ECE4634
Digital Communications
Fall 2007

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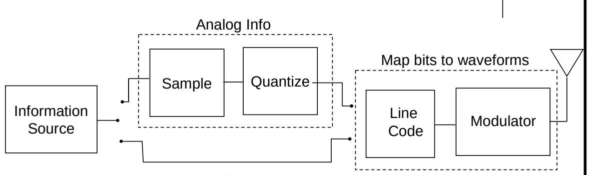
Lecture #18: Multilevel or *M*-ary Modulation

PSK and ASK

Overview

- Today we expand our discussion of digital modulation from binary to *M*-ary modulation
- *M*-ary modulation *can (but not necessarily)* reduce the bandwidth requirements of a modulation scheme by reducing the symbol rate
- We will specifically examine *M*-ary PSK and ASK modulation and will examine M-FSK next class
- What to read – Section 7.5, 7.7

Communication System (Transmitter)



- We've talked extensively about converting analog information into digital waveforms (PCM)
- Modulation is using the digital information to *modulate* a sinusoidal carrier.
- Modulation can either be *binary* or *M-ary*.

Multilevel Modulation

In *binary* modulation each bit corresponds to one symbol.

$$b = \{0,1,1,1,0,0,1,0,1,0,0,1,0\ldots,\}$$

$$s = \{s_0, s_1, s_1, s_1, s_0, s_0, s_1, s_0, s_1, \ldots,\}$$

2 Possible symbols $\{s_0, s_1\}$ e.g. $\{\cos(\omega_c t), -\cos(\omega_c t)\}$

In *multi-level* modulation each $log_2(M)$ bits corresponds to one symbol. Example: $M = 8$, $log_2(M) = 3$

$$b = \{0,1,1,1,0,0,1,0,1,0,0,1,0,0,0,\ldots,\}$$

$$s = \{s_3, s_4, s_5, s_1, s_0, \ldots,\}$$

8 Possible symbols $\{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$

Multilevel Modulation

- Let $m(t)$ be the information message
- Binary Signaling: $m(t) \in \{0,1\}$
- M-ary Signaling: $m(t) \in \{0,1,\ldots, M-1\}$
  - message signal takes one of M values
    $$M = 2^l$$
  - l = # of bits/symbol
- Examples:
  - M different phases (M-ary PSK)
  - M different amplitudes (M-ary ASK)
  - Combinations: Quadrature Amplitude Modulation (QAM)

Advantage of Multilevel Signaling: Saves on Bandwidth *(usually)*

- Let
  - T_b be the duration of one bit
  - T_s be the duration of one symbol
- Then
  - $R_b = 1/T_b$ is the bit rate
  - $R_s = 1/T_s = R_b/l = 1/lT_b$ is the symbol rate
- Information is transmitted at the bit rate
- Bandwidth is proportional to the symbol rate
  - Only one pulse is sent for each symbol
- **Exception**: M-FSK – This scheme requires more bandwidth than BFSK. However, M-FSK doesn't suffer in energy efficiency as M-PSK and M-QAM do.

Quadrature Phase Shift Keying

- Recall that with BPSK, the I/Q version of the signal can be written as

$$v(t) = x(t)\cos(\omega_c t)$$

where *x(t)* is a polar NRZ line code.

- Now, since the I and Q channels are orthogonal, in the same bandwidth we could send the signal

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

where y*(t)* is also a polar NRZ line code carrying independent information.

QPSK – cont.

$$v(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

- Converting this signal to Magnitude/Phase form:

$$v(t) = \sqrt{x^2(t) + y^2(t)}\cos\left(\omega_c t + \tan^{-1}\left\{\frac{y(t)}{x(t)}\right\}\right)$$

- Now, since x(t) and y(t) are both polar NRZ line codes (assumed to use square pulses), there are four separate situations that can arise:

$$x(t) = rect\left(\frac{t}{T}\right), y(t) = rect\left(\frac{t}{T}\right) \quad R(t) = \sqrt{2}rect\left(\frac{t}{T}\right) \quad \theta(t) = \frac{\pi}{4}rect\left(\frac{t}{T}\right)$$

$$x(t) = rect\left(\frac{t}{T}\right), y(t) = -rect\left(\frac{t}{T}\right) \quad R(t) = \sqrt{2}rect\left(\frac{t}{T}\right) \quad \theta(t) = -\frac{\pi}{4}rect\left(\frac{t}{T}\right)$$

$$x(t) = -rect\left(\frac{t}{T}\right), y(t) = rect\left(\frac{t}{T}\right) \quad R(t) = \sqrt{2}rect\left(\frac{t}{T}\right) \quad \theta(t) = \frac{3\pi}{4}rect\left(\frac{t}{T}\right)$$

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QPSK – cont.

- Thus, the resulting signal is constant amplitude with one of four different phases
- This can be thought of as 4-ary PSK, but is usually termed Quadrature Phase Shift Keying or QPSK due to the use of the quadrature channel
- The signal can then be written as

$$s(t) = A_c\cos\left[\omega_c t + \frac{\pi}{4} + \frac{\pi}{2}m(t)\right]$$

$$m(t) = \sum_{i=-\infty}^{\infty}(m_i)rect\left(\frac{t - iT_s}{T_s}\right)$$

$$m_i \in \{0,1,2,3\}$$

M-ary Phase Shift Keying (*M*-PSK)

- Magnitude and Phase Representation:

$$s(t) = A_c \cos\left[\omega_c t + \frac{2\pi}{M} m(t)\right]$$

  - $m(t) = \sum_{i=-\infty}^{\infty} (m_i)\, rect\left(\frac{t - iT_s}{T_s}\right)$

    $m_i \in \{0, 1, \ldots, M-1\}$
  - A_c is a constant representing amplitude
- **Special Case:** M=2 corresponds to BPSK

$$s(t) = \begin{cases} A_c \cos(\omega_c t), & m_i = 0 \\ A_c \cos(\omega_c t + \pi) = -A_c \cos(\omega_c t), & m_i = 1 \end{cases}$$

Other Representations of MPSK

- Complex Envelope:

$$s(t) = \mathrm{Re}\left\{g(t) e^{j\omega_c t}\right\}$$

$$g(t) = A_c e^{j\theta(t)} = A_c \cos[\theta(t)] + jA_c \sin[\theta(t)]$$

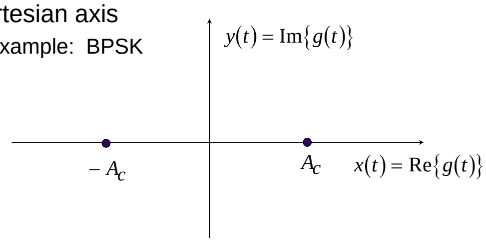
$$\theta(t) = \frac{2\pi}{M} m(t)$$

- In-Phase/Quadrature (I/Q) Representation:

$$s(t) = \underbrace{A_c \cos\left(\frac{2\pi}{M} m(t)\right)}_{x(t)} \cos(\omega_c t) - \underbrace{A_c \sin\left(\frac{2\pi}{M} m(t)\right)}_{y(t)} \sin(\omega_c t)$$

Signal Constellation Representation of Signals

- Plot the real and imaginary parts of $g(t)$ on Cartesian axis
  - Example: BPSK



Signal Constellation Representation of Signals

- Plot the real and imaginary parts of $g(t)$ on Cartesian axis
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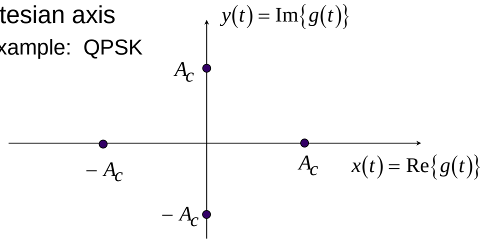

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Physical Interpretation of Signal Constellation Diagram

- Real axis shows what modulates cosine wave
- Imaginary axis shows what modulates sine wave
- Distance from origin corresponds to amplitude
  - Distance squared is energy
- Separation of points shows how “far apart” signals are
  - This tells us how likely it is to mistake one signal for another
- Note that different phase offsets can produce the same constellation rotated about the axis
  - example: $s(t) = A_c \cos\left[\omega_c t + \frac{2\pi}{M} m(t) + \frac{\pi}{4}\right]$

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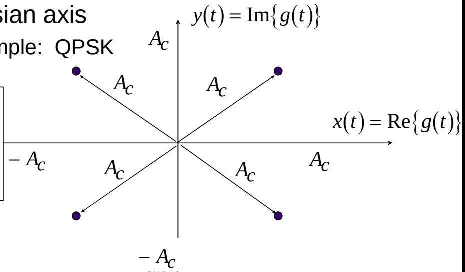
Signal Constellation Representation of Signals

- Plot the real and imaginary parts of $g(t)$ on Cartesian axis
  - Example: QPSK

Note: To keep the energy constant, the distance from the origin must be the same



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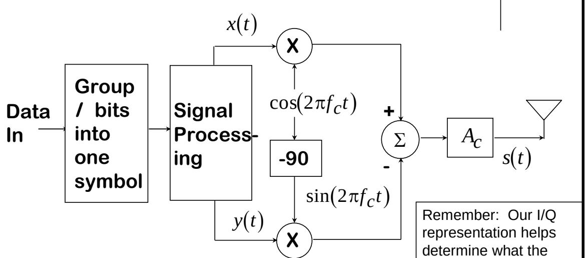
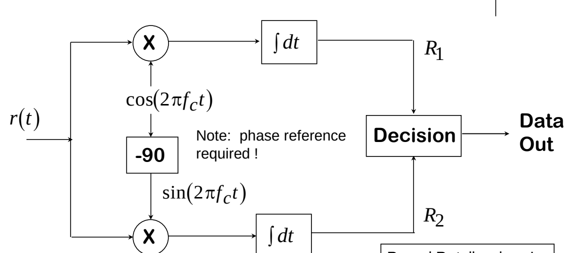
Transmitter for M-ary PSK



Remember: Our I/Q representation helps determine what the transmitter might look like

Receiver for M-ary PSK



R_1 and R_2 tell us how In-Phase and Quadrature were modulated respectively

Note: Integrators are low pass filters. Are they optimal?

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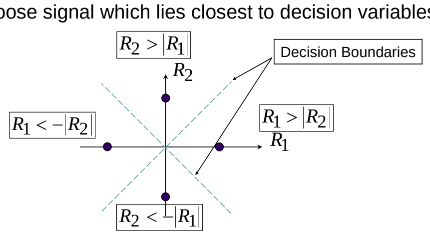
Decision Criteria

- Generate two variables R_1 and R_2 from correlation
- Decision can be made from constellation diagram
  - Plot the decision variables R_1 and R_2
  - Choose signal which lies closest to decision variables

Four decision regions Corresponding to four possible symbols



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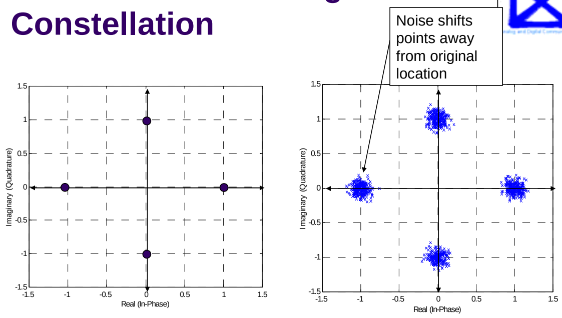
Effect of Noise on Signal Constellation



Original Signal Constellation

Signal Constellation for 1000 received symbols with noise

Quadrature Amplitude Modulation

- Representation
  - Quadrature: $s(t) = A_c x(t)\cos(\omega_c t) - A_c y(t)\sin(\omega_c t)$
  - Complex Envelope: $s(t) = \mathrm{Re}\left\{ g(t) e^{j\omega_c t} \right\},\ g(t) = x(t) + jy(t)$
- *x*(*t*) and *y*(*t*) are both multilevel signals
- Once again:

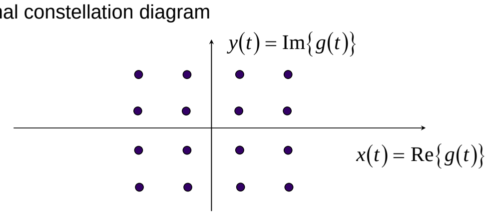
  $M =$ number of possible signals

  $l = \log_2(M) =$ number of bits / symbol
- Bandwidth of QAM:
  - Rectangular Pulses (first null-to-null): $BW_{null} = 2R_s = 2R_b/l = 2n \cdot f_s/l$
  - Raised Cosine (absolute): $BW = (1+r)R_s = (1+r)R_b/l$

    $= (1+r)n \cdot f_s/l$

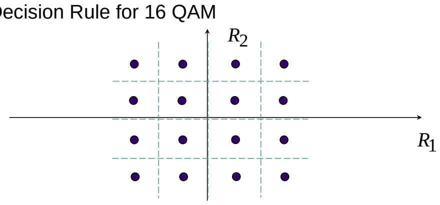
Example: 16 QAM

- *M*=16 different signal combinations, *l* = 4 bits/symbol
  - *x*(*t*) can take on four different values
  - *y*(*t*) can take on four different values
- Signal constellation diagram



Implementation of QAM

- Transmitter and Receiver Representations are identical to MPSK
  - Different signal processing and decision rules are used
- Decision Rule for 16 QAM



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Comparison of QAM and MPSK

- MPSK has constant amplitude. This has two advantages:
  - More efficient “Class C” power amplifiers may be used
  - MPSK is much less vulnerable to amplitude fading
- QAM signals need not be confined to a circle in the signal constellation diagram
  - More energy efficient since signal constellation more spread out
    - As a result QAM may contain many more levels (as many as 256)
    - For the same transmit power - Potential for higher data rates

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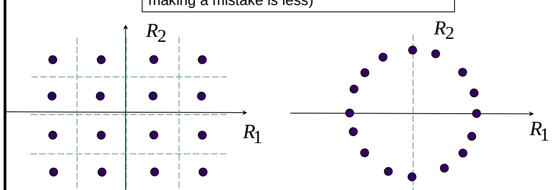
Comparison of QAM and MPSK

Note that QAM has more distance between points, thus more energy efficient. (i.e., for the same average energy the probability of making a mistake is less)



16-QAM

16-PSK

Both transmit 4 bits per symbol, thus BW efficiency equal.

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Spectral Characteristics of MPSK and QAM

- Power Spectral Densities for MPSK and QAM are the same. For square pulses:

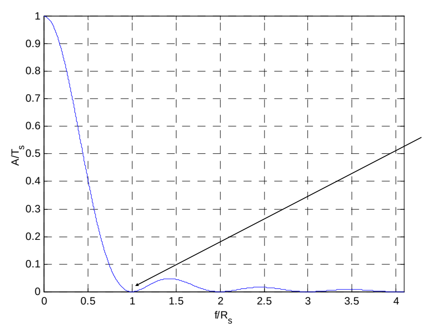
$$P_g(f) = A_c^2 T_s \left(\frac{\sin(\pi f T_s)}{\pi f T_s} \right)^2$$

$$= A_c^2 T_s \operatorname{sinc}^2(f T_s) \qquad \text{PSD for complex baseband}$$

$$T_s = \frac{1}{R_s} = \frac{l}{R_b} \quad \leftarrow \text{Bits per symbol}$$

First null bandwidth: first null occurs when $P(f) = 0$, i.e, when $f = 1/T_s = R_s$

PSD of MPSK and QAM



Complex Baseband $P_g(f)$

First Null = R_s

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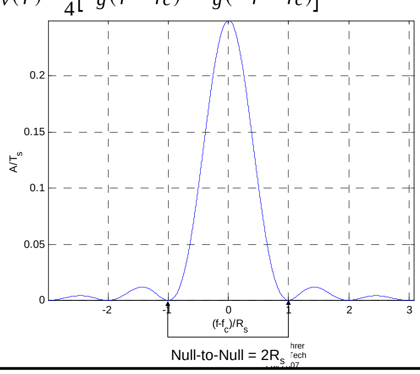
PSD of MPSK and QAM

$$P_V(f) = \frac{1}{4}\left[P_g(f - f_c) + P_g(-f - f_c) \right]$$



Bandpass Spectrum

Bandwidth of MPSK/QAM

- Rectangular Pulses (Null-to-Null)

$$BW = 2R_s$$
$$= 2R_b/l$$
$$= 2n \cdot f_s/l$$

For analog information signals
n = bits from quantizer
f_s = sampling rate

- Raised Cosine Pulses (Absolute)

$$BW = (1+r)R_s$$
$$= (1+r)R_b/l$$
$$= (1+r)n \cdot f_s/l$$

Roll-off factor
$0 \le r \le 1$

Summary

- Today we have investigated two methods of digital bandpass modulation that improve the overall bandwidth efficiency over the previous techniques
- This improvement comes from mapping multiple bits to a single symbol thus reducing the symbol rate (and thus the bandwidth)
- This comes at the price of degraded energy efficiency as we will see
  - This can be guessed from the fact that signal constellation points are closer together
- Note that not all *M*-ary schemes improve bandwidth efficiency (we will see this next class)