

ECE4634 Digital Communications Fall 2007

Instructor: R. Michael Buehrer
Lecture #16: Bandpass Signals
Bandpass Modulation - BPSK



Lecture Objective



- The objective of this lecture is to introduce the concept of Binary Phase Shift Modulation, a form of digital bandpass modulation.

Modulation



- Baseband signals $w(t)$ may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t)\cos(\omega_c t + \theta) \Leftrightarrow \frac{1}{2} [e^{j\theta}W(f - f_c) + e^{-j\theta}W(f + f_c)]$$

- Most transmitted signals are modulated onto a carrier because
 - Modulated signals propagate well through the atmosphere
 - Modulation allows many signals with different carrier frequencies to share the spectrum
 - Efficient antennas can be built at a reasonable size

Three Ways of Representing Bandpass Signals



- Magnitude and Phase

$$v(t) = R(t) \cos[\omega_c t + \theta(t)]$$

- In Phase and Quadrature

$$v(t) = x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \text{Re} \left[g(t) e^{j\omega_c t} \right]$$

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Transmitting Information



- Previously we saw that information (bits) can be transmitted by changing the amplitude of pulses.
 - Baseband communications
- With bandpass communications information is transmitted by modulating (changing) the phase, frequency or amplitude of a carrier (sinusoid)
- Modulation can be envisioned in the time domain as changing or modulating the sinusoid with time
- Modulation in the frequency domain can be envisioned as shifting the frequency components of the signal from baseband to bandpass around some center frequency.

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Bandpass Modulation - BPSK



- Binary Phase Shift Keying
- Basic Idea:
 - Data determines the **phase** of transmit sinusoid over

$$\begin{aligned}
 s(t) &= \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_i) \\
 1 &\Rightarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + 0) \\
 0 &\Rightarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)
 \end{aligned}
 \quad 0 \leq t < T_s$$

- Since $\cos(\pi) = -1$, this is equivalent to:

$$\begin{aligned}
 1 &\Rightarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \Big|_0^{T_s} \\
 0 &\Rightarrow -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \Big|_0^{T_s}
 \end{aligned}$$

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Remember: We can transmit information by varying either the (1) amplitude, (2) phase, or (3) frequency. Here we use the phase.

BPSK - Magnitude and Phase Representation



- $s(t) = R(t) \cos[\omega_c t + \theta(t)]$ where

- $R(t) = \sqrt{\frac{2E_b}{T_b}}$ (constant envelope)

- $1 \Rightarrow \theta(t) = 0 \Big|_0^{T_s}$

- $0 \Rightarrow \theta(t) = \pi \Big|_0^{T_s}$

Note: Constant envelope signals are beneficial in fading channels because fading corrupts the amplitude, but here the amplitude carries no information.

- All of the information is in the phase

BPSK - I & Q Representation



$$v(t) = x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t)$$

$$x(t) = R(t) \cos(\theta(t))$$

$$y(t) = R(t) \sin(\theta(t))$$

$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} & \theta = 0 \\ -\sqrt{\frac{2E_b}{T_b}} & \theta = \pi \end{cases} \quad 0 \leq t \leq T_s$$

$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} \approx 0 & \theta = 0 \\ \sqrt{\frac{2E_b}{T_b}} \approx 0 & \theta = \pi \end{cases} \quad 0 \leq t \leq T_s$$

$$v(t) = \underbrace{x(t)}_{\text{polar NRZ signal}} \cos(\omega_c t)$$

- No Q component
- I component is just a polar NRZ signal

Note: Since there is no signal in the Q component we could send an additional BPSK signal in the Q portion. This is termed Q (Quadrature) PSK signal.

BPSK - Complex Envelope Representation



- $v(t) = \text{Re}[g(t)e^{j\omega_c t}]$ where

- $g(t) = x(t) + j \cdot y(t)$

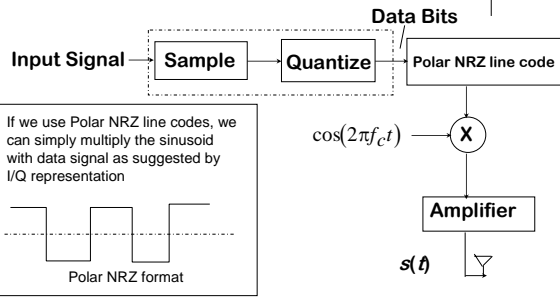
$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} & \theta = 0 \\ -\sqrt{\frac{2E_b}{T_b}} & \theta = \pi \end{cases}$$

$$g(t) = x(t) = \sqrt{\frac{2E_b}{T_b}} \sum_k (2b_k - 1) \text{rect}\left(\frac{t - kT_s}{T_s}\right)$$

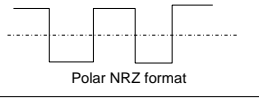
polar NRZ
linecode

- Complex envelope is entirely real
- Complex envelope is equivalent to polar NRZ signaling
- Imaginary portion of complex envelope corresponds to Q component

Transmitter for BPSK



If we use Polar NRZ line codes, we can simply multiply the sinusoid with data signal as suggested by I/Q representation



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Alternative Representation for BPSK



- We can also think of BPSK as a binary modulated pulse stream multiplied a carrier

$$v(t) = \sqrt{\frac{2E_b}{T_b}} \left(\sum_k d_k p(t - kT_s) \right) \cos(\omega_c t)$$

where

$$p(t) = \text{rect}\left(\frac{t - T_s/2}{T_s}\right) \quad \text{Rectangular pulses}$$

$$d_k = 2b_k - 1 \quad \text{Convert bits to } \{+1, -1\}$$

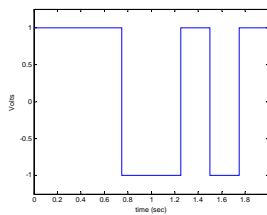
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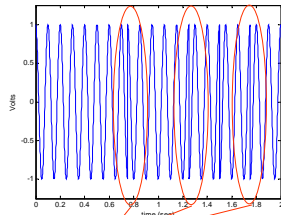
Example Signal



- Baseband signal



- Modulated Signal



Phase changes

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Power Spectral Density of BPSK



- Recall that the PSD of a bandpass signal can be related to the PSD of the complex envelope:

$$P_V(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

- Thus, since
 - we know the complex baseband signal is NRZ signal and
 - we know the PSD of a polar NRZ signal,
 we can determine the PSD of the BPSK signal. The
- The complex envelope representation is useful for determining the PSD of bandpass signals.

Power Spectral Density for BPSK



We have shown for polar NRZ signaling:

$$P_g(f) = A^2 T_b \left(\frac{\sin(\pi f T_b)}{\pi f T_b} \right)^2$$

Thus,

$$P_V(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

$$= \frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c) T_b)}{\pi(f - f_c) T_b} \right)^2 + \left(\frac{\sin(\pi(-f - f_c) T_b)}{\pi(-f - f_c) T_b} \right)^2 \right]$$

$$= \frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c) T_b)}{\pi(f - f_c) T_b} \right)^2 + \left(\frac{-\sin(\pi(f + f_c) T_b)}{\pi(-f - f_c) T_b} \right)^2 \right]$$

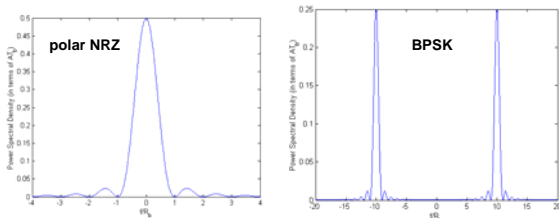
$$= \frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c) T_b)}{\pi(f - f_c) T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c) T_b)}{\pi(f + f_c) T_b} \right)^2 \right]$$

$$A = \sqrt{\frac{2E_b}{T_b}}$$

Power Spectral Density of BPSK



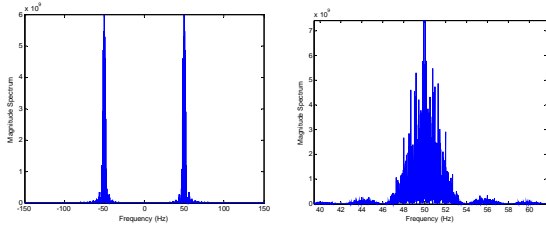
- Complex envelope of BPSK is polar NRZ



Example Signal (random data)



- $f_c = 50\text{Hz}$, $R_s = R_b = 4\text{bps}$



What is the difference between this plot and the PSD on the previous page??

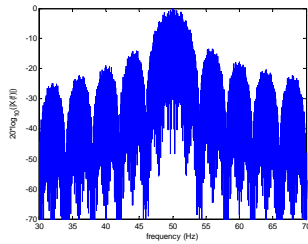
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Example – cont.



- Log plot

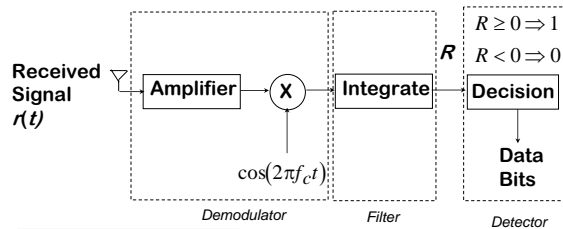


Spectrum identical to what we expect with a NRZ line code with square pulses. (Notice first sidelobes are -13dB down)

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Receiver for BPSK



Note: We will discuss optimal receiver design shortly. This implementation is optimal only for square (rectangular) pulses.

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Receiver for BPSK



- Multiply by a sinusoid
 - phase must be aligned with incoming signal (coherent)
- Integrate over duration of one bit
 - Integration is a form of low pass filtering (what is the optimal filter?)
- Compare result with a threshold

- Decision Variable R :

$$R = \frac{1}{T_s} \int_0^{T_s} y(t) \cos(2\pi f_c t) dt = \frac{1}{T_s} \int_0^{T_s} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) dt$$

$$= \frac{1}{T_s} \int_0^{T_s} x(t) \frac{1}{2} [\cos(0) + \cos(4\pi f_c t)] dt \approx \sqrt{\frac{E_b}{2T_b}} d_k$$

Decision rule: $R \geq 0 \Rightarrow 1, R < 0 \Rightarrow 0$

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Why is this rule optimal?

Quadrature Phase Shift Keying



- Note:
 - The receiver is coherent – i.e., we must track the phase of the incoming carrier
 - The in-phase and quadrature channels are orthogonal provided that we demodulate the signal coherently
 - In BPSK we haven't transmitted anything in the Q channel
- Thus, we have an opportunity to transmit more data in the Q channel – this is termed Quadrature Phase Shift Keying (QPSK)

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Analogy to AM



- BASK is simply AM transmission with a unipolar NRZ line code as the message and a sensitivity factor of $k_a = 1$
 - Like AM, non-coherent detection is possible
- BPSK is Double Sideband Suppressed Carrier AM with a polar NRZ line code as the message
 - Like DSBSC a coherent receiver is required
- Like AM vs DSBSC, BPSK is more energy efficient than BASK due to the fact that it does not transmit the unmodulated carrier
 - This will become more clear when we examine the BER performance

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Pulse Shaping



- Recall that the Power Spectral Density is

$$P_s(f) = \frac{A^2 T_b}{4} \left[\left(\frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left(\frac{\sin(\pi(f + f_c)T_b)}{\pi(f + f_c)T_b} \right)^2 \right]$$

- Which is dominated by the pulse shape

$$P(f) = \left(\frac{\sin(\pi f T_b)}{\pi f T_b} \right)$$

- Thus, if we replace the pulse shape in the line code, we can control the PSD of BPSK

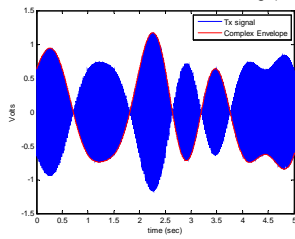
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Pulse Shaping – Example



- Consider sinc pulses with a truncation length of 20 symbols
- Note that the signal is no longer constant amplitude (i.e., the information is still in the phase, but the amplitude variation must be considered in the receiver design)



$$v(t) = \sqrt{\frac{2E_b}{T_b}} \left(\sum_k d_k p(t - kT_s) \right) \cos(\omega_c t)$$

$$p(t) = \text{sinc}\left(\frac{t - T_s/2}{T_s}\right)$$

$$d_k = 2b_k - 1$$

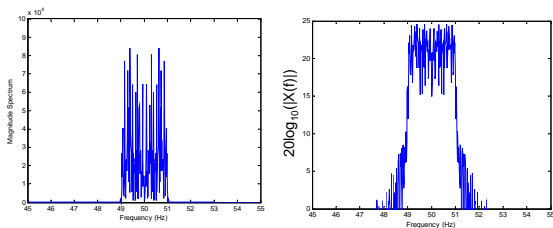
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Pulse Shaping – cont.



- Example – $f_c = 50\text{Hz}$, $R_s = R_b = 2\text{bps}$



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In class drill



- If time

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Summary



- In this lecture we have expanded our knowledge of sinusoidal modulation techniques by examining binary phase shift keying (BPSK)
- BPSK is analogous to DSBSC AM and is more energy efficient than BASK which we studied last time
- However, the receiver for BPSK must be coherent, unlike BASK
- Since the modulation is *linear* the PSD is directly controlled by the baseband line code used
- Pulse shaping can be employed to modify the spectrum

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