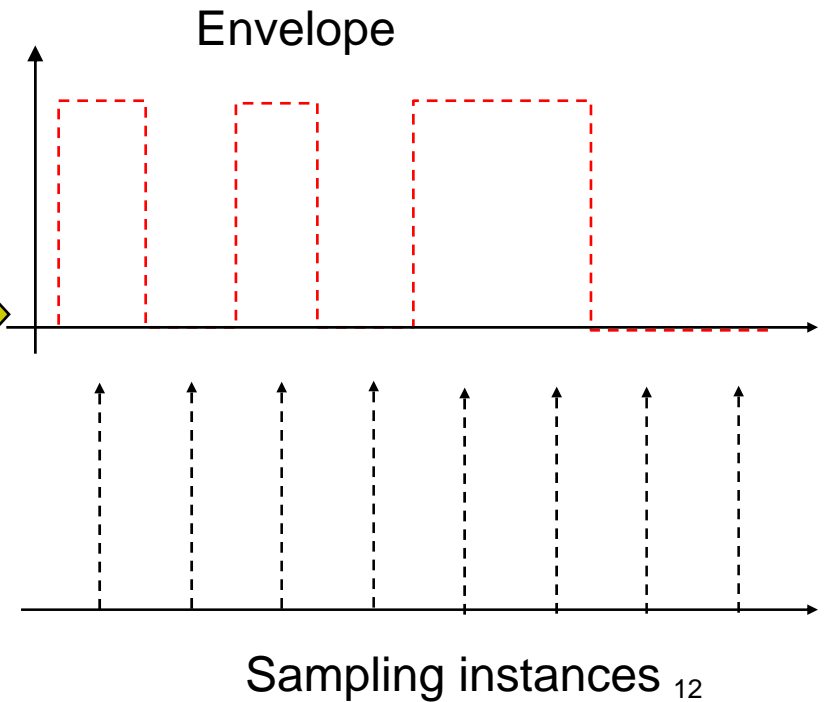
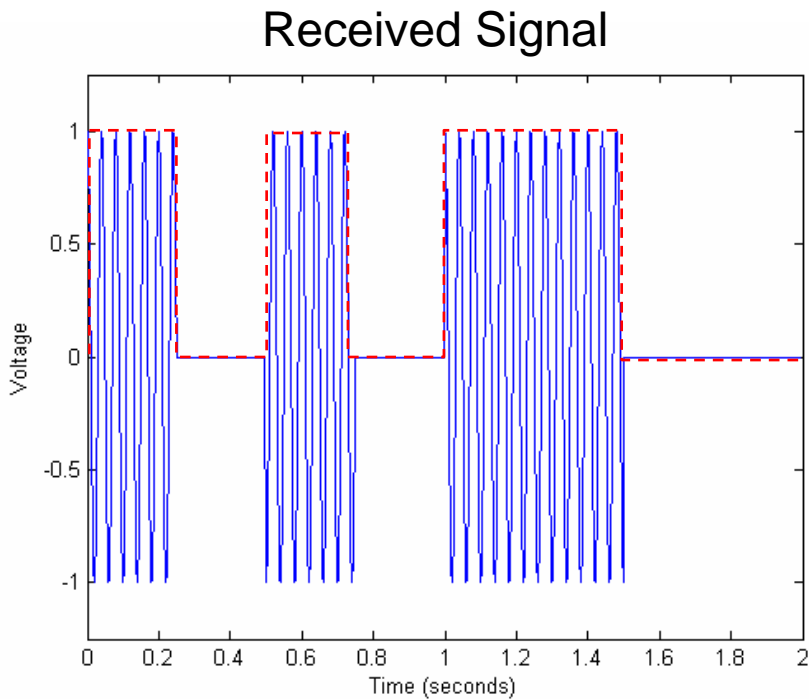


Receivers for BASK

- A digital receiver can be broken down into to two general functions
 - Converting the bandpass signal into a baseband signal (often termed *demodulation*)
 - Determining the data bits from the baseband signal (often termed *detection*)
- Receiver structures for BASK can be found by recognizing the similarity between BASK and AM
 - Envelope detection
 - Product detector
 - Non-coherent product detector
- Sampling vs. Filtering
 - Determining the data bits can be done by simply sampling the bandpass signal and making a decision
 - However in the presence of noise better performance can be obtained by filtering or integrating the signal first and then sampling

Envelope Detector

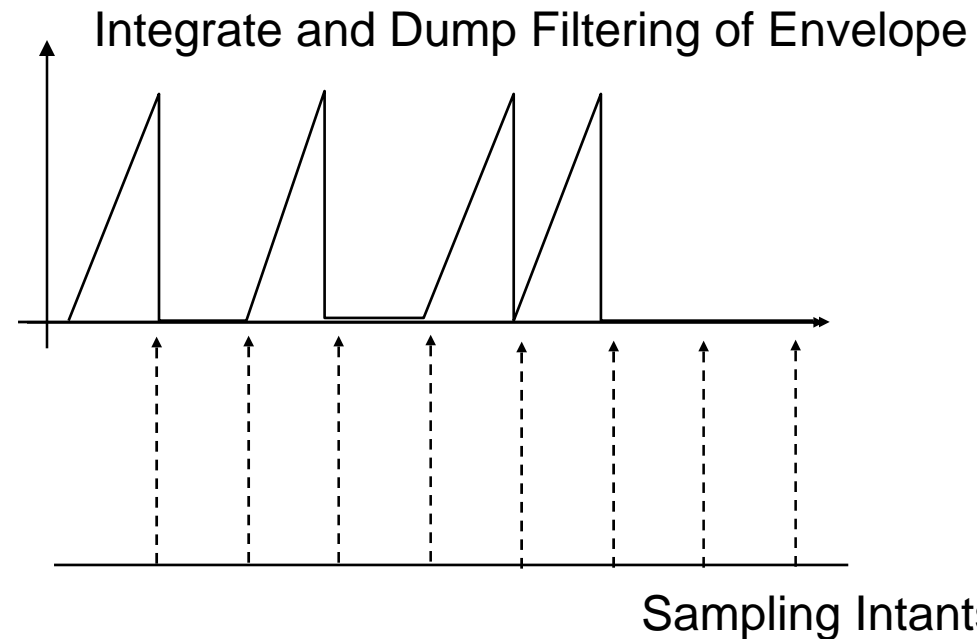
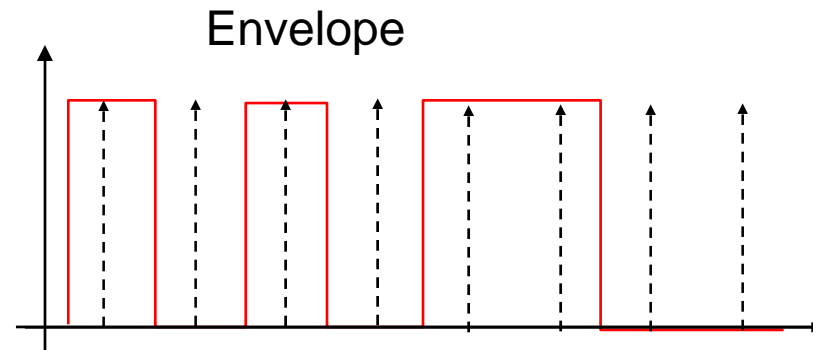
- The simplest receiver is an envelope detector





Sampling vs. Filtering

- Integration improves SNR (as we will see later in the course) but may require more accurate timing



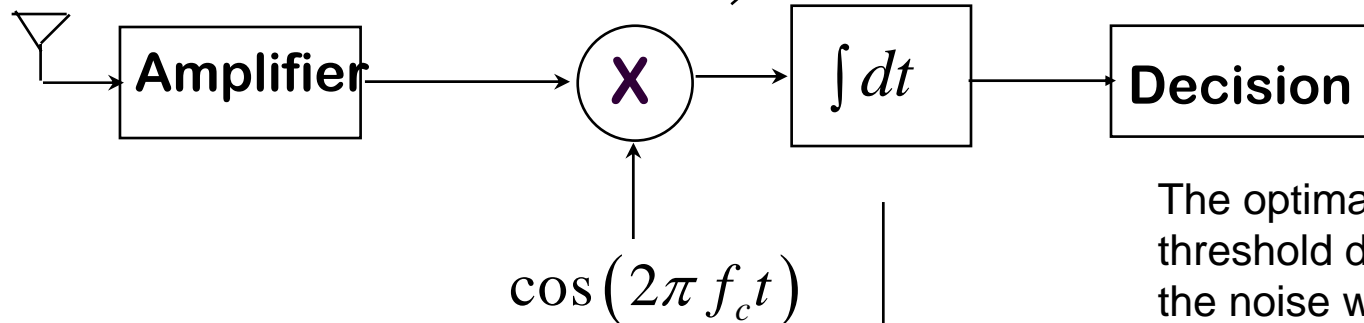


Coherent Product Detector

$$r(t) = \sqrt{\frac{2E_b}{T_b}} \left(\sum_{n=-\infty}^{\infty} d_n p(t - nT_s) \right) \cos(\omega_c t)$$

$$d_n = \begin{cases} 1 & b = 1 \\ 0 & b = 0 \end{cases}$$

Received
Signal
 $r(t)$



Mixer shifts the spectrum down to DC and up to $2f_c$

$$Z \geq \sqrt{\frac{E_b}{8T_b}} \Rightarrow 1$$
$$Z < \sqrt{\frac{E_b}{8T_b}} \Rightarrow 0$$

The optimal decision threshold depends on the noise which we will examine in a few weeks.

Note that we ASK receiver does not necessarily need coherent phase reference. However, using a coherent phase reference improves performance as we will see.

Integration removes double frequency term and integrates baseband signal providing SNR benefit.



Receiver for ASK

- Decision Variable Z (assume that $b=1$):

$$\begin{aligned} Z &= \frac{1}{T_s} \int_0^{T_s} r(t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_s} \int_0^{T_s} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_s} \int_0^{T_s} x(t) \frac{1}{2} [1 + \cos(4\pi f_c t)] dt \\ &\approx \sqrt{\frac{E_b}{2T_b}} \end{aligned}$$

Integrates to ~ 0

$$x(t) = \sqrt{\frac{2E_b}{T_b}} p(t - nT_s) \cos(\omega_c t)$$

- Decision Variable Z (assume that $b=0$):

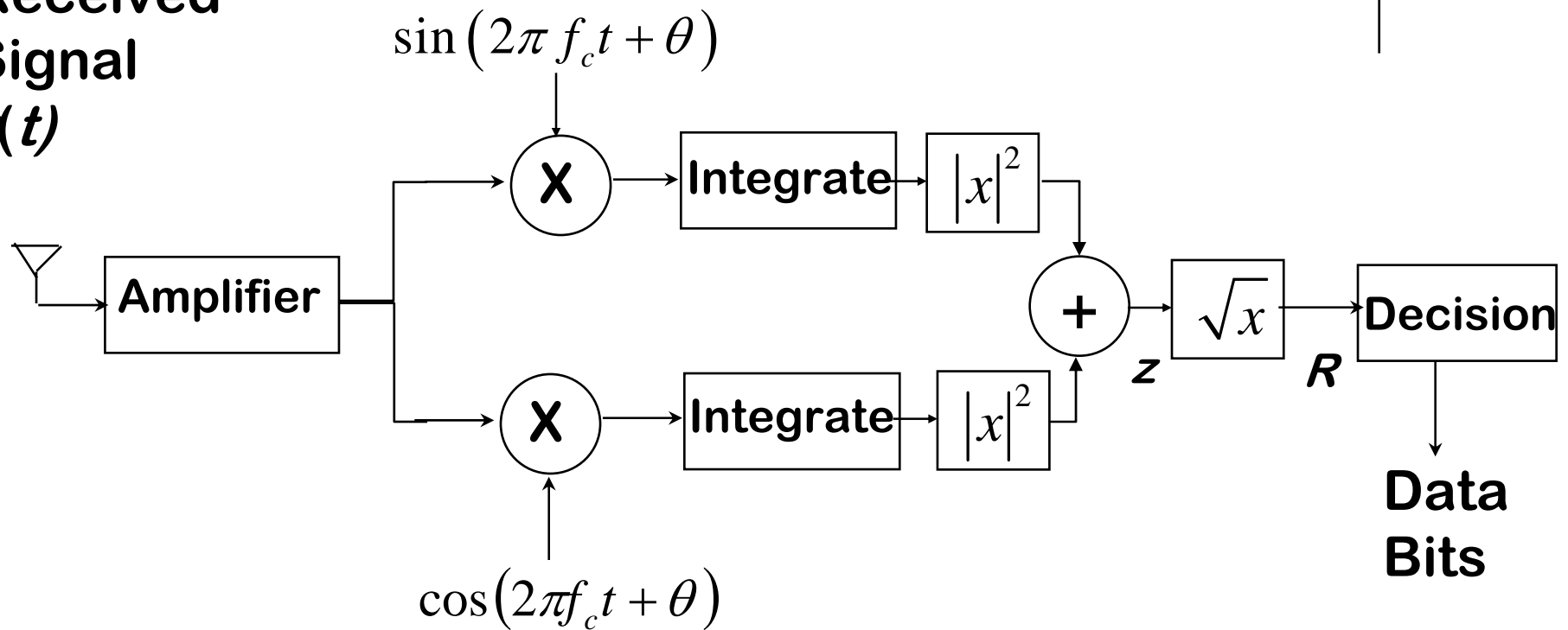
$$\begin{aligned} Z &= \frac{1}{T_s} \int_0^{T_s} r(t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_s} \int_0^{T_s} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) dt \\ &\approx 0 \end{aligned}$$

Non-coherent Product Detector



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Received
Signal
 $r(t)$



Note that this ASK receiver does not need a coherent phase reference. In other words, the value of θ is irrelevant.

$$R \geq \sqrt{\frac{E_b}{8T_b}} \Rightarrow 1$$

$$R < \sqrt{\frac{E_b}{8T_b}} \Rightarrow 0$$



Receiver for ASK

- Decision Variable R (ignoring noise for now) :

$$\begin{aligned} Z &= \left(\frac{1}{T_s} \int_0^{T_s} r(t) \cos(2\pi f_c t + \theta) dt \right)^2 + \left(\frac{1}{T_s} \int_0^{T_s} r(t) \sin(2\pi f_c t + \theta) dt \right)^2 \\ &= \left(\frac{1}{T_s} \int_0^{T_s} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \theta) dt \right)^2 + \left(\frac{1}{T_s} \int_0^{T_s} x(t) \cos(2\pi f_c t) \sin(2\pi f_c t + \theta) dt \right)^2 \\ &= \left(\frac{1}{T_s} \int_0^{T_s} x(t) \frac{1}{2} [\cos(\theta) + \cos(4\pi f_c t)] dt \right)^2 + \left(\frac{1}{T_s} \int_0^{T_s} x(t) \frac{1}{2} [\sin(\theta) + \sin(4\pi f_c t)] dt \right)^2 \\ &\approx \left(\sqrt{\frac{E_b}{2T_b}} \right)^2 (\cos^2(\theta) + \sin^2(\theta)) \end{aligned}$$

$$R = \sqrt{Z}$$

$$= \sqrt{\frac{E_b}{2T_b}}$$

$$R \geq \sqrt{\frac{E_b}{8T_b}} \Rightarrow 1, R < \sqrt{\frac{E_b}{8T_b}} \Rightarrow 0$$

$$x(t) = \sqrt{\frac{2E_b}{T_b}} p(t - nT_s) \cos(\omega_c t)$$

Note: In the absence of noise, we get the same result for coherent or non-coherent reception. However, when noise is added to the equation, we will see a difference. ¹⁷



Power Spectral Density

- The power spectral density can be determined by examining the complex baseband representation

$$P_v(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

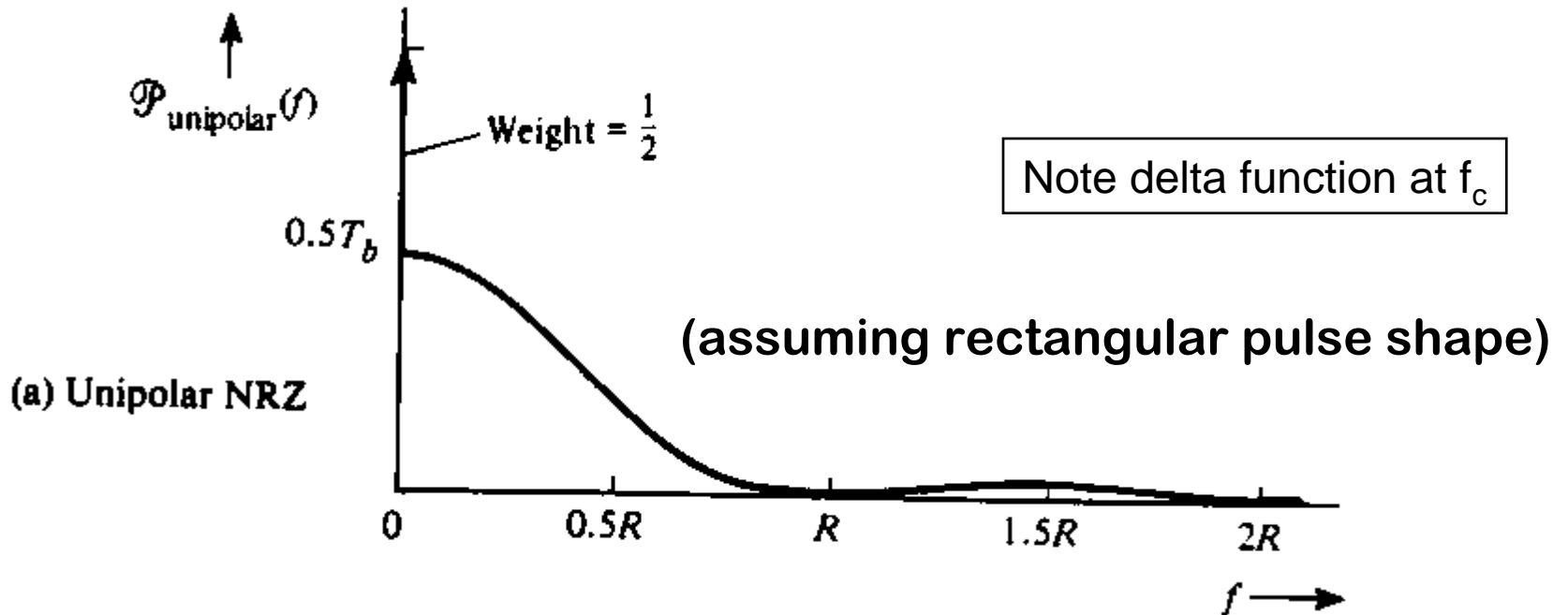
- However, we know that the complex baseband is simply a unipolar NRZ line code. We derived the PSD for this earlier in the course. Thus we can directly borrow the result.

Power Spectral Density of ASK



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- Complex envelope $g(t)$ of ASK is unipolar NRZ



- Can relate power spectral density of ASK:

$$P_v(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

Power Spectral Density for ASK

We have shown for unipolar NRZ signaling:

$$P_g(f) = \frac{A^2 T_b}{4} \underbrace{\left(\frac{\sin(\pi f T_b)}{\pi f T_b} \right)^2}_{\text{due to pulse shape}} \left[1 + \underbrace{\frac{1}{T_s} \delta(f)}_{\text{due to DC component}} \right]$$

Thus,

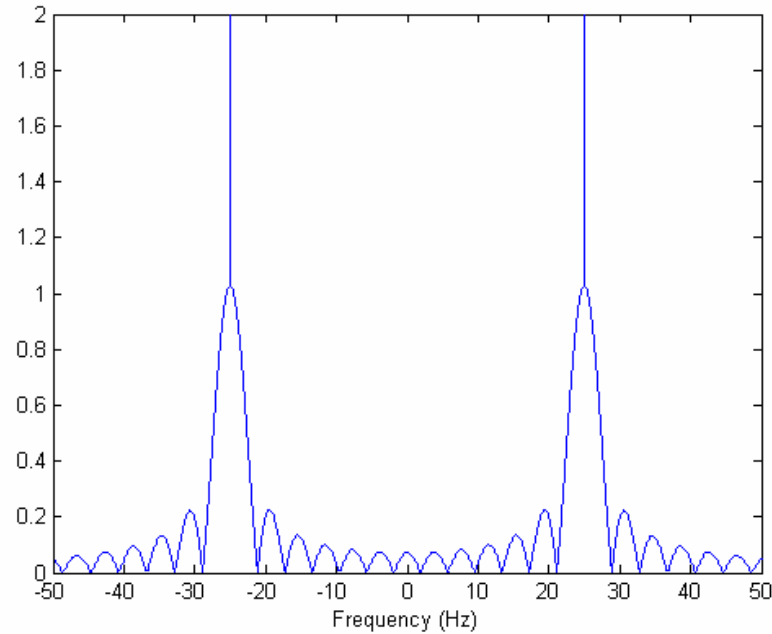
$$\begin{aligned} P_v(f) &= \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)] \\ &= \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(-f - f_c)T_b)}{\pi(-f - f_c)T_b} \right)^2 + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(-f - f_c) \right] \\ &= \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{-\sin(\pi(f + f_c)T_b)}{\pi(-f - f_c)T_b} \right)^2 + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(f + f_c) \right] \\ &= \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c)T_b)}{\pi(f + f_c)T_b} \right)^2 + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(f + f_c) \right] \end{aligned}$$

$$A = \sqrt{\frac{2E_b}{T_b}}$$

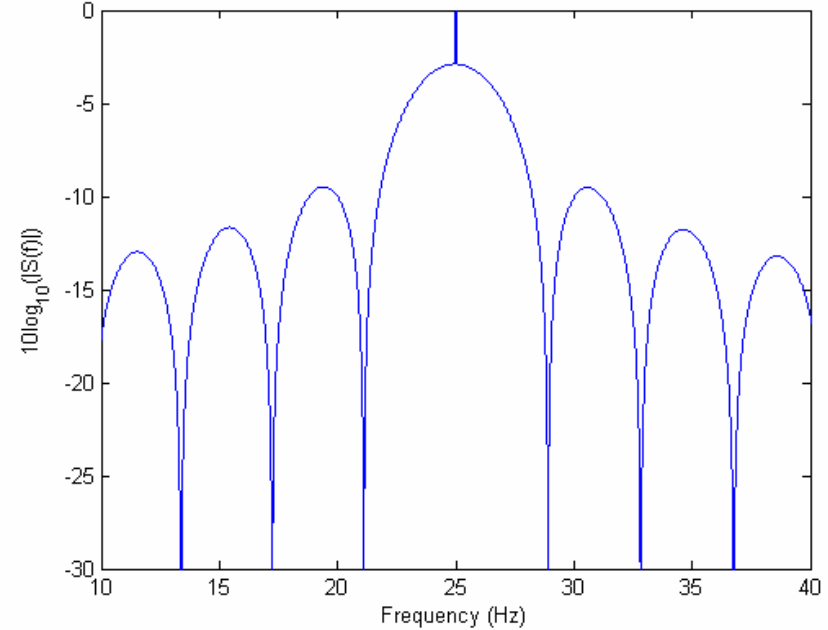


Example - Theory

- $f_c = 25\text{Hz}$, $R_b = 4\text{bps}$



Linear Plot

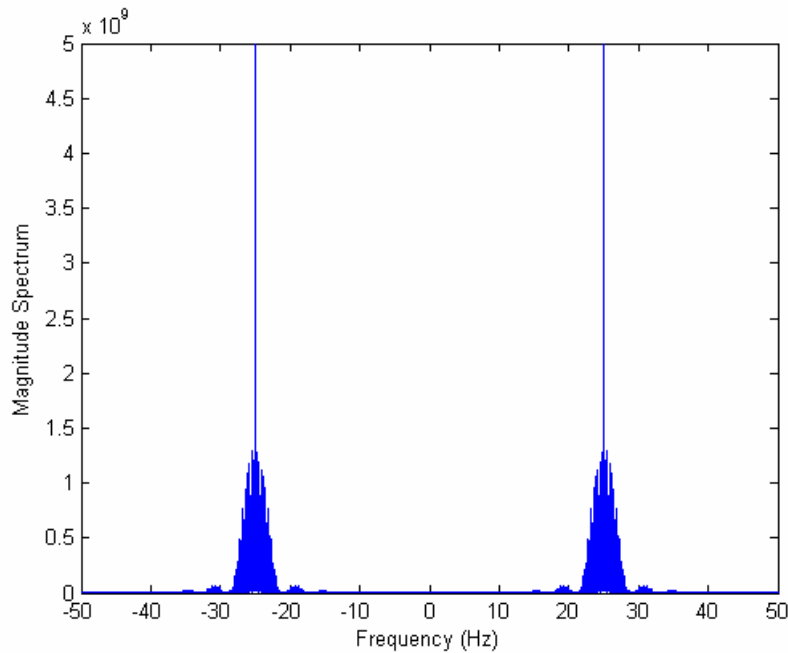


Log Plot
(zoomed to 25Hz)

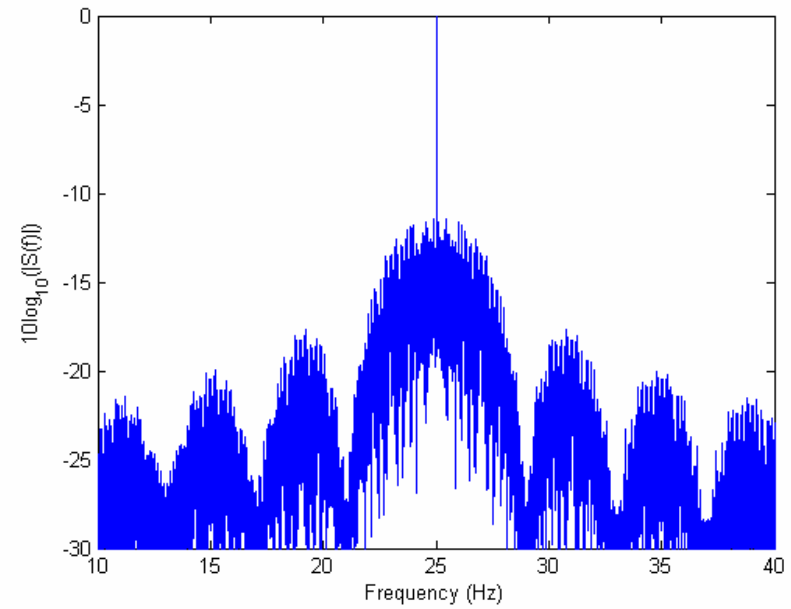


Example – Measured Signal

- $f_c = 25\text{Hz}$, $R_b = 4\text{bps}$



Linear Plot



Log Plot
(zoomed to 25Hz)



Pulse Shaping

- Recall that the power spectral density of BASK is dominated by the square pulse from the NRZ line code:

$$P_v(f) = \frac{A^2 T_b}{16} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c)T_b)}{\pi(f + f_c)T_b} \right)^2 + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(f + f_c) \right]$$

- Thus, we can improve the spectral characteristics of BASK by changing the pulse shape $p(t)$ associated with the NRZ line code:

$$x(t) = \sqrt{\frac{2E_b}{T_b}} \underbrace{\sum_k b_k p\left(\frac{t - kT_s}{T_s}\right)}_{\text{unipolar NRZ linecode}}$$

Pulse Shape

$$p(t) \iff P(f)$$

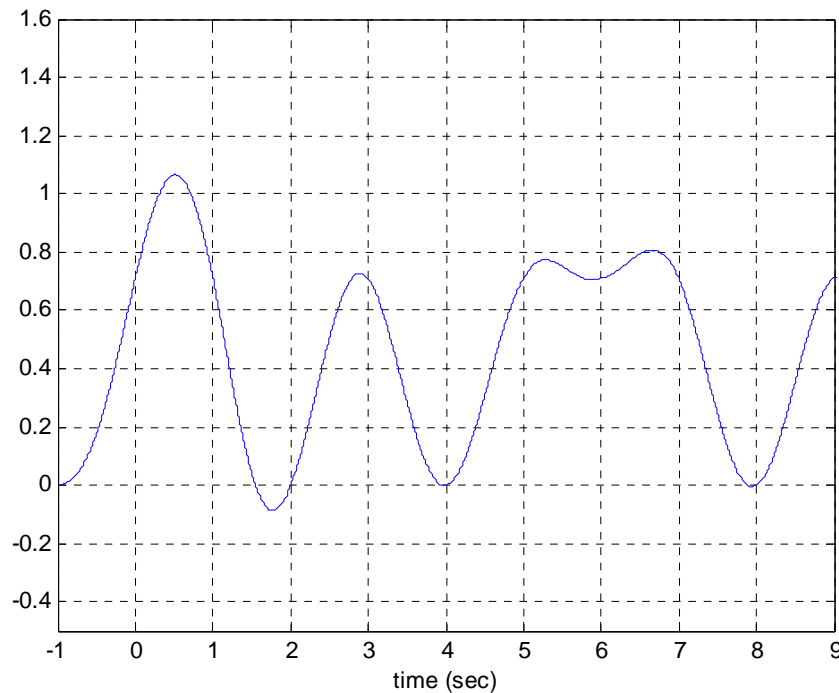
- And the new PSD is

$$P_v(f) = \frac{A^2 T_b}{16} \left[P^2(f) + P^2(f) + \frac{1}{T_b} \delta(f - f_c) + \frac{1}{T_b} \delta(f + f_c) \right]$$

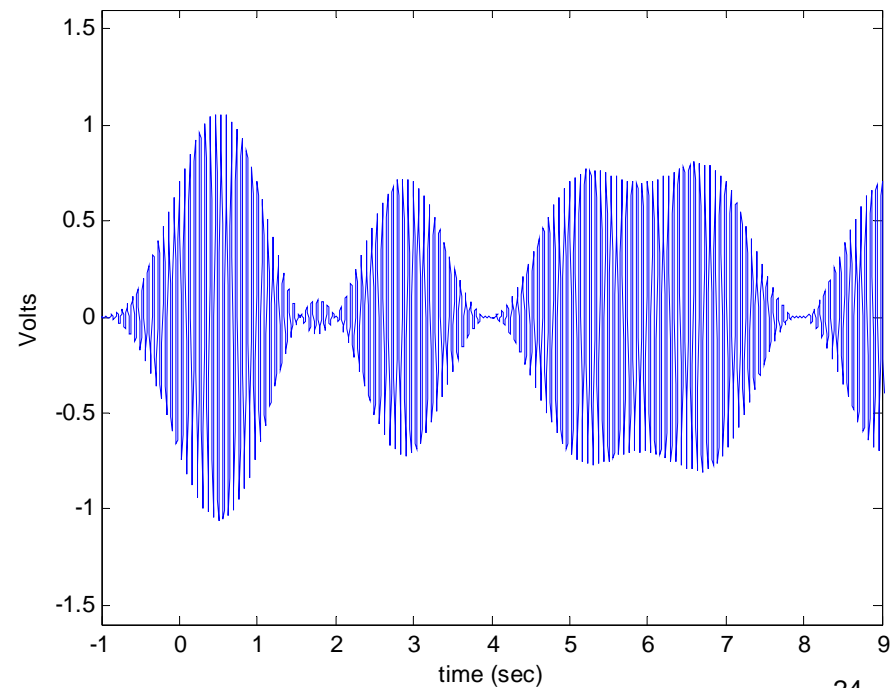


Example – Sinc Functions

- Truncation length – 20 symbols
- Carrier frequency – 15Hz
- Data rate – 1sps



baseband signal



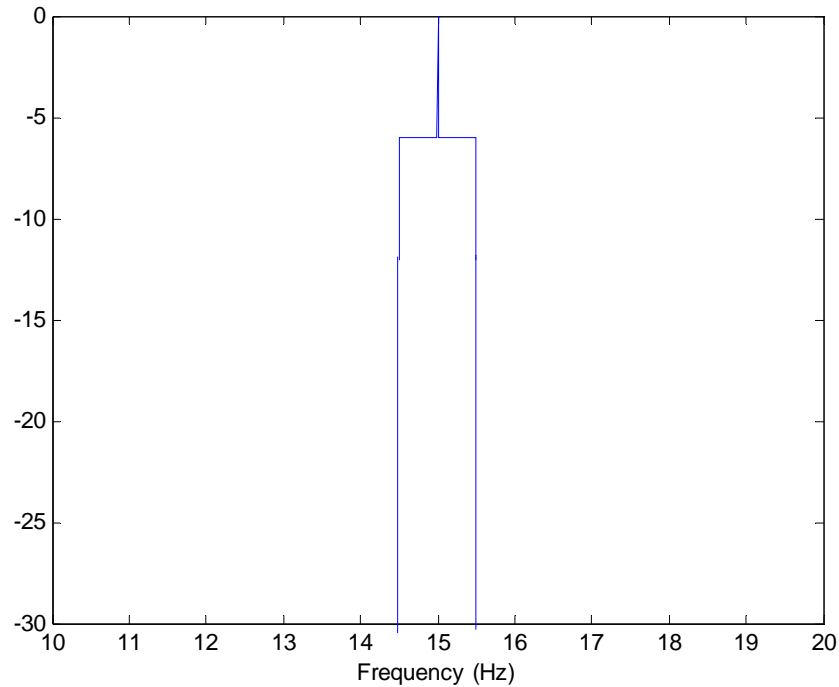
BASK signal

Example – PSD.

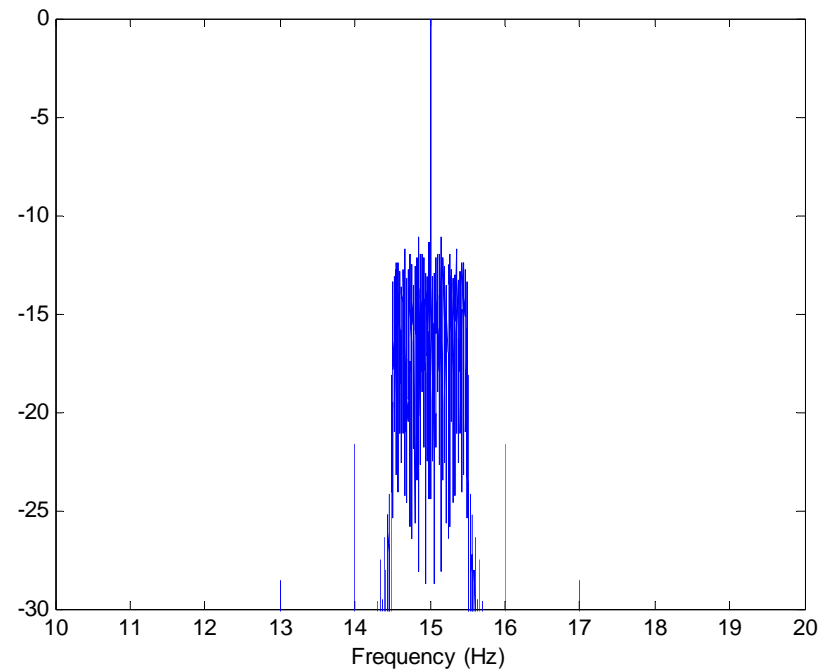


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Theory



Measured



In class Drill

- If time permits...



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Summary



- We have begun our investigation of binary digital bandpass modulation schemes with perhaps the simplest scheme Binary Amplitude Shift Keying (BASK)
- BASK can be viewed as a simple binary NRZ line code linearly modulating a sinusoid
- The transmitter and receiver are similar to those for analog Amplitude Modulation
- The spectral properties of BASK can be easily modified through changing the pulse shape in the NRZ line code