

# ECE4634

## Digital Communications

### Fall 2007

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Instructor: R. Michael Buehrer  
Lecture #16: Bandpass Signals  
Bandpass Modulation - BPSK



Analog and Digital Communications

# Lecture Objective



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- The objective of this lecture is to introduce the concept of Binary Phase Shift Modulation, a form of digital bandpass modulation.

# Modulation



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- Baseband signals  $w(t)$  may be transformed into bandpass signals through multiplication by a sinusoid:

$$w(t)\cos(\omega_c t + \theta) \Leftrightarrow \frac{1}{2}\left[e^{j\theta}W(f - f_c) + e^{-j\theta}W(f + f_c)\right]$$

- Most transmitted signals are modulated onto a carrier because
  - Modulated signals propagate well through the atmosphere
  - Modulation allows many signals with different carrier frequencies to share the spectrum
  - Efficient antennas can be built at a reasonable size

# Three Ways of Representing Bandpass Signals



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- Magnitude and Phase

$$v(t) = R(t) \cos[\omega_c t + \theta(t)]$$

- In Phase and Quadrature

$$v(t) = x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t)$$

- Complex Envelope

$$v(t) = \text{Re} \left[ g(t) e^{j\omega_c t} \right]$$



# Transmitting Information

- Previously we saw that information (bits) can be transmitted by changing the amplitude of pulses.
  - Baseband communications
- With bandpass communications information is transmitted by modulating (changing) the phase, frequency or amplitude of a carrier (sinusoid)
- Modulation can be envisioned in the time domain as changing or modulating the sinusoid with time
- Modulation in the frequency domain can be envisioned as shifting the frequency components of the signal from baseband to bandpass around some center frequency.

# Bandpass Modulation - BPSK



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- Binary Phase Shift Keying
- Basic Idea:
  - Data determines the **phase** of transmit sinusoid over

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_i)$$
$$1 \Rightarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + 0)$$
$$0 \Rightarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$$
$$0 \leq t < T_s$$

- Since  $\cos(\pi) = -1$ , this is equivalent to:

$$1 \Rightarrow \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \Big|_0^{T_s}$$
$$0 \Rightarrow -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \Big|_0^{T_s}$$

Remember: We can transmit information by varying either the (1) amplitude, (2) phase, or (3) frequency. Here we use the phase.

# BPSK - Magnitude and Phase Representation



- $s(t) = R(t) \cos[\omega_c t + \theta(t)]$  where

- $R(t) = \sqrt{\frac{2E_b}{T_b}}$  (constant envelope)

- $1 \Rightarrow \theta(t) = 0 \Big|_0^{T_s}$

- $0 \Rightarrow \theta(t) = \pi \Big|_0^{T_s}$

Note: Constant envelope signals are beneficial in fading channels because fading corrupts the amplitude, but here the amplitude carries no information.

- All of the information is in the phase



# BPSK - I & Q Representation

$$v(t) = x(t) \cos(\omega_c t) - y(t) \sin(\omega_c t)$$

$$x(t) = R(t) \cos(\theta(t))$$

$$y(t) = R(t) \sin(\theta(t))$$

$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} & \theta = 0 \\ -\sqrt{\frac{2E_b}{T_b}} & \theta = \pi \end{cases} \quad 0 \leq t \leq T_s$$

$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} * 0 & \theta = 0 \\ \sqrt{\frac{2E_b}{T_b}} * 0 & \theta = \pi \\ = 0 \end{cases} \quad 0 \leq t \leq T_s$$

$$v(t) = \underbrace{x(t)}_{\text{polar NRZ signal}} \cos(\omega_c t)$$

- No Q component
- I component is just a polar NRZ signal

Note: Since there is no signal in the Q component we could send an additional BPSK signal in the Q portion. This is termed Q (Quadrature) PSK signal <sup>8</sup>

# BPSK - Complex Envelope Representation



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- $v(t) = \text{Re}\left[g(t)e^{j\omega_c t}\right]$  where

- $g(t) = x(t) + j \cdot y(t)$

$$= \begin{cases} \sqrt{\frac{2E_b}{T_b}} & \theta = 0 \\ -\sqrt{\frac{2E_b}{T_b}} & \theta = \pi \end{cases}$$

$$g(t) = x(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}} \sum_k (2b_k - 1) \text{rect}\left(\frac{t - kT_s}{T_s}\right)}_{\text{polar NRZ linecode}}$$

- Complex envelope is entirely real
- Complex envelope is equivalent to polar NRZ signaling
- Imaginary portion of complex envelope corresponds to Q component

# Transmitter for BPSK



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Data Bits

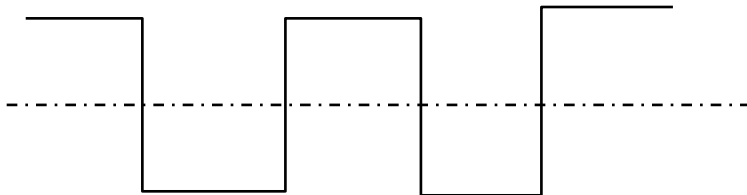
Input Signal

Sample

Quantize

Polar NRZ line code

If we use Polar NRZ line codes, we can simply multiply the sinusoid with data signal as suggested by I/Q representation



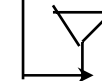
Polar NRZ format

$$\cos(2\pi f_c t)$$

X

Amplifier

$s(t)$



# Alternative Representation for BPSK



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- We can also think of BPSK as a binary modulated pulse stream multiplied a carrier

$$v(t) = \underbrace{\sqrt{\frac{2E_b}{T_b}} \left( \sum_k d_k p(t - kT_s) \right)}_{x(t)} \cos(\omega_c t)$$

where

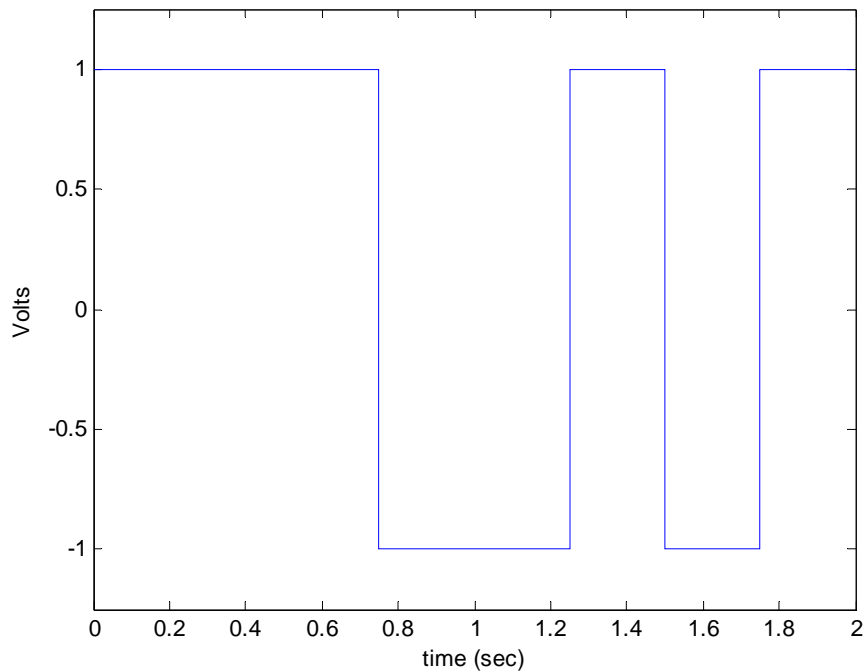
$$p(t) = \text{rect} \left( \frac{t - T_s / 2}{T_s} \right) \quad \text{Rectangular pulses}$$

$$d_k = 2b_k - 1 \quad \text{Convert bits to } \{+1, -1\}$$

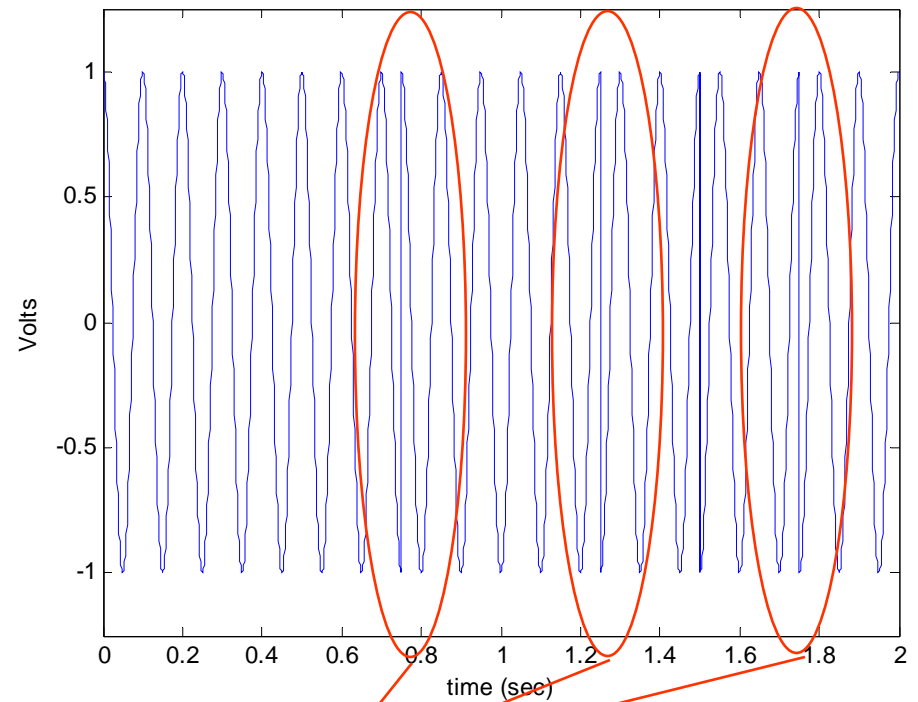


# Example Signal

- Baseband signal



## Modulated Signal



Phase changes



# Power Spectral Density of BPSK

- Recall that the PSD of a bandpass signal can be related to the PSD of the complex envelope:

$$P_v(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

- Thus, since
  - we know the complex baseband signal is NRZ signal and
  - we know the PSD of a polar NRZ signal,we can determine the PSD of the BPSK signal. The
- The complex envelope representation is useful for determining the PSD of bandpass signals.



# Power Spectral Density for BPSK

We have shown for  
polar NRZ signaling:

$$P_g(f) = A^2 T_b \left( \frac{\sin(\pi f T_b)}{\pi f T_b} \right)^2$$

Thus, 
$$P_v(f) = \frac{1}{4} [P_g(f - f_c) + P_g(-f - f_c)]$$

$$= \frac{A^2 T_b}{4} \left[ \left( \frac{\sin(\pi (f - f_c) T_b)}{\pi (f - f_c) T_b} \right)^2 + \left( \frac{\sin(\pi (-f - f_c) T_b)}{\pi (-f - f_c) T_b} \right)^2 \right]$$

$$= \frac{A^2 T_b}{4} \left[ \left( \frac{\sin(\pi (f - f_c) T_b)}{\pi (f - f_c) T_b} \right)^2 + \left( \frac{-\sin(\pi (f + f_c) T_b)}{\pi (-f - f_c) T_b} \right)^2 \right]$$

$$\frac{A^2 T_b}{4} \left[ \left( \frac{\sin(\pi (f - f_c) T_b)}{\pi (f - f_c) T_b} \right)^2 + \left( \frac{\sin(\pi (f + f_c) T_b)}{\pi (f + f_c) T_b} \right)^2 \right]$$

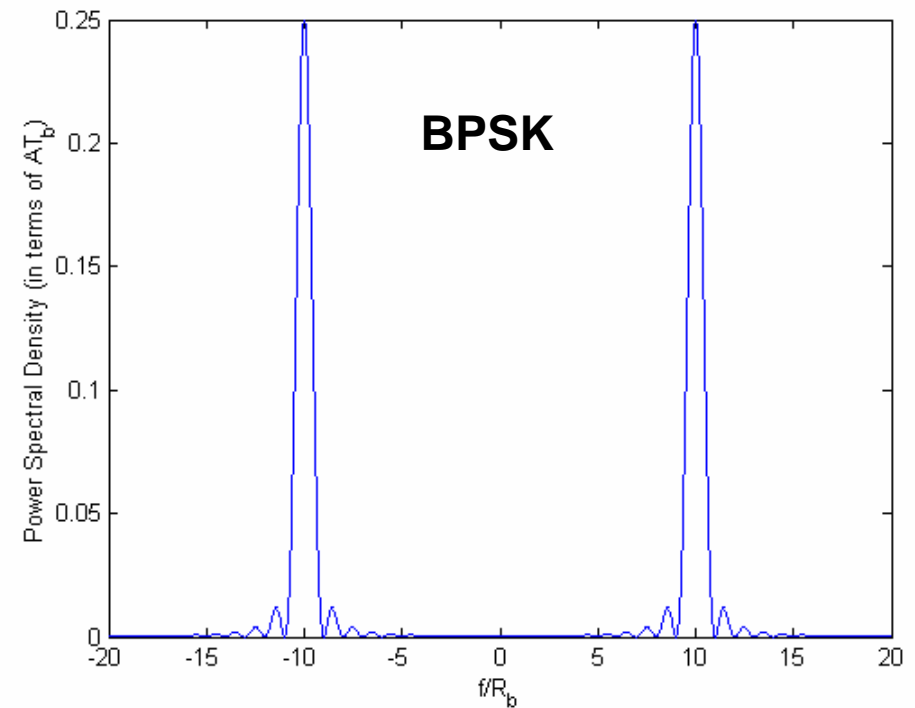
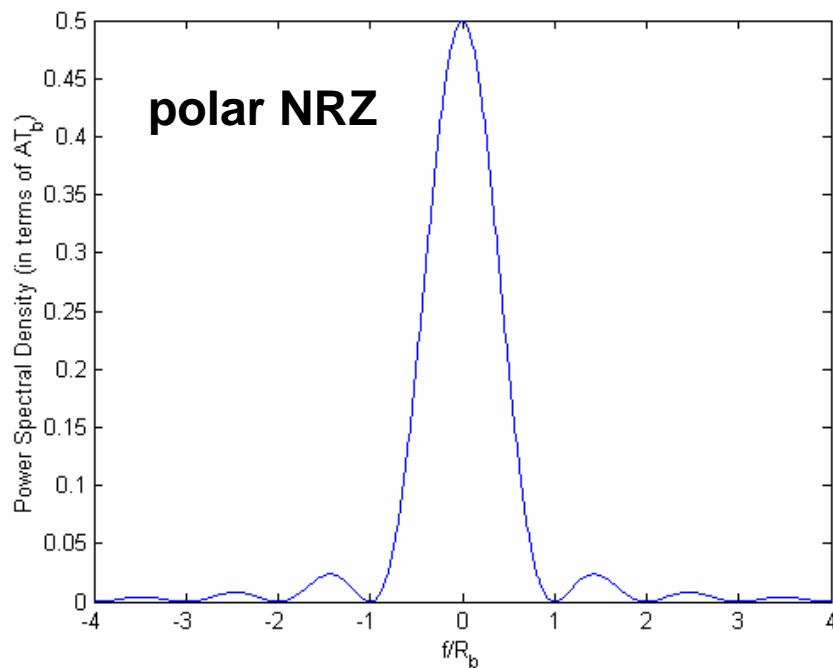
$$A = \sqrt{\frac{2E_b}{T_b}}$$

# Power Spectral Density of BPSK



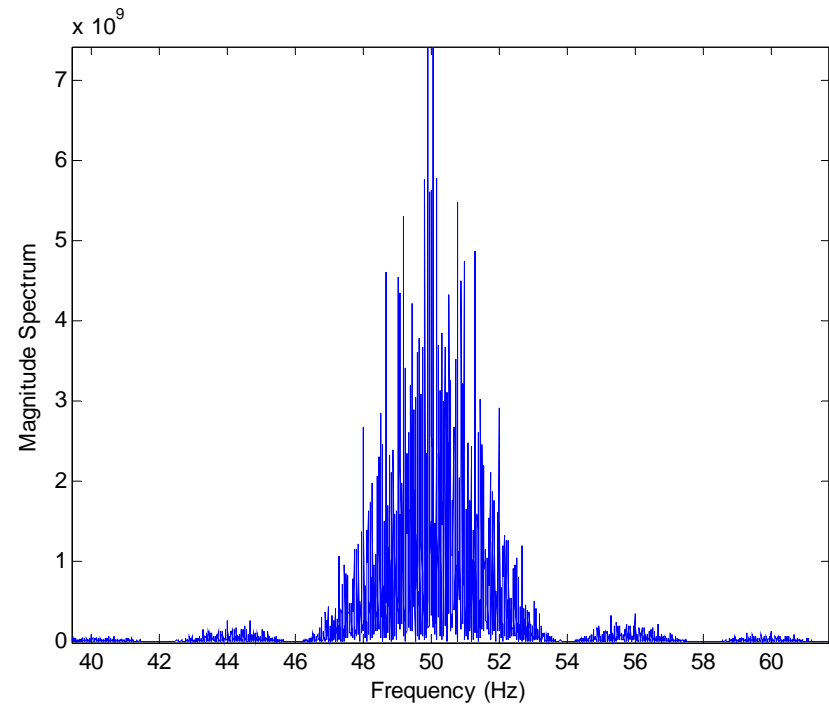
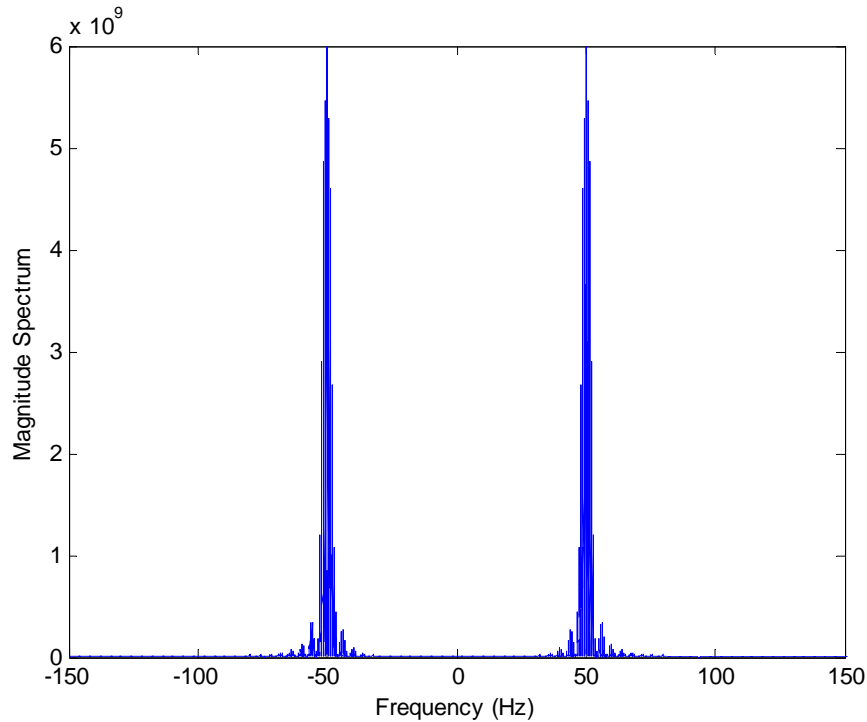
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- Complex envelope of BPSK is polar NRZ



# Example Signal (random data)

- $f_c = 50\text{Hz}$ ,  $R_s = R_b = 4\text{bps}$

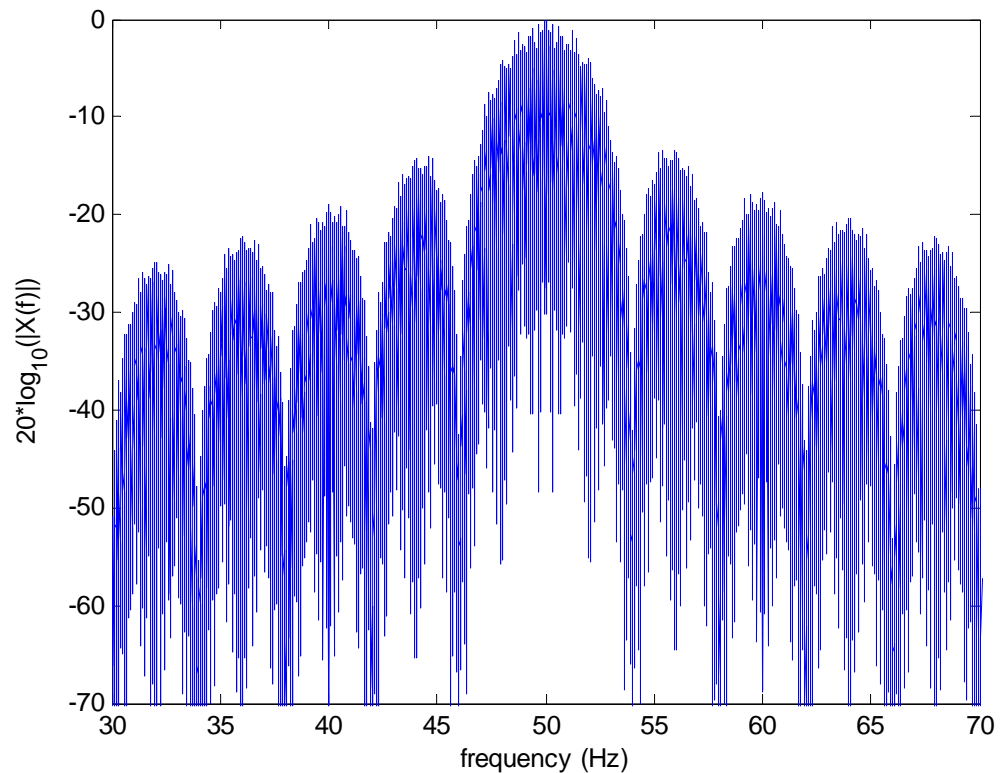


What is the difference between this plot and the PSD on the previous page??



# Example – cont.

- Log plot

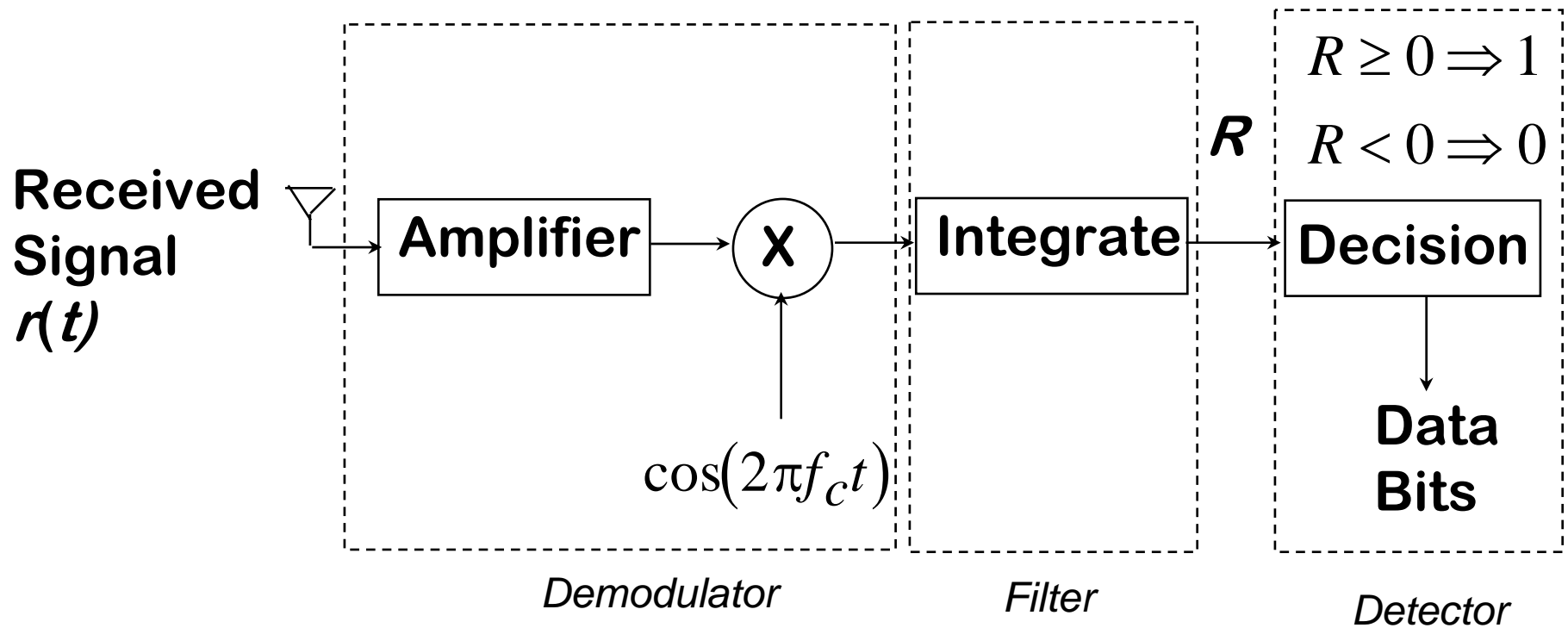


Spectrum identical to what we expect with a NRZ line code with square pulses. (Notice first sidelobes are ~13dB down)

# Receiver for BPSK



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Note: We will discuss optimal receiver design shortly. This implementation is optimal only for *square (rectangular) pulses*.

# Receiver for BPSK



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- Multiply by a sinusoid
  - phase must be aligned with incoming signal (coherent)
- Integrate over duration of one bit
  - Integration is a form of low pass filtering (what is the optimal filter?)
- Compare result with a threshold
- Decision Variable  $R$  :

$$\begin{aligned} R &= \frac{1}{T_s} \int_0^{T_s} v(t) \cos(2\pi f_c t) dt = \frac{1}{T_s} \int_0^{T_s} x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) dt \\ &= \frac{1}{T_s} \int_0^{T_s} x(t) \frac{1}{2} [\cos(0) + \cos(4\pi f_c t)] dt \approx \sqrt{\frac{E_b}{2T_b}} d_k \end{aligned}$$

Decision rule:  $R \geq 0 \Rightarrow 1, R < 0 \Rightarrow 0$

Why is this rule optimal?



# Quadrature Phase Shift Keying

- Note:
  - The receiver is coherent – i.e., we must track the phase of the incoming carrier
  - The in-phase and quadrature channels are orthogonal provided that we demodulate the signal coherently
  - In BPSK we haven't transmitted anything in the Q channel
- Thus, we have an opportunity to transmit more data in the Q channel – this is termed Quadrature Phase Shift Keying (QPSK)



# Analogy to AM

- BASK is simply AM transmission with a unipolar NRZ line code as the message and a sensitivity factor of  $k_a = 1$ 
  - Like AM, non-coherent detection is possible
- BPSK is Double Sideband Suppressed Carrier AM with a polar NRZ line code as the message
  - Like DSBSC a coherent receiver is required
- Like AM vs DSBSC, BPSK is more energy efficient than BASK due to the fact that it does not transmit the unmodulated carrier
  - This will become more clear when we examine the BER performance



# Pulse Shaping

- Recall that the Power Spectral Density is

$$P_v(f) = \frac{A^2 T_b}{4} \left[ \left( \frac{\sin(\pi(f - f_c)T_b)}{\pi(f - f_c)T_b} \right)^2 + \left( \frac{\sin(\pi(f + f_c)T_b)}{\pi(f + f_c)T_b} \right)^2 \right]$$

- Which is dominated by the pulse shape

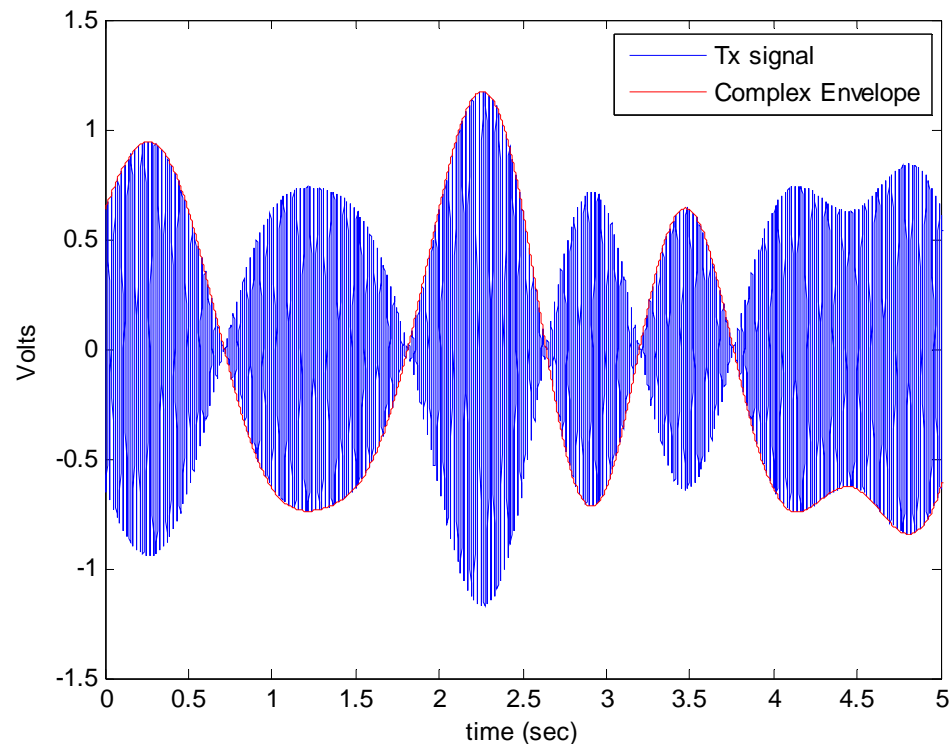
$$P(f) = \left( \frac{\sin(\pi f T_b)}{\pi f T_b} \right)$$

- Thus, if we replace the pulse shape in the line code, we can control the PSD of BPSK



# Pulse Shaping – Example

- Consider sinc pulses with a truncation length of 20 symbols
- Note that the signal is no longer constant amplitude (i.e., the information is still in the phase, but the amplitude variation must be considered in the receiver design)



$$v(t) = \sqrt{\frac{2E_b}{T_b}} \underbrace{\left( \sum_k d_k p(t - kT_s) \right)}_{x(t)} \cos(\omega_c t)$$

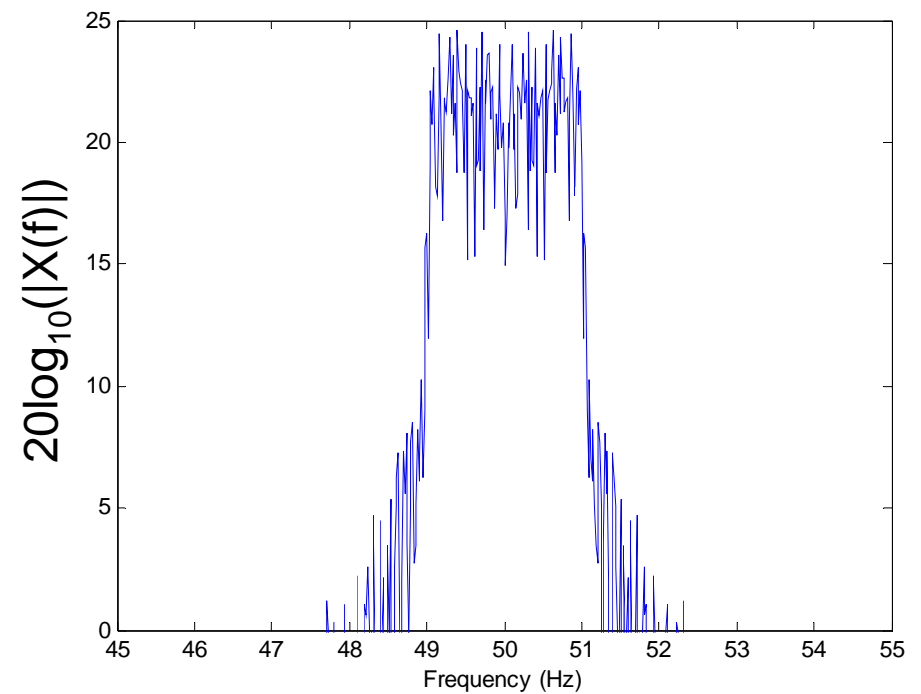
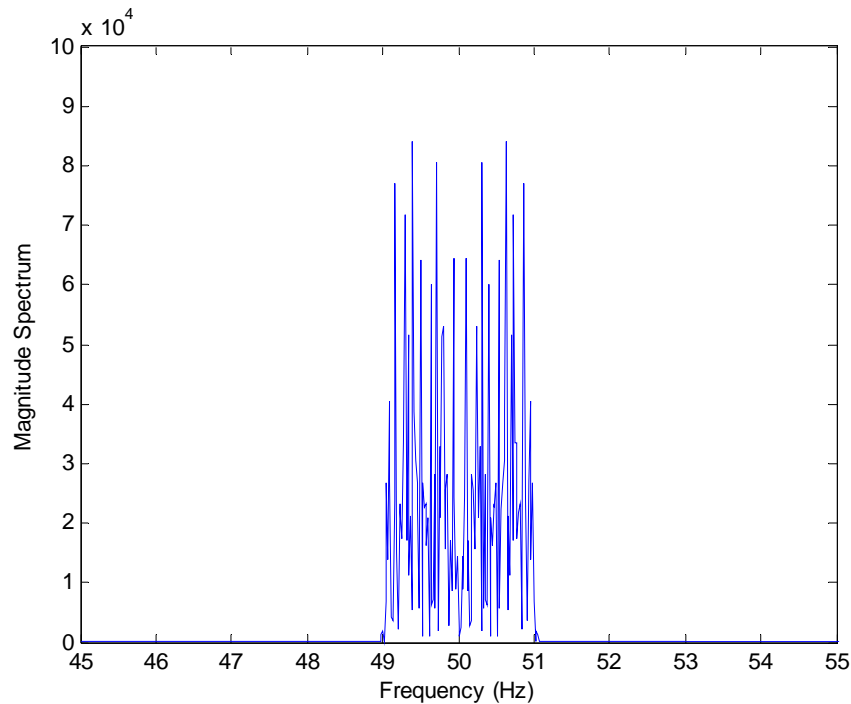
$$p(t) = \text{sinc} \left( \frac{t - T_s/2}{T_s} \right)$$

$$d_k = 2b_k - 1$$



# Pulse Shaping – cont.

- Example –  $f_c = 50\text{Hz}$ ,  $R_s = R_b = 2\text{bps}$



# In class drill

- If time



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# Summary

- In this lecture we have expanded our knowledge of sinusoidal modulation techniques by examining binary phase shift keying (BPSK)
- BPSK is analogous to DSBSC AM and is more energy efficient than BASK which we studied last time
- However, the receiver for BPSK must be coherent, unlike BASK
- Since the modulation is *linear* the PSD is directly controlled by the baseband line code used
- Pulse shaping can be employed to modify the spectrum