

ECE4634

Digital Communications

Fall 2007

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Lecture #19: *M*-ary FSK
Pulse shaping



Analog and Digital Communications



Overview

- Previously we examined two methods of multilevel (M -ary) modulation M -PSK and QAM
 - Allows for improved bandwidth efficiency but degraded energy efficiency
- Today we extend our discussion to M -FSK modulation
 - Does not improve bandwidth efficiency (in fact degrades bandwidth efficiency) but improves energy efficiency
- What to read – Sections 7.5, 7.7

Bandpass Modulation - FSK



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- Binary Frequency Shift Keying (BFSK)
- We modulate or change the frequency depending on the data bit to be sent
- Basic Idea:
 - Send one tone f_1 for a 1
 - Send another tone f_2 for a 0
 - Then we transmit the signal $s(t)$:

$$1 \Rightarrow s(t) = \cos(2\pi f_1 t + \theta_1) \Big|_0^{T_s}$$

$$0 \Rightarrow s(t) = \cos(2\pi f_2 t + \theta_2) \Big|_0^{T_s}$$

θ_1 and θ_2 are arbitrary constants that simply reflect the fact that the two oscillators are not phase locked. Better spectral properties are achieved if $\theta_1 = \theta_2$ and if the phase is continuous.

3



Continuous Phase BFSK

- We can also create a BFSK signal using a frequency modulator. Such a scheme keeps the phase a continuous function (i.e., there are no phase jumps).
- This type of FSK has better spectral properties and is called *continuous phase FSK*

$$s(t) = A_c \cos \left[\omega_c t + D_f \int_{-\infty}^t m(\lambda) d\lambda \right]$$
$$= \text{Re} \left\{ g(t) e^{j\omega_c t} \right\}$$

$$g(t) = A_c e^{j\theta(t)}$$

$$\theta(t) = D_f \int_{-\infty}^t m(\lambda) d\lambda$$

$$m(t) = \text{polar NRZ line code}$$
$$D_f = 2\pi\Delta f$$



Minimum Shift Keying

- Can be viewed as a pulse shaped version of Offset QPSK with Sinusoidal pulse shaping

$$s(t) = A \left\{ \left[\sum_{n=-\infty}^{\infty} I_{2n} g(t - nT_s) \right] \cos(2\pi f_c t) + \left[\sum_{n=-\infty}^{\infty} I_{2n+1} g\left(t - nT_s - \frac{T_s}{2}\right) \right] \sin(2\pi f_c t) \right\}$$

$$I_n \in \{+1, -1\}$$

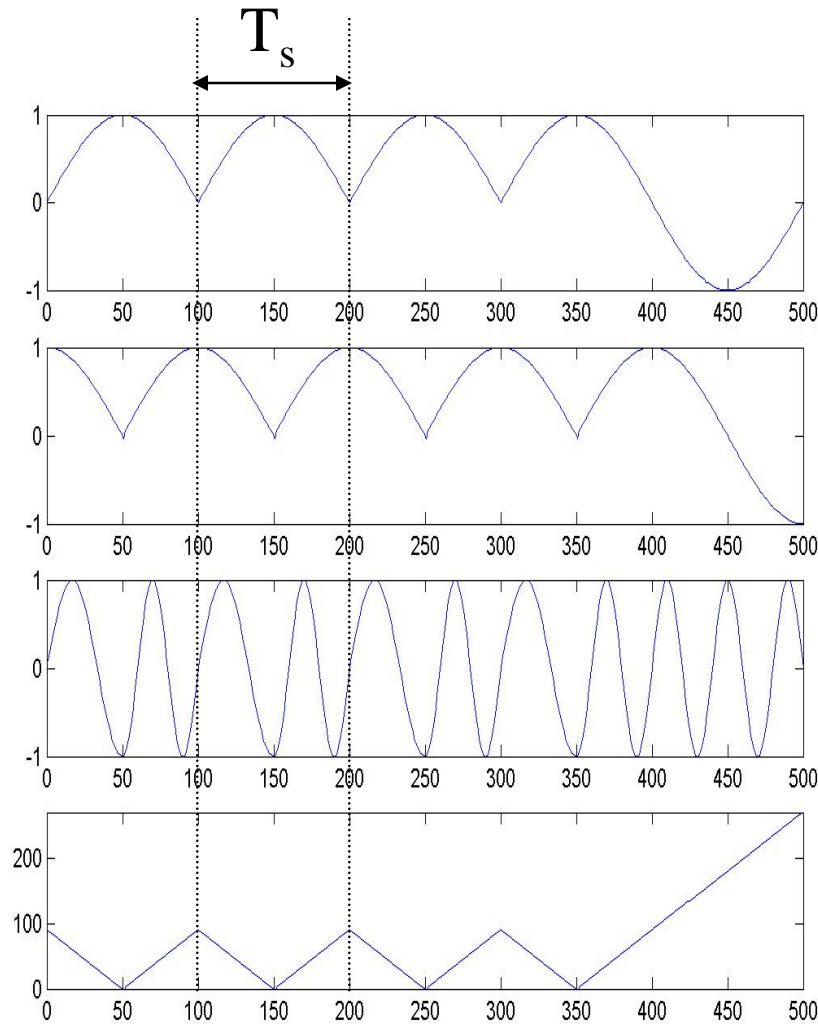
$$I_n = (-1)^{b_n}$$

$$g(t) = \begin{cases} \sin\left(\frac{\pi t}{T_s}\right) & 0 \leq t \leq T_s \\ 0 & \text{else} \end{cases}$$

- Can also be viewed as a special case of FSK
 - With minimum coherent frequency separation (thus the name *Minimum* Shift Keying)



Minimum Shift Keying (MSK)



I channel bits - pulse shaped with $\sin(\pi t/T_s)$

Q channel bits - pulse shaped with $\sin(\pi t/T_s)$ and staggered by $T_s/2$

Tx Signal

Phase

Note: Phase is continuous



MSK vs BFSK

- When viewed/demodulated bit by bit, we have an BFSK signal. However, because of the phase constraints imposed, we can view the signal over a two bit interval as offset QPSK with sinusoidal pulse shaping.

Each bit changes the direction of the phase rotation

Binary Data	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}
I-channel	b_1	b_3	b_5	b_7	b_9	b_{11}	b_{13}							
Q-channel		b_2	b_4	b_6	b_8	b_{10}	b_{12}	b_{14}						

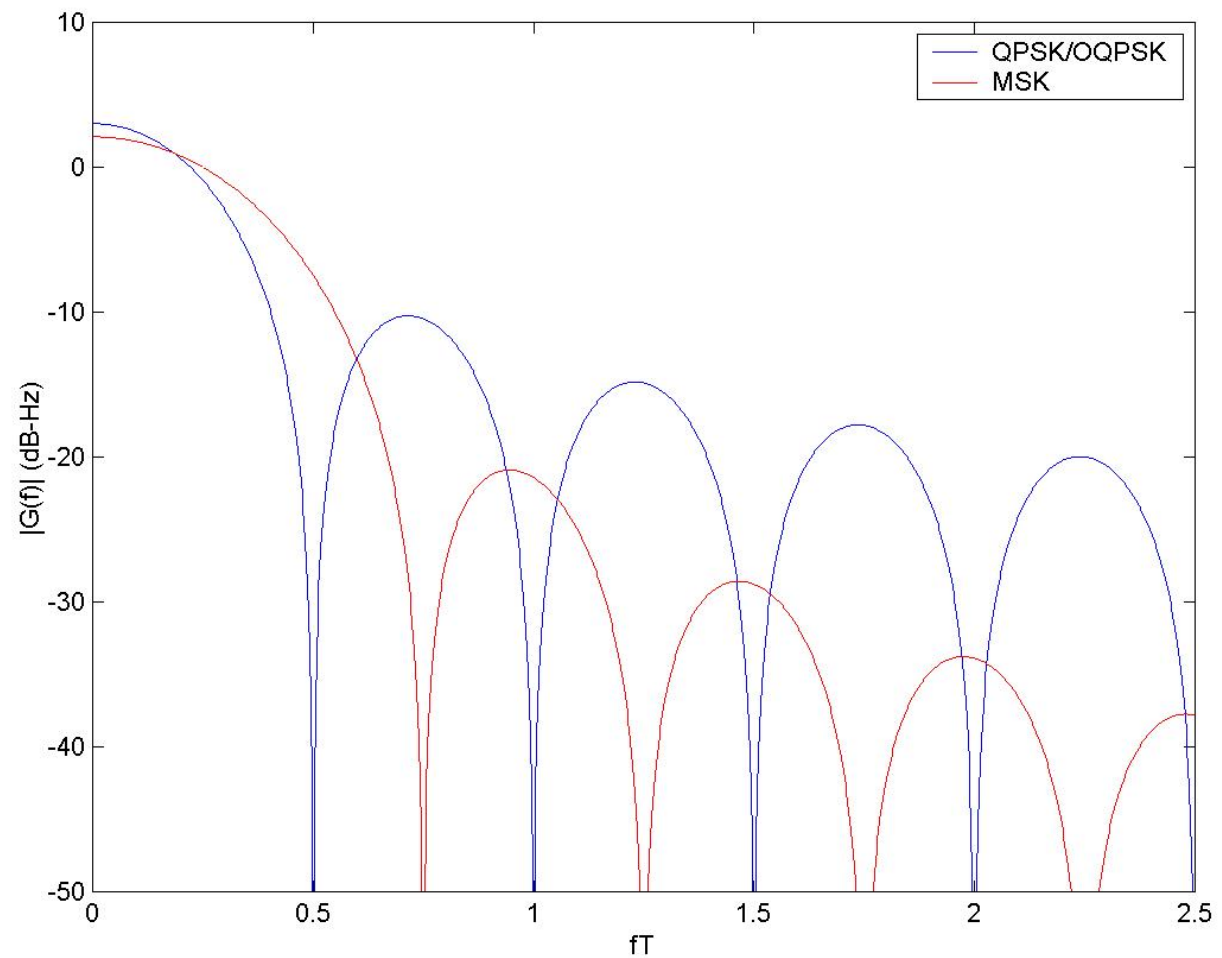
I/Q functions stay constant over a 2 bit interval

Spectrum of MSK and O/QPSK



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Main lobe is wider
Side-lobes fall off faster





M-ary FSK

- To extend FSK to $M > 2$ we simply add frequencies
- Example 4-FSK

$$s_1(t) = A \cos(\omega_1 t) \Big|_0^T$$

$$s_2(t) = A \cos(\omega_2 t) \Big|_0^T$$

$$s_3(t) = A \cos(\omega_3 t) \Big|_0^T$$

$$s_4(t) = A \cos(\omega_4 t) \Big|_0^T$$

Four orthogonal carriers
Four symbols
Two bits per symbol



Bandwidth Efficiency

- Non-coherent carriers
 - Recall that the minimum frequency spacing to maintain orthogonality between carriers is $2\Delta f_{min} = R_s$
 - Further, if we assume square pulses, the first null bandwidth on either side of the first and last carrier is R_s .
 - Thus, the total bandwidth (null-to-null) is thus

$$\begin{aligned} B &= R_s + (M - 1)(f_1 - f_2) + R_s \\ &= 2R_s + (M - 1)2\Delta f \\ &= 2R_s + (M - 1)R_s \\ &= (M + 1)R_s \end{aligned}$$



Bandwidth Efficiency

- Coherent Carriers

- Recall that the minimum frequency spacing to maintain orthogonality between carriers is $2\Delta f_{min} = R_s/2$
- Further, if we assume square pulses, the first null bandwidth on either side of the first and last carrier is R_s .
- Thus, the total bandwidth (null-to-null) is

$$\begin{aligned} B &= R_s + (M - 1)(f_1 - f_2) + R_s \\ &= 2R_s + (M - 1)2\Delta f \\ &= 2R_s + (M - 1)R_s / 2 \\ &= \left(\frac{M + 3}{2} \right) R_s \end{aligned}$$



Bandwidth Efficiency

- We typically refer to bandwidth efficiency η_{BW} as the bit rate over the bandwidth
- Thus for non-coherent carriers:

$$\begin{aligned} B &= (M + 1) R_s \\ R_b &= (\log_2 M) R_s \\ \eta_{BW} &= \frac{R_b}{B} = \frac{(\log_2 M) R_s}{(M + 1) R_s} \\ &= \frac{\log_2 M}{M + 1} \end{aligned}$$

Thus, the bandwidth efficiency of FSK decreases with M . This is in contrast to PSK, QAM, ASK.



Bandwidth Efficiency

Modulation Scheme	First null-to-null bandwidth efficiency
M -PSK	$\eta_{BW} = \frac{\log_2 M}{2}$
M -ASK	$\eta_{BW} = \frac{\log_2 M}{2}$
QAM	$\eta_{BW} = \frac{\log_2 M}{2}$
M -FSK (non-coherent carriers)	$\eta_{BW} = \frac{\log_2 M}{M + 1}$



Orthogonal Signaling

- FSK is one form of a family of modulation schemes known as orthogonal modulation schemes
- With orthogonal modulation schemes all M symbols are orthogonal
 - Requires M “dimensions” – we will discuss this more in a week or so
 - Increasing the number of dimensions improves performance but drastically reduces bandwidth efficiency



Conclusions - MFSK

- Frequency Shift Keying can be analyzed just as any other modulation scheme
- It is slightly different due to the fact that as M increases, the number of dimensions (i.e., the number of orthogonal components) increases.
 - This is in contrast to PSK, ASK and QAM which have a constant number of dimensions.
- The result is that bandwidth efficiency goes down with M and energy efficiency goes up with M



Pulse shaping

- So far we have discussed controlling bandwidth by using multi-level modulation
 - Reducing the symbol rate without reducing the bit rate reduces the bandwidth
 - Has implications on performance
- We can also reduce the bandwidth of bandpass modulation schemes through pulse shaping
 - Analogous to baseband case



Pulse shaping

- Recall that both M -PSK, M -ASK and QAM can be represented as

$$s(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t)$$

Square pulse

- where

$$x(t) = \sum_{i=-\infty}^{\infty} a_i p(t - iT_s)$$

$$y(t) = \sum_{i=-\infty}^{\infty} b_i p(t - iT_s)$$

$$\text{PSK} \rightarrow a_i = \cos\left(\frac{2\pi}{M}i\right) \quad b_i = \sin\left(\frac{2\pi}{M}i\right)$$

$$\text{QAM} \rightarrow a_i, b_i \in \{\pm 1, \pm 2, \dots, \pm \sqrt{M}\}$$

$$\text{ASK} \rightarrow a_i \in \{0, 1, 2, \dots, M-1\}$$

$$b_i = 0$$



Power Spectral Density

- Thus, the complex baseband version is

$$g(t) = x(t) + jy(t)$$
$$= \sum_{i=-\infty}^{\infty} (a_i + jb_i) p(t - iT)$$

- We know from the modulation principle that

$$P_v(f) = \frac{1}{4} \left[P_g(f - f_c) + P_g(-f - f_c) \right]$$

- Further, for a digitally modulated pulse stream

$$P_x(f) = \underbrace{\frac{\sigma_a^2}{T_s} |F(f)|^2}_{\text{continuous}} + \underbrace{\frac{m_a^2}{T_s} \sum_{k=-\infty}^{\infty} \left| F\left(\frac{n}{T_s}\right) \right|^2 \delta\left(f - \frac{n}{T_s}\right)}_{\text{discrete}}$$

Pulse spectrum

Data properties

Power Spectral Density

- Assume equal *average* power for all schemes

$$E\{|a_i + jb_i|^2\} = P_{avg}$$

$$= \begin{cases} \sigma^2 & PSK, QAM \\ \sigma^2 + A_{avg}^2 & ASK \end{cases}$$

$$P_g(f) = \frac{P_{avg}}{T_s} |F(f)|^2 \quad \text{PSK, QAM}$$

$$P_g(f) = \frac{P_{avg} - A_{avg}^2}{T_s} |F(f)|^2 + \frac{A_{avg}^2}{T_s} \sum_{k=-\infty}^{\infty} \left| F\left(\frac{n}{T_s}\right) \right|^2 \delta\left(f - \frac{n}{T_s}\right) \quad \text{ASK}$$

- Thus we can control the power spectral density through the pulse shape $F(f)$



Pulse Shaping

- Pulse shaping for bandpass systems (when PSK, QAM, ASK are used) is identical to pulse shaping for baseband systems
- Raised cosine (and sinc) pulses are preferable since they have controlled bandwidth properties and have zero ISI
- Since a matched filter is desired at the receiver, we actually use *square root raised cosine* pulses

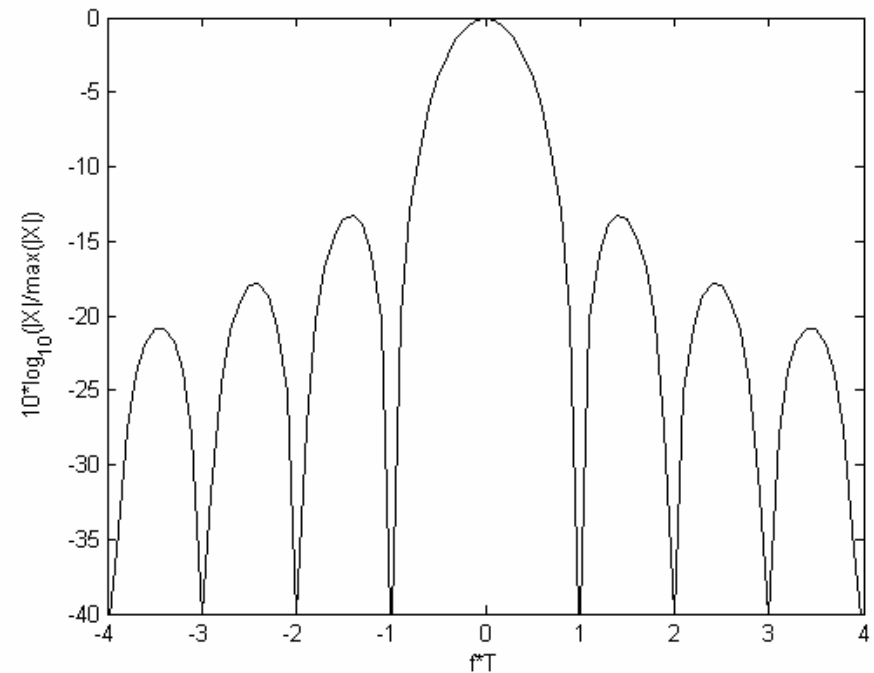
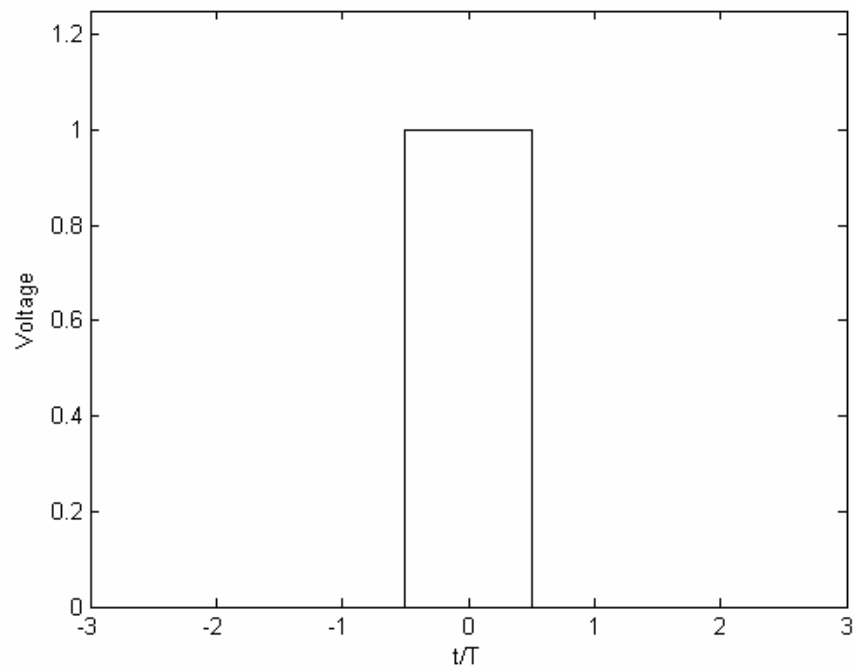
Square Pulses



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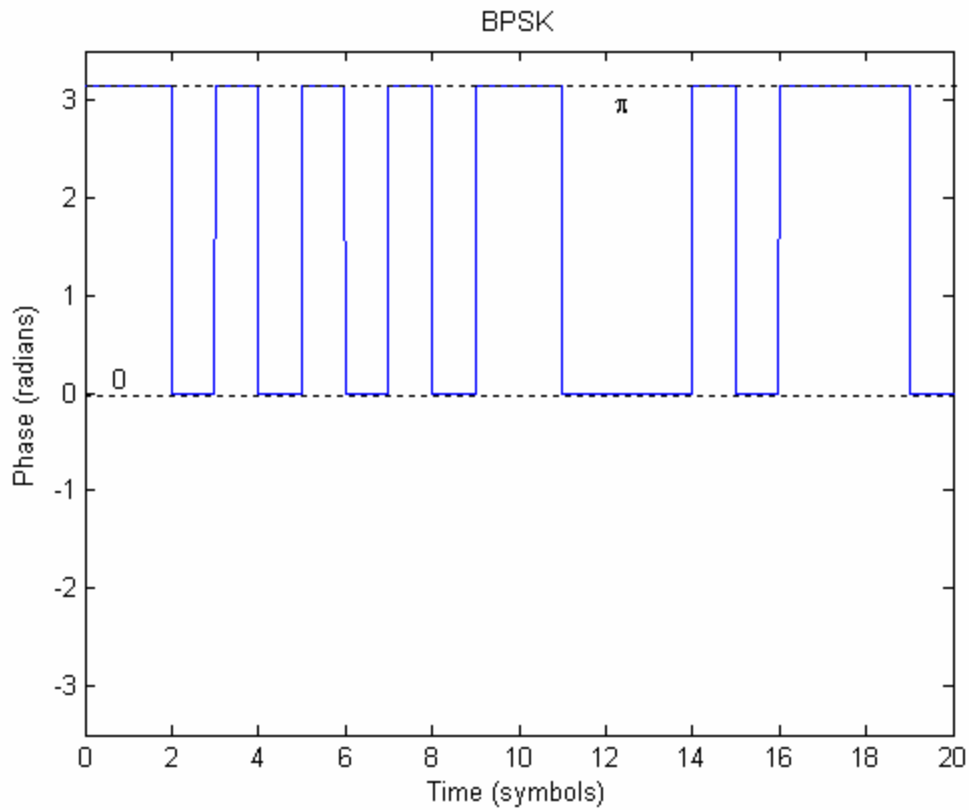
- Pulse shape

Spectrum

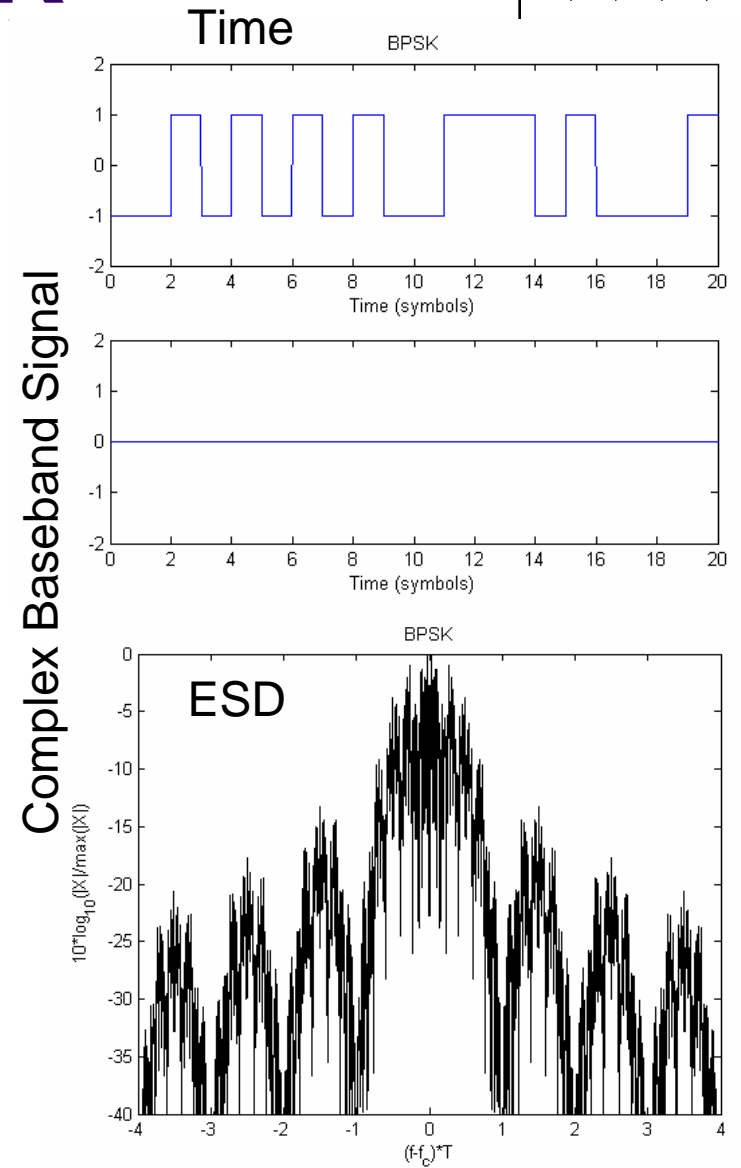




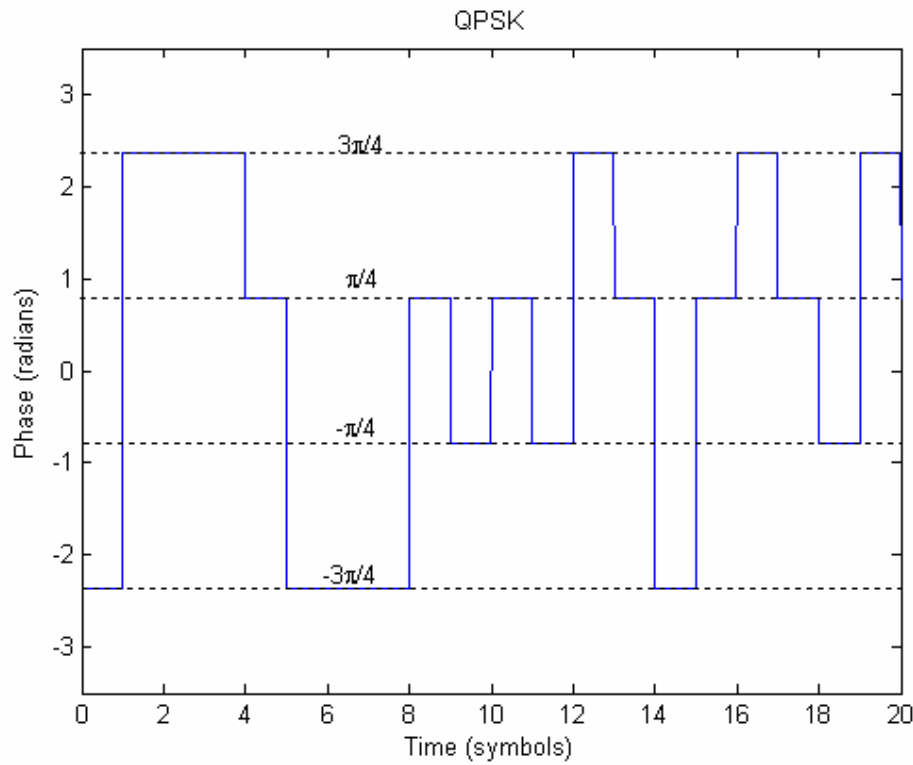
Square Pulses – BPSK



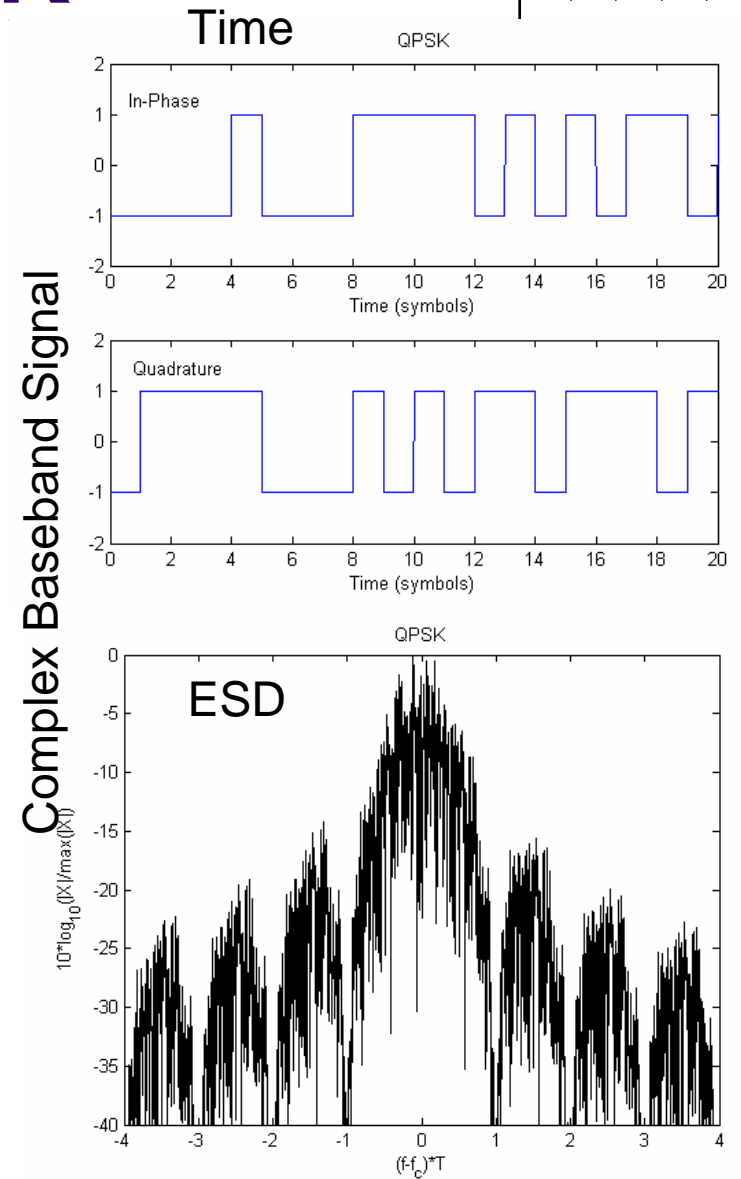
Phase of Carrier



Square Pulses – QPSK



Phase of Carrier

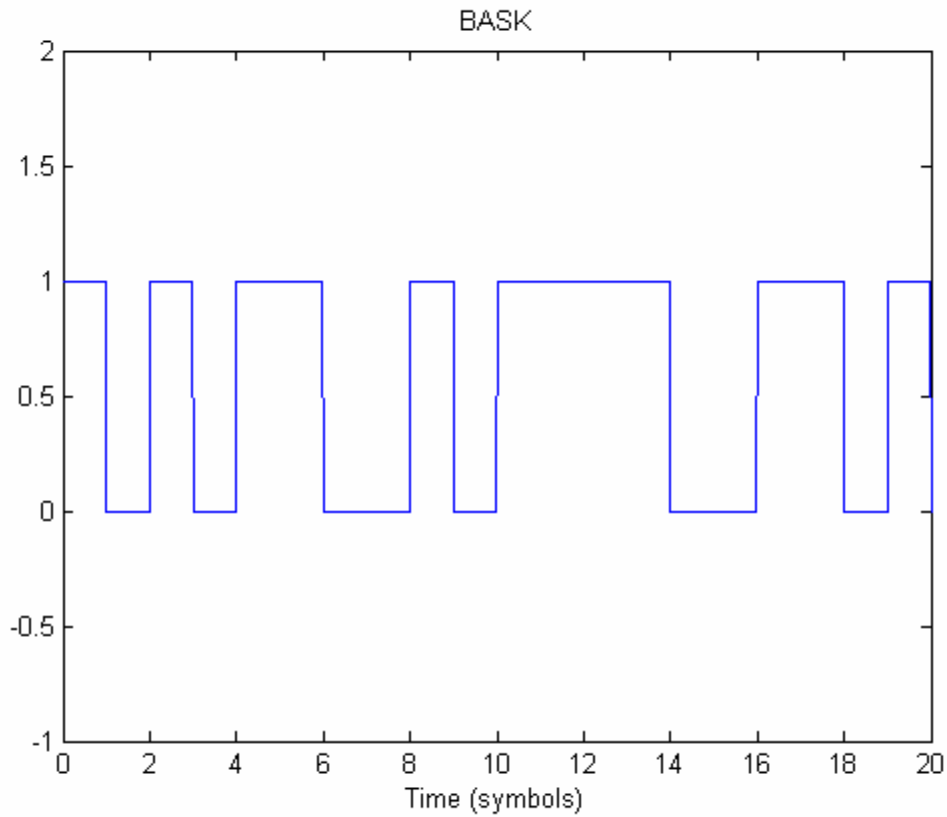


Square Pulses – BASK

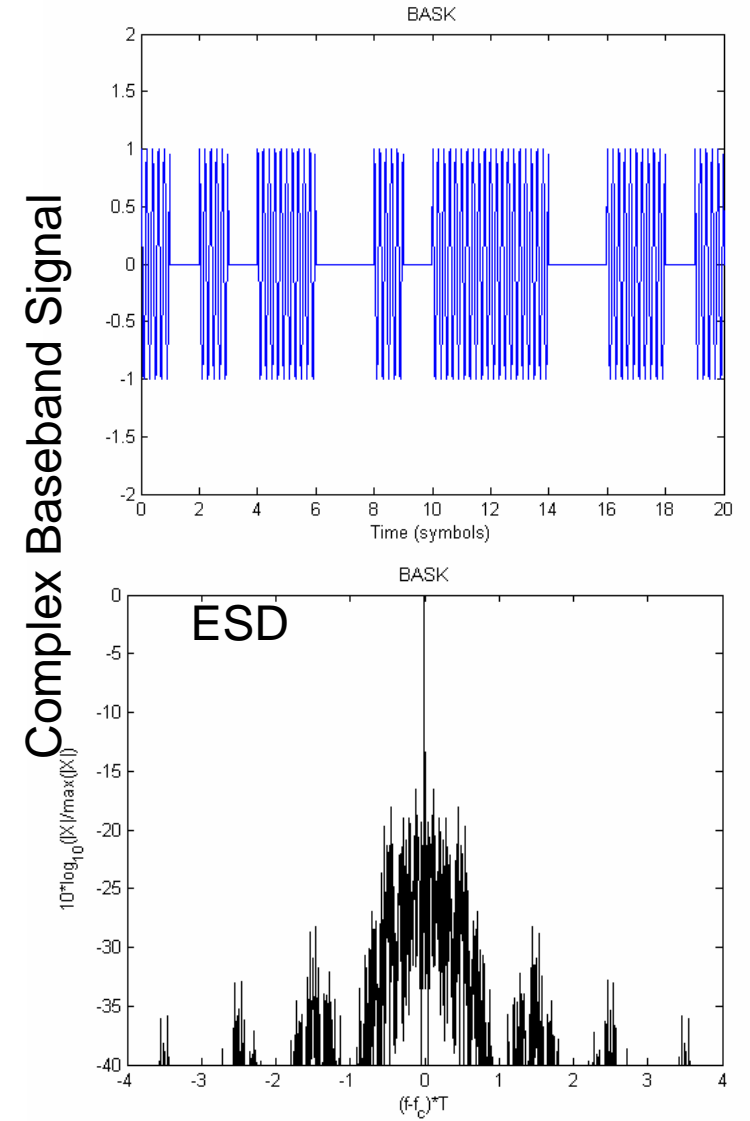


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Time



Amplitude of Carrier



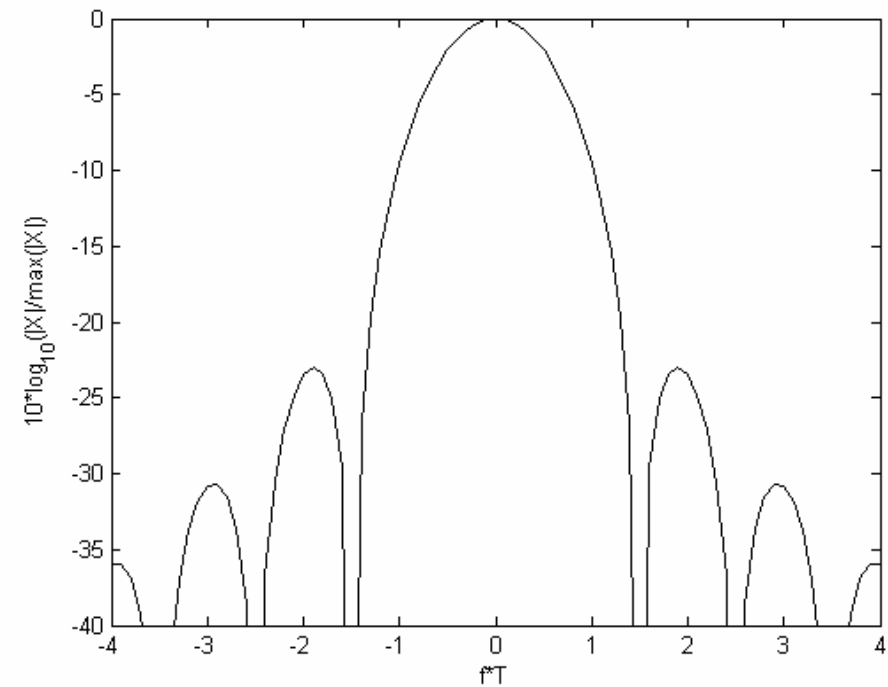
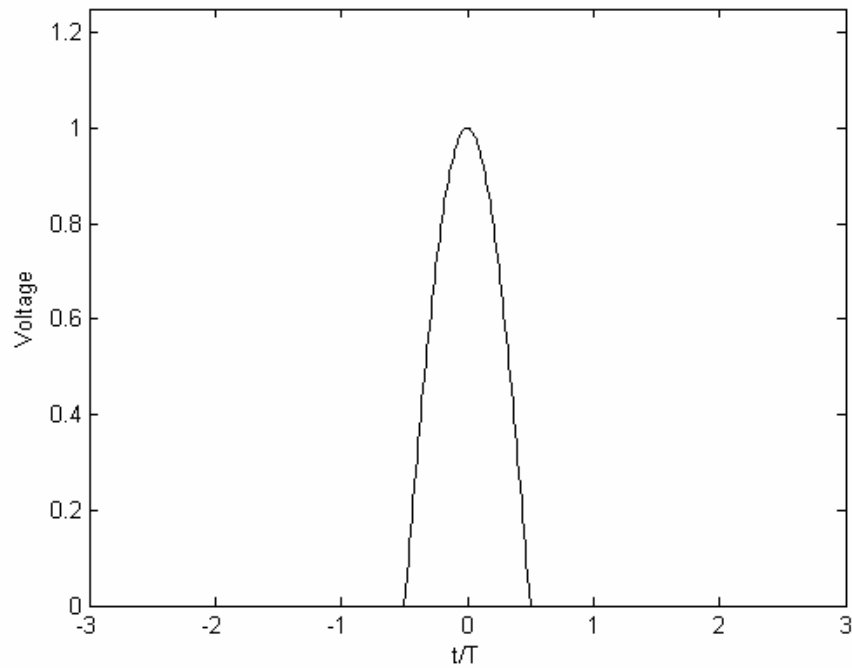
Cosine Pulses



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- Pulse shape

Spectrum

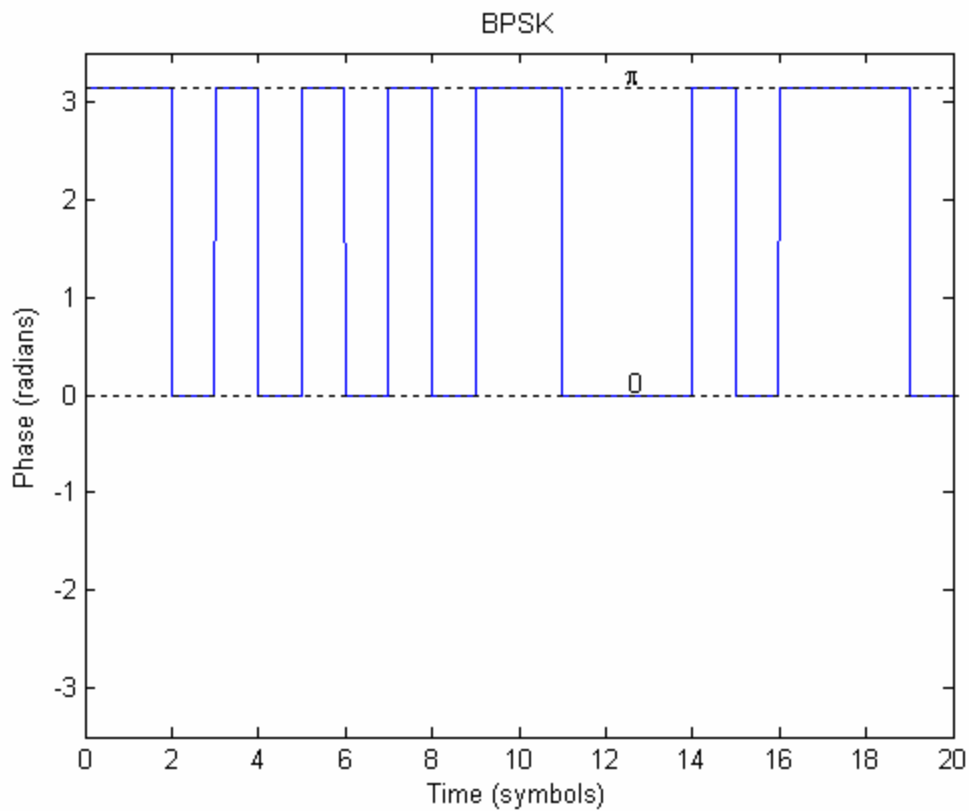


Cosine Pulses – BPSK

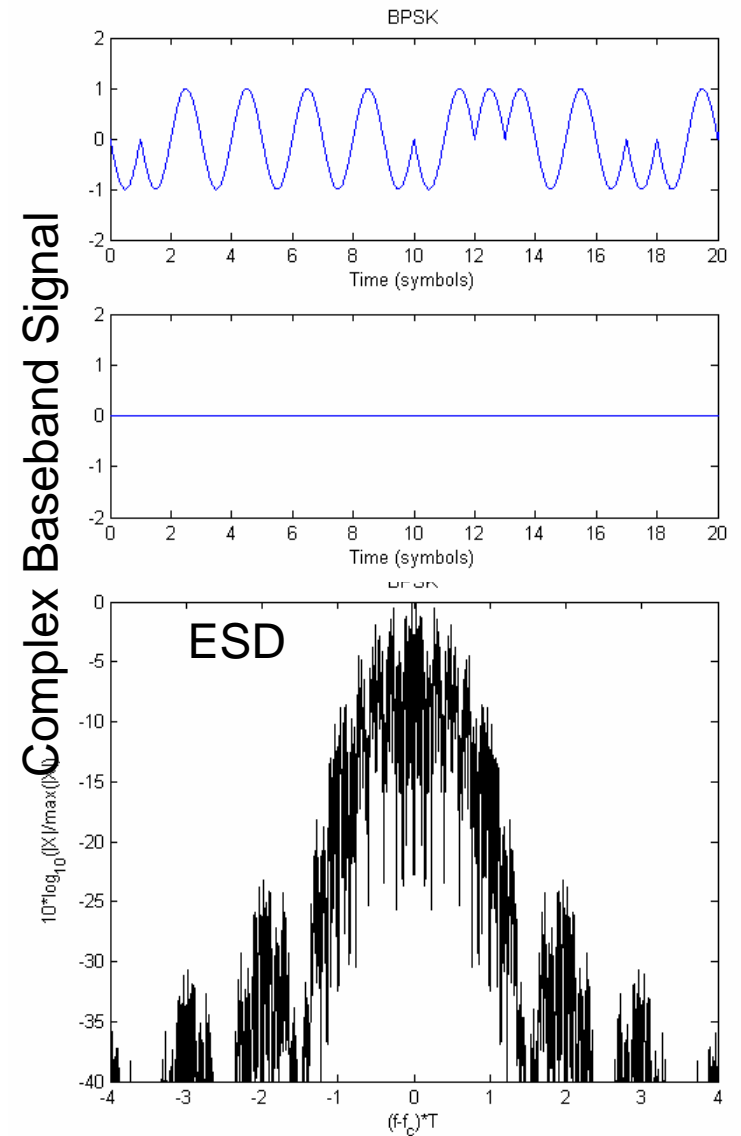


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Time

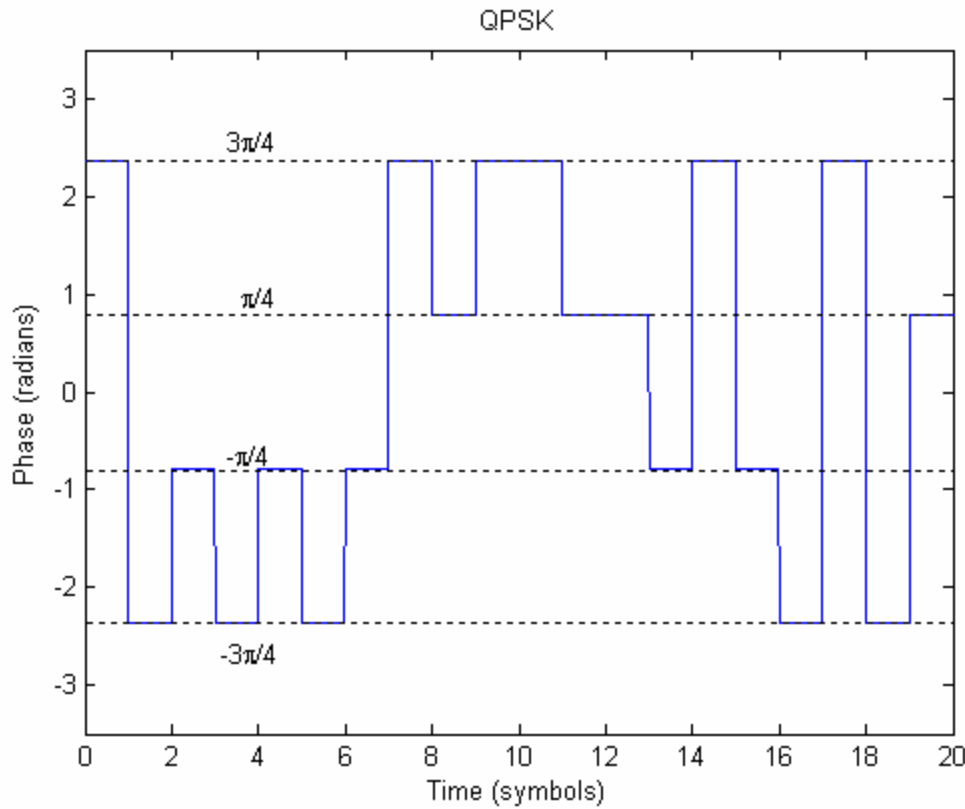


Phase of Carrier



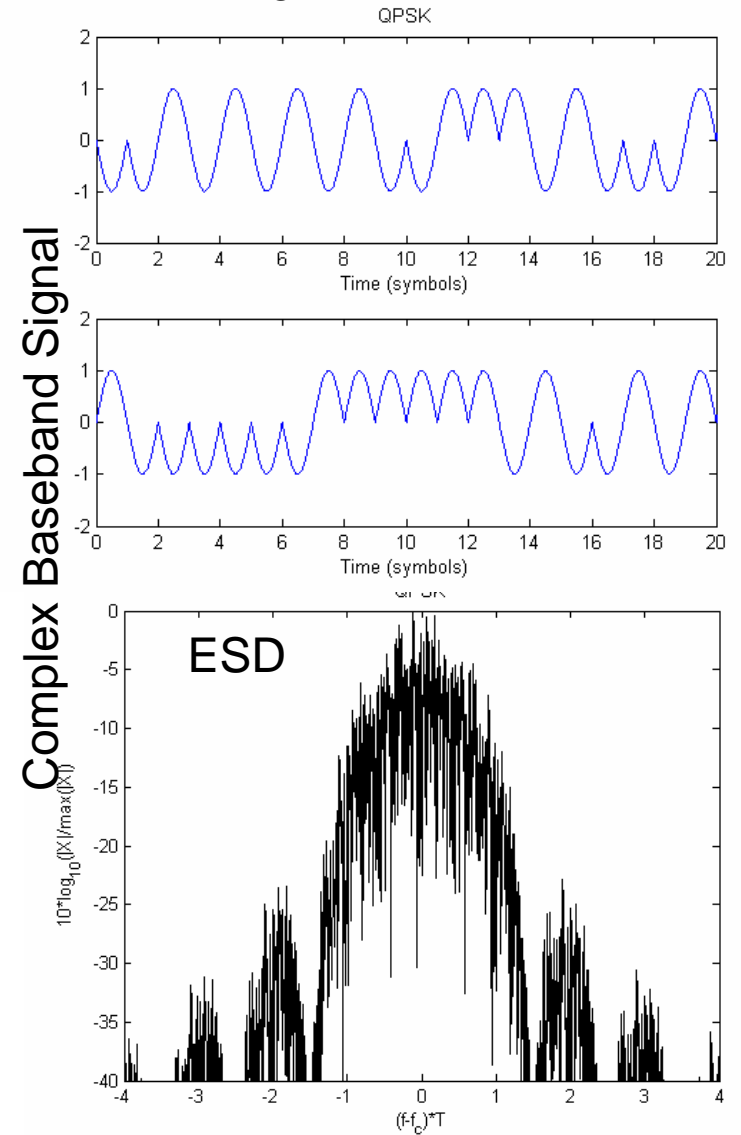


Cosine Pulses – QPSK



Phase of Carrier

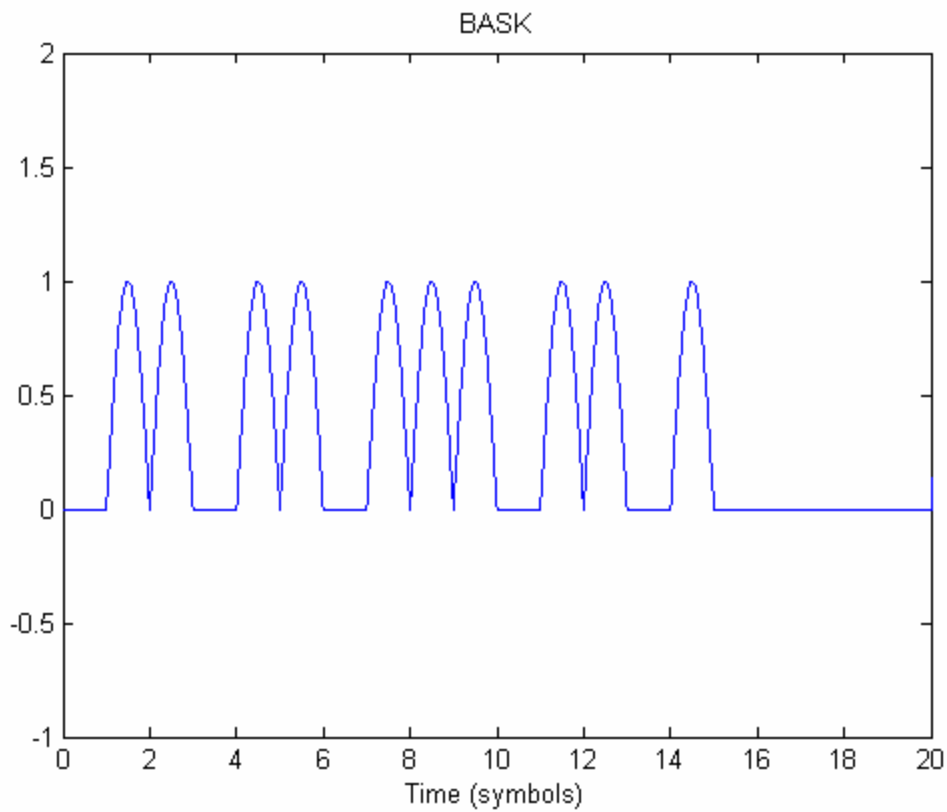
Time



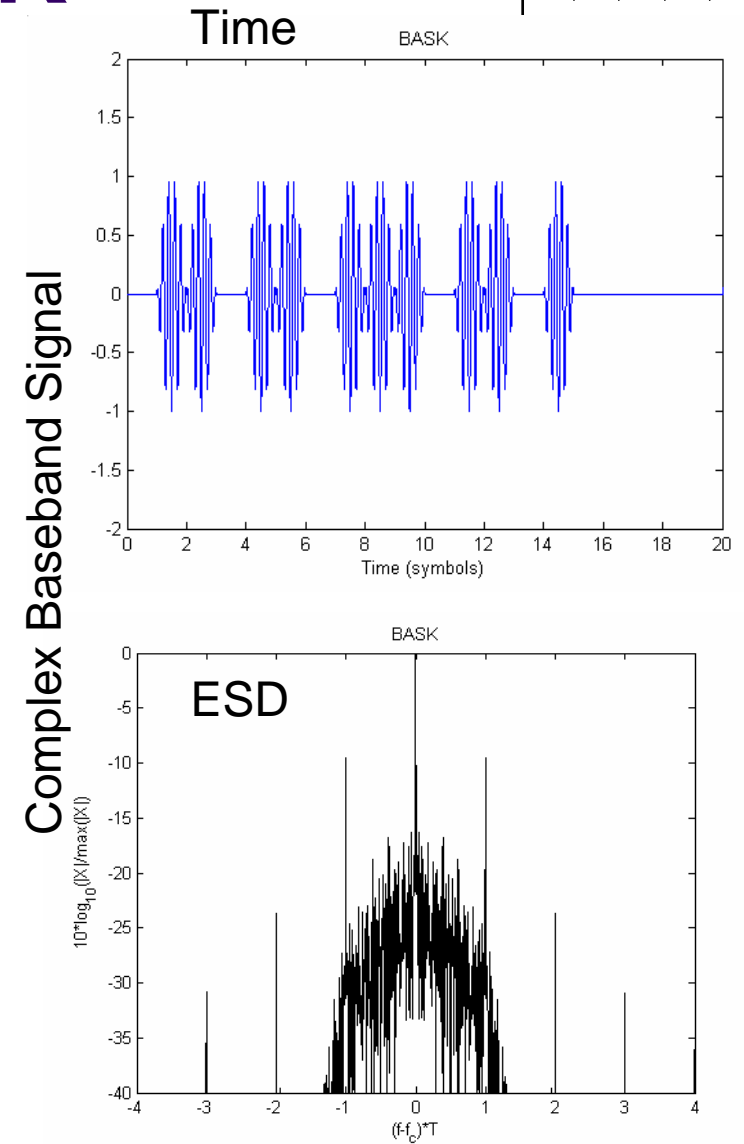
Cosine Pulses – BASK



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Amplitude of Carrier





Conclusions

- Pulse shaping does not affect the phase modulation but does affect amplitude modulation
 - Can be ignored with proper sampling time
- Pulse shaping does affect the spectral characteristics
- Using non-square pulses results in non-constant envelope
- We will find that the performance is unaffected by pulse shape provided
 - No ISI introduced
 - Matched filter used