ECE4634
Introduction to
Digital Communications
Fall 2007

Instructor: R. Michael Buehrer

Lecture #26: Bandpass Receivers

Analog and Digital Communications

Receiver

Analog and Digital Communications

- Assuming a benign channel, at the receiver we observe the transmitted signal plus additive white Gaussian noise

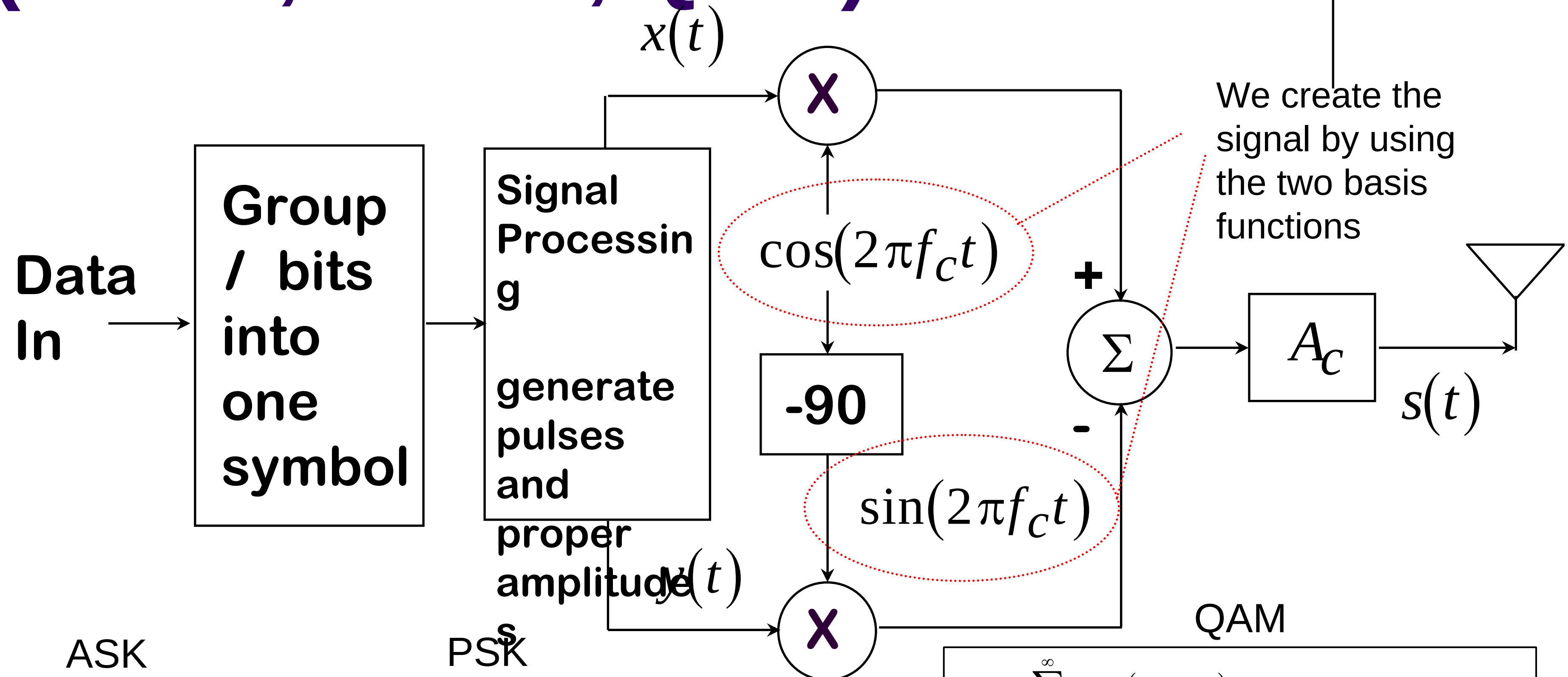
$$r(t) = x(t) + n(t)$$

- Both the transmitted signal x(t) and the noise signal *n(t)* are *a priori* unknown.
- Since there are a finite number of possible transmitted signals, the job of the receiver is to determine the most likely transmitted signal based on the observed receive signal

Receiver (cont.)

- Using the signal space representation of modulation the receiver consists of
  - Correlating the received signal with each basis function
  - Sampling each correlator output to create a single point in *K*-dimensional space
  - Creating the symbol estimate as the closest symbol in terms of Euclidean distance
- To maximize SNR we use *matched filters*
  - Correlation with the basis function can be viewed as matched filtering

Generic Transmitter (MPSK, MASK, QAM)



ASK

$$x(t)=\sum_{n=-\infty}^{\infty} A_n p(t-nT_s)$$

$$A_n = A_1, A_2, \ldots A_M$$

$$y(t)=0$$

PSK

$$x(t)=\sum_{n=-\infty}^{\infty} A_n p(t-nT_s)$$

$$A_n=\cos(\theta_1),\cos(\theta_2),\ldots\cos(\theta_M)$$

$$y(t)=\sum_{n=-\infty}^{\infty} B_n p(t-nT_s)$$

$$B_n=\sin(\theta_1),\sin(\theta_2),\ldots\sin(\theta_M)$$

QAM

$$x(t)=\sum_{n=-\infty}^{\infty} A_n p(t-nT_s)$$

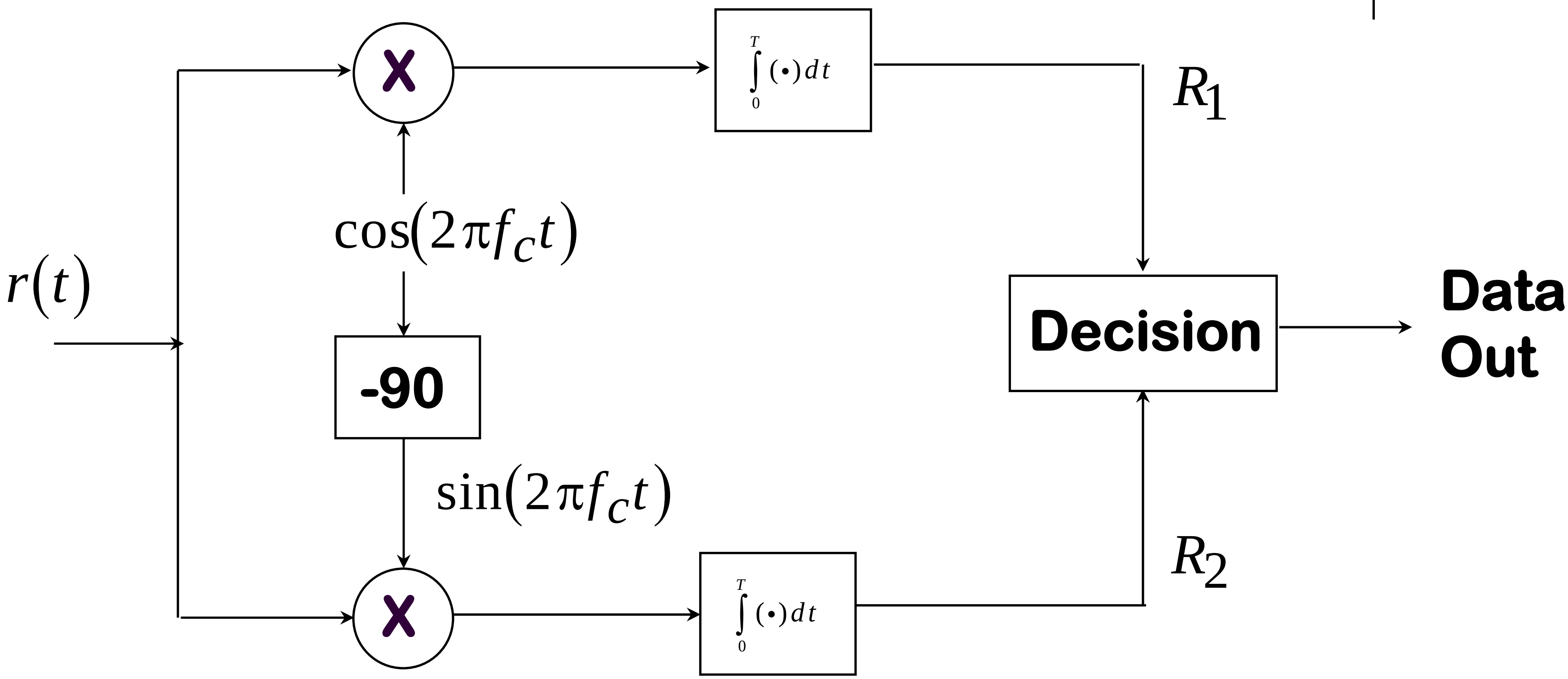
$$A_n=\frac{1}{\sqrt{E_{avg}}}\left\{-\sqrt{M}+1,-\sqrt{M}+3,\ldots\sqrt{M}-3,\sqrt{M}-1\right\}$$

$$y(t)=\sum_{n=-\infty}^{\infty} B_n p(t-nT_s)$$

$$B_n=\frac{1}{\sqrt{E_{avg}}}\left\{-\sqrt{M}+1,-\sqrt{M}+3,\ldots\sqrt{M}-3,\sqrt{M}-1\right\}$$

Generic *Coherent* Receiver
(MPSK, MASK, QAM)

Analog and Digital Communications



R_1 and R_2 tell us how In-Phase and Quadrature are modulated respectively. To determine the bits we simply map to the closest constellation point. **Note: Integration assumes square pulses!**

Received Signal

- The received signal for MPSK, QAM, MASK

$$r(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)$$

$$= \sum_{n=-\infty}^{\infty} A_n p(t - nT_s)\cos(2\pi f_c t) - \sum_{n=-\infty}^{\infty} B_n p(t - nT_s)\sin(2\pi f_c t)$$

- The first step is demodulation (In-Phase branch) and low pass (matched) filtering

$$\int_0^T r(t)\cos(2\pi f_c t)dt = \int_0^T \left\{x(t)\cos^2(2\pi f_c t) - y(t)\sin(2\pi f_c t)\cos(2\pi f_c t)\right\} dt$$

$$= \int_0^T A_1 \frac{1}{2}\left[1 + \underbrace{\cos(4\pi f_c t)}_{\text{eliminated by int.}}\right] dt - \int_0^T B_1 \underbrace{\sin(2\pi f_c t)\cos(2\pi f_c t)}_{\text{orthogonal}} dt$$

$$\approx \frac{A_1 T}{2}$$

Assumes square pulses

Received Signal (cont.)

Analog and Digital Communications

- Similarly on the Quadrature branch

$$\int_0^T r(t)\sin(2\pi f_c t)dt = \int_0^T \left\{ x(t)\cos(2\pi f_c t)\sin(2\pi f_c t) - y(t)\sin^2(2\pi f_c t)\right\} dt$$

$$= \int_0^T A_1 \frac{1}{2}\underbrace{\sin(2\pi f_c t)\cos(2\pi f_c t)}_{\text{orthogonal}} dt - \int_0^T B_1 \frac{1}{2}\left[1 - \underbrace{\cos(4\pi f_c t)}_{\text{eliminated by int.}}\right] dt$$

$$\approx -\frac{B_1 T}{2}$$

- Thus, if we negate the Q-channel output, we have a scaled version of the signal space point.

Matched Filtering

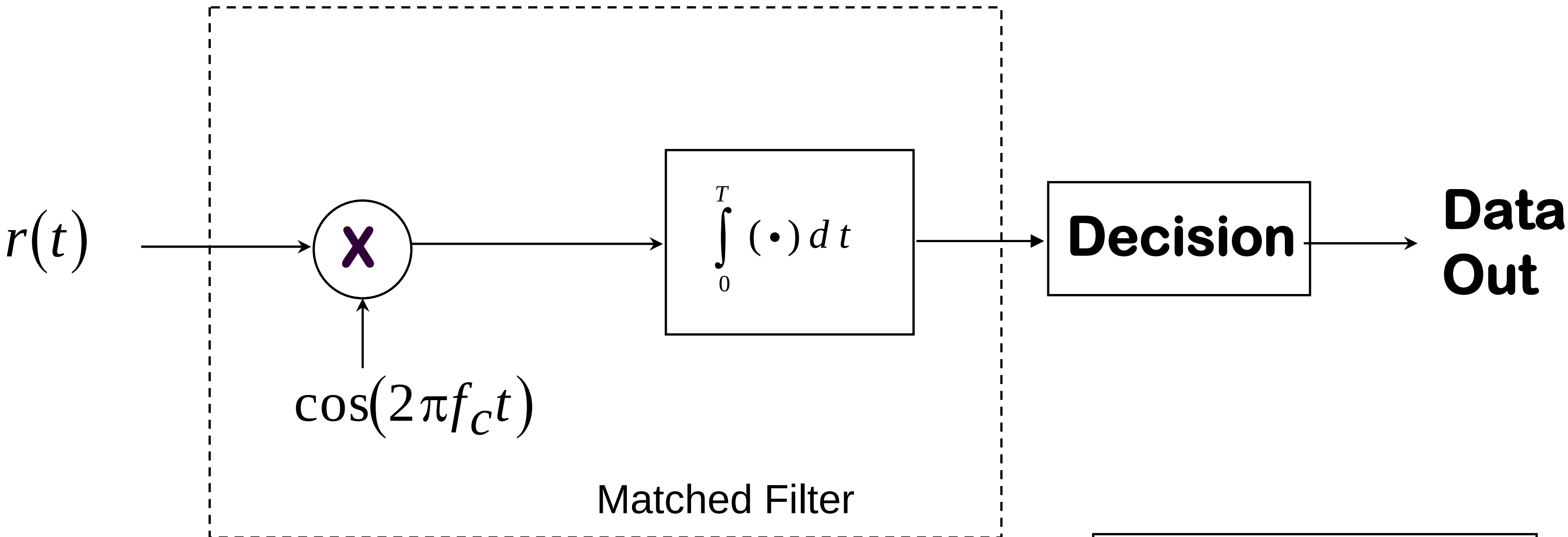
- The previous receiver structure is also an implementation of the matched filter
- Consider BPSK

$$s_1(t) = \Pi\left(\frac{t-T}{T}\right)\cos(2\pi f_c t)$$

$$s_2(t) = -\Pi\left(\frac{t-T}{T}\right)\cos(2\pi f_c t)$$

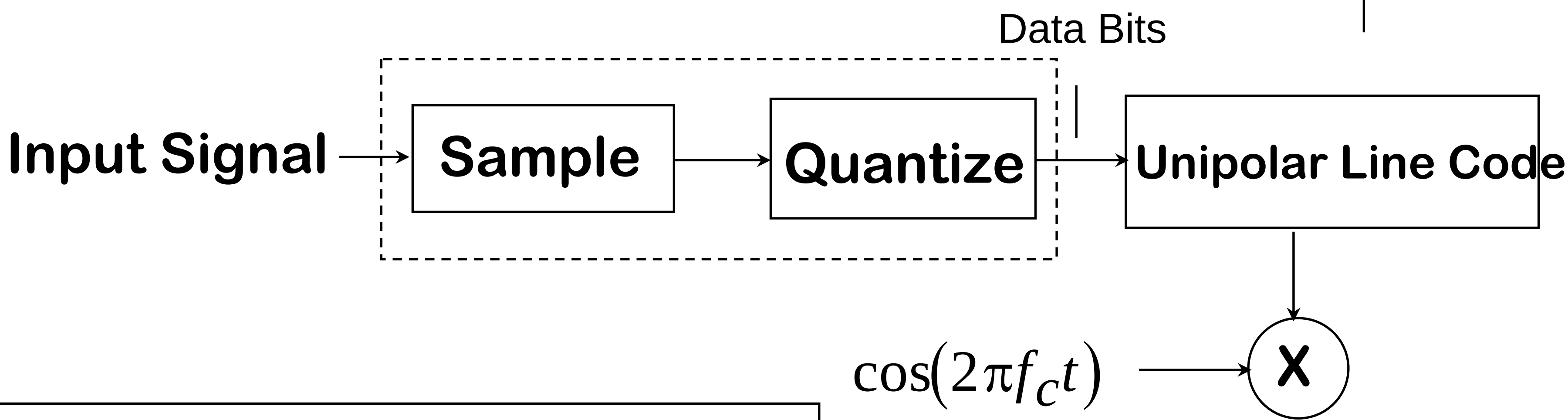
- Thus, we are modulating the pulse $\Pi\left(\frac{t-T}{T}\right)\cos(2\pi f_c t)$
- The receiver should simply correlate with this pulse to maximize SNR

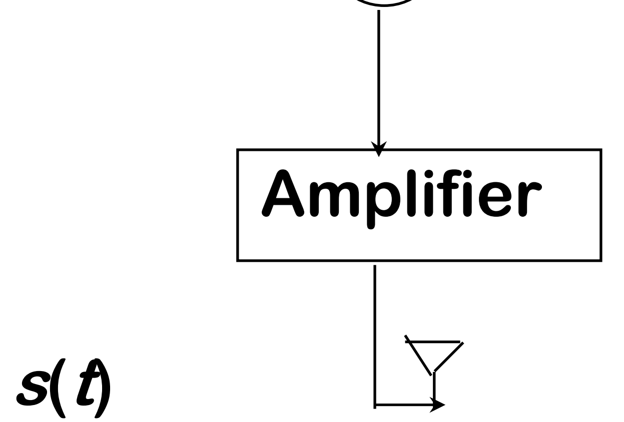
BPSK Receiver



The combination of a mixer and integrator are a matched filter for BPSK with square pusles

BASK Transmitter





Again note: We have used the I/Q representation to lead us to a transmitter design. We simply create the I channel (unipolar signal) and modulate a sinusoid.

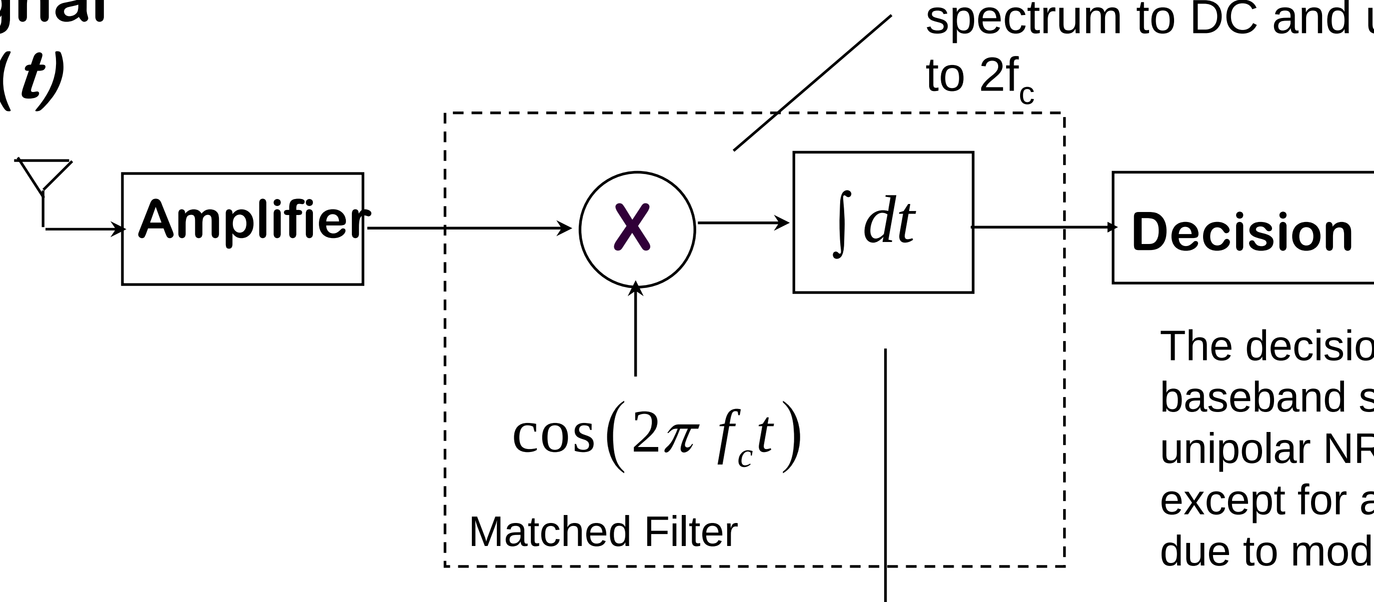
Coherent Receiver for ASK

$$r(t) = \left(\sum_{n=-\infty}^{\infty} d_n p(t - nT_s) \right) \cos(\omega_c t) + n(t)$$

$$d_n = \begin{cases} A & b = 1 \\ 0 & b = 0 \end{cases}$$

Received Signal *r(t)*

Mixer shifts the spectrum to DC and up to $2f_c$



$$Z \geq A/4 \Rightarrow 1$$
$$Z < A/4 \Rightarrow 0$$

The decision is similar to baseband signaling with unipolar NRZ except for a factor of 2 due to modulation

Note that we ASK receiver does not necessarily need coherent phase reference as we have seen previously. For the sake of illustration, we will assume coherent reception here.

Integration removes double frequency term and integrates baseband signal providng same benefit as in baseband receiver.

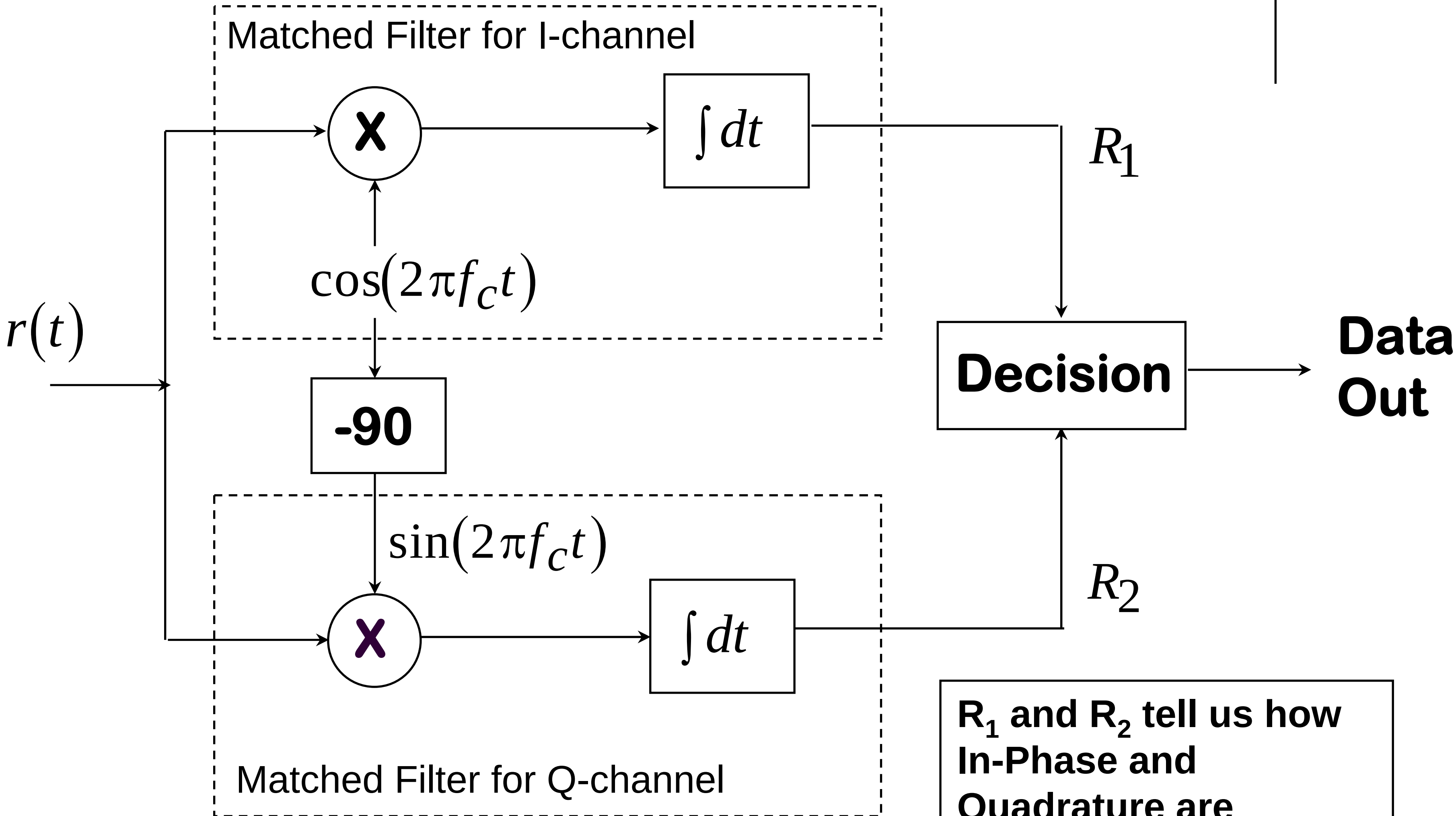
Receiver for ASK

- Decision Variable *Z (assume that b=1)*:

$$x(t) = A\, p\left(t - nT_s\right)\cos\left(\omega_c t\right)$$

$$
\begin{aligned}
Z &= \frac{1}{T_s}\int_0^{T_s} r(t)\cos\left(2\pi f_c t\right)dt \\
&= \frac{1}{T_s}\int_0^{T_s} \Big(x(t)\cos\left(2\pi f_c t\right) + n(t)\Big)\cos\left(2\pi f_c t\right)dt \\
&= \frac{1}{T_s}\int_0^{T_s} x(t)\frac{1}{2}\left[1 + \cos\left(4\pi f_c t\right)\right]dt + \int_0^{T} n_b(t)\,dt \\
&\approx \frac{A}{2} + N
\end{aligned}
$$

Receiver for M-ary PSK



R_1 and R_2 tell us how In-Phase and Quadrature are modulated respectively

Receiver for MPSK

Analog and Digital Communications

- Decision Variable $Z=R_1+jR_2$

$$R_1 = \frac{1}{T_s}\int_0^{T_s} r(t)\cos(2\pi f_c t)\,dt$$
$$= \frac{1}{T_s}\int_0^{T_s} \left(x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) + n(t)\right)\cos(2\pi f_c t)\,dt$$
$$= \frac{1}{T_s}\int_0^{T_s} x(t)\frac{1}{2}\left[1+\cos(4\pi f_c t)\right]dt - \frac{1}{T_s}\int_0^{T_s} y(t)\sin(2\pi f_c t)\cos(2\pi f_c t)\,dt + \int_0^{T} n_i(t)\,dt$$
$$\approx \frac{A_n}{2} + N_I$$

$$A_n = \cos(\theta_1), \cos(\theta_2), \ldots \cos(\theta_M)$$
$$B_n = \sin(\theta_1), \sin(\theta_2), \ldots \sin(\theta_M)$$

$$R_2 = \frac{1}{T_s}\int_0^{T_s} r(t)\sin(2\pi f_c t)\,dt$$
$$= \frac{1}{T_s}\int_0^{T_s} \left(x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) + n(t)\right)\sin(2\pi f_c t)\,dt$$
$$= \frac{1}{T_s}\int_0^{T_s} x(t)\cos(2\pi f_c t)\sin(2\pi f_c t)\,dt - \frac{1}{T_s}\int_0^{T_s} y(t)\frac{1}{2}\left[1-\cos(4\pi f_c t)\right]dt + \int_0^{T} n_q(t)\,dt$$
$$\approx -\frac{B_n}{2} + N_Q$$

Pulse shaping

- The transmit signal for ASK with pulse shaping is

$$x(t)=\left(\sum_{n=-\infty}^{\infty} d_n p\left(t-nT_s\right)\right)\cos\left(\omega_c t\right)$$

$$d_n=\begin{cases} A & b=1 \\ 0 & b=0 \end{cases}$$

  - Where p(t) is now any pulse shape rather than a square pulse.
- The transmit signal for PSK with pulse shaping is

$$x(t)=\left(\sum_{n=-\infty}^{\infty} A_n p\left(t-nT_s\right)\right)\cos\left(\omega_c t\right)-\left(\sum_{n=-\infty}^{\infty} B_n p\left(t-nT_s\right)\right)\sin\left(\omega_c t\right)$$

$$A_n=\cos\left(\theta_n\right) \qquad \theta_n \in \left\{0,\frac{\pi}{M},\frac{2\pi}{M},\ldots\frac{\left(M-1\right)\pi}{M}\right\}$$
$$B_n=\sin\left(\theta_n\right)$$

Pulse Shaping Receiver

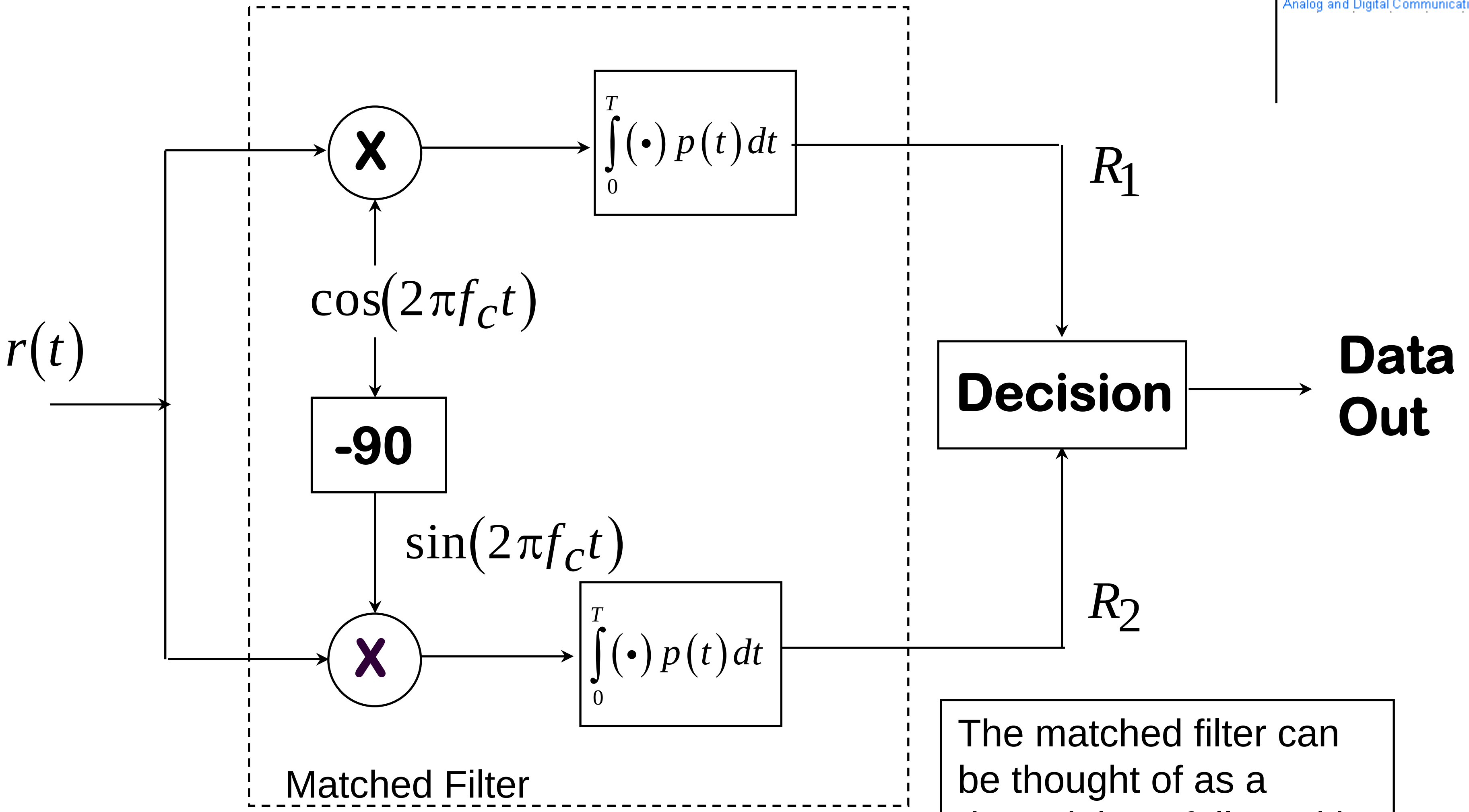
- Again we must use a receiver matched to the basis functions which includes the pulse shape
- For BPSK or *M*-ary ASK we have a single matched filter

$$p(t)\cos(2\pi f_c t)$$

- For *M*-PSK we have two matched filters

$$p(t)\cos(2\pi f_c t)$$
$$p(t)\sin(2\pi f_c t)$$

Ex: Receiver for M-ary PSK



The matched filter can be thought of as a demodulator followed by a baseband matched filter.

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Virginia Tech
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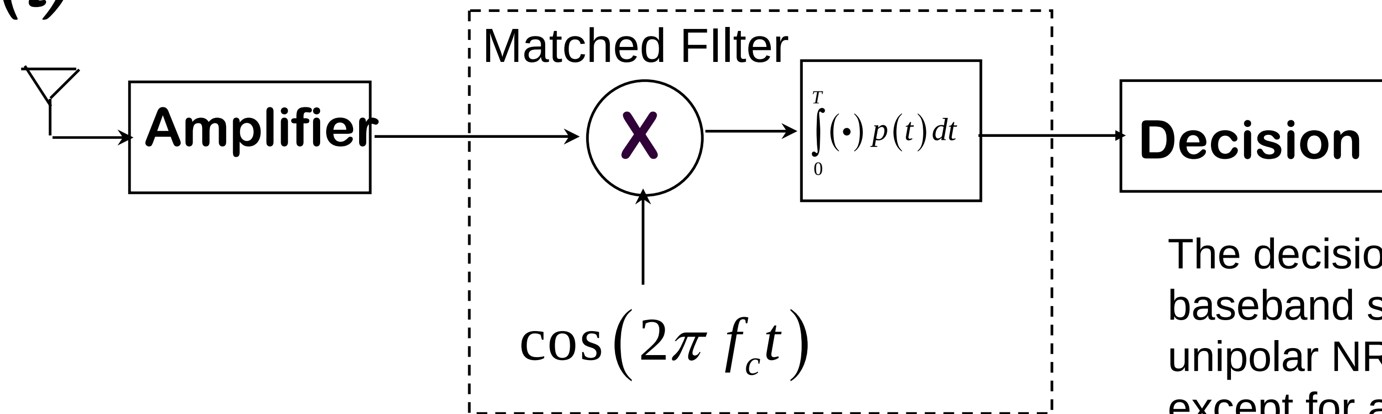
Ex: Coherent Receiver for ASK

$$r(t)=\left(\sum_{n=-\infty}^{\infty} d_n p\left(t-nT_s\right)\right)\cos\left(\omega_c t\right)+n(t)$$

Received Signal ***r(t)***

$$d_n=\begin{cases}A & b=1\\ 0 & b=0\end{cases}$$



$$Z \geq A/4 \Rightarrow 1$$
$$Z < A/4 \Rightarrow 0$$

The decision is similar to baseband signaling with unipolar NRZ
except for a factor of 2 due to modulation

Inter-symbol Interference

- Recall that the for zero ISI we must satisfy the Nyquist criterion
- Pulses such as square pulses, sinc pulses, and raised cosine pulses satisfy this criterion
- However
  - Square pulses require too much bandwidth
  - Sinc pulses require infinite time delay
- Truncated sinc pulses are a practical alternative
- Truncated raised cosine pulses are even more practical since they can be truncated to shorter lengths
- These can also be used with bandpass modulation

Raised Cosine pulses and Matched Filters

Analog and Digital Communications

- If we transmit raised cosine pulses, in order to maximize SNR we must use a matched filter (i.e., a raised cosine pulse filter) at the receiver
- If we apply a raised cosine pulse filter twice (once on transmit and once at the receiver) the result will not be a raised cosine pulse and thus will have ISI
- To avoid this, we must apply a square root raised cosine pulse ($p_{sq}(t)$) at the transmitter and at the receiver

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Square Root Raised Cosine Filters

- We actually want the zero ISI to occur at the receiver
  - We want the spectrum out of the matched filter to have a raised cosine shape
- Therefore we use:
  - Pulse shape at transmitter: $S(f)=\sqrt{P_{rc}(f)}$
  - Pulse shape of matched filter: $H(f)=\sqrt{P_{rc}(f)}$
- Overall result is a raised cosine pulse at output of receiver matched filter

$$R(f)=S(f)H(f)=\sqrt{P_{rc}(f)}\sqrt{P_{rc}(f)}=P_{rc}(f)$$

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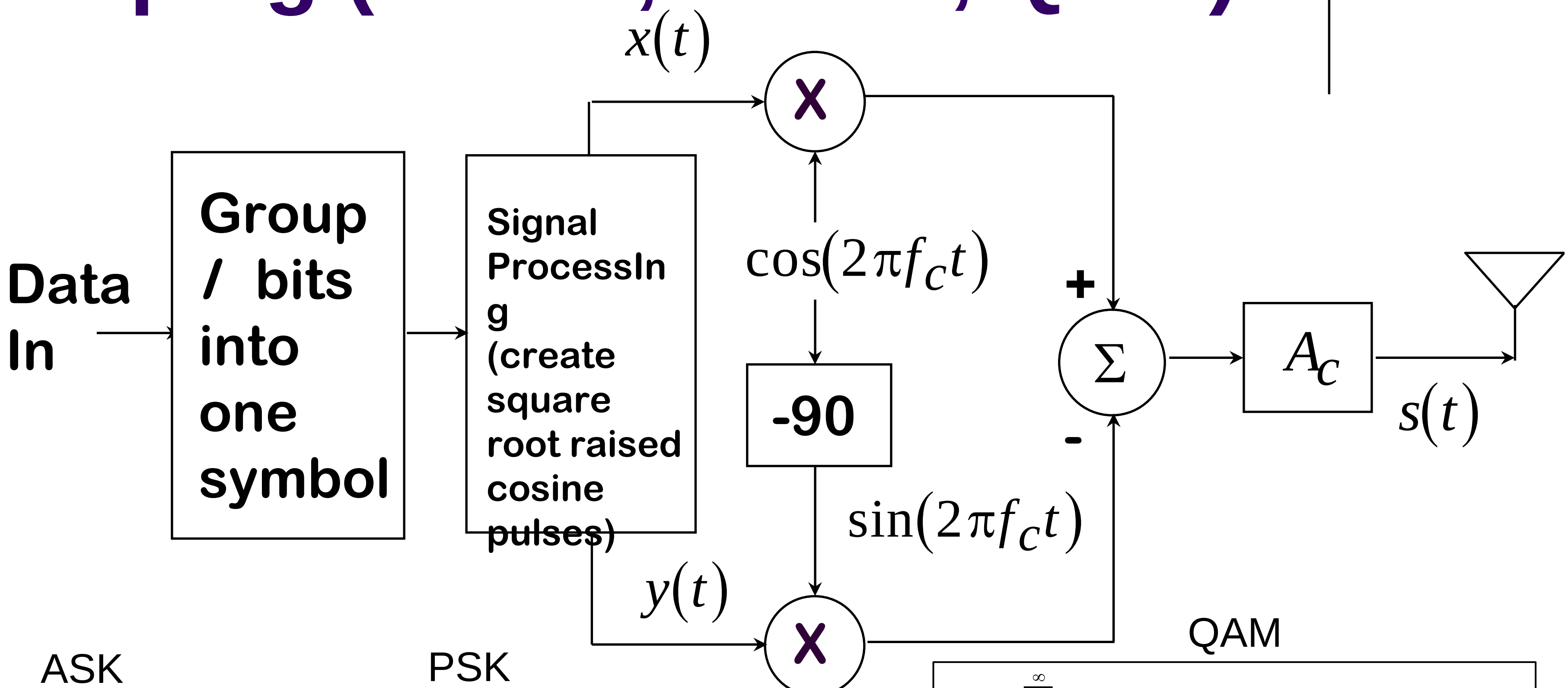
Generic Transmitter with Pulse Shaping (MPSK, MASK, QAM)



ASK

$$x(t)=\sum_{n=-\infty}^{\infty} A_n p_{sq}\left(t-nT_s\right)$$

$$A_n = A_1, A_2, \ldots A_M$$

$$y(t)=0$$

PSK

$$x(t)=\sum_{n=-\infty}^{\infty} A_n p_{sq}\left(t-nT_s\right)$$

$$A_n=\cos\left(\theta_1\right),\cos\left(\theta_2\right),\ldots\cos\left(\theta_M\right)$$

$$y(t)=\sum_{n=-\infty}^{\infty} B_n p_{sq}\left(t-nT_s\right)$$

$$B_n=\sin\left(\theta_1\right),\sin\left(\theta_2\right),\ldots\sin\left(\theta_M\right)$$

QAM

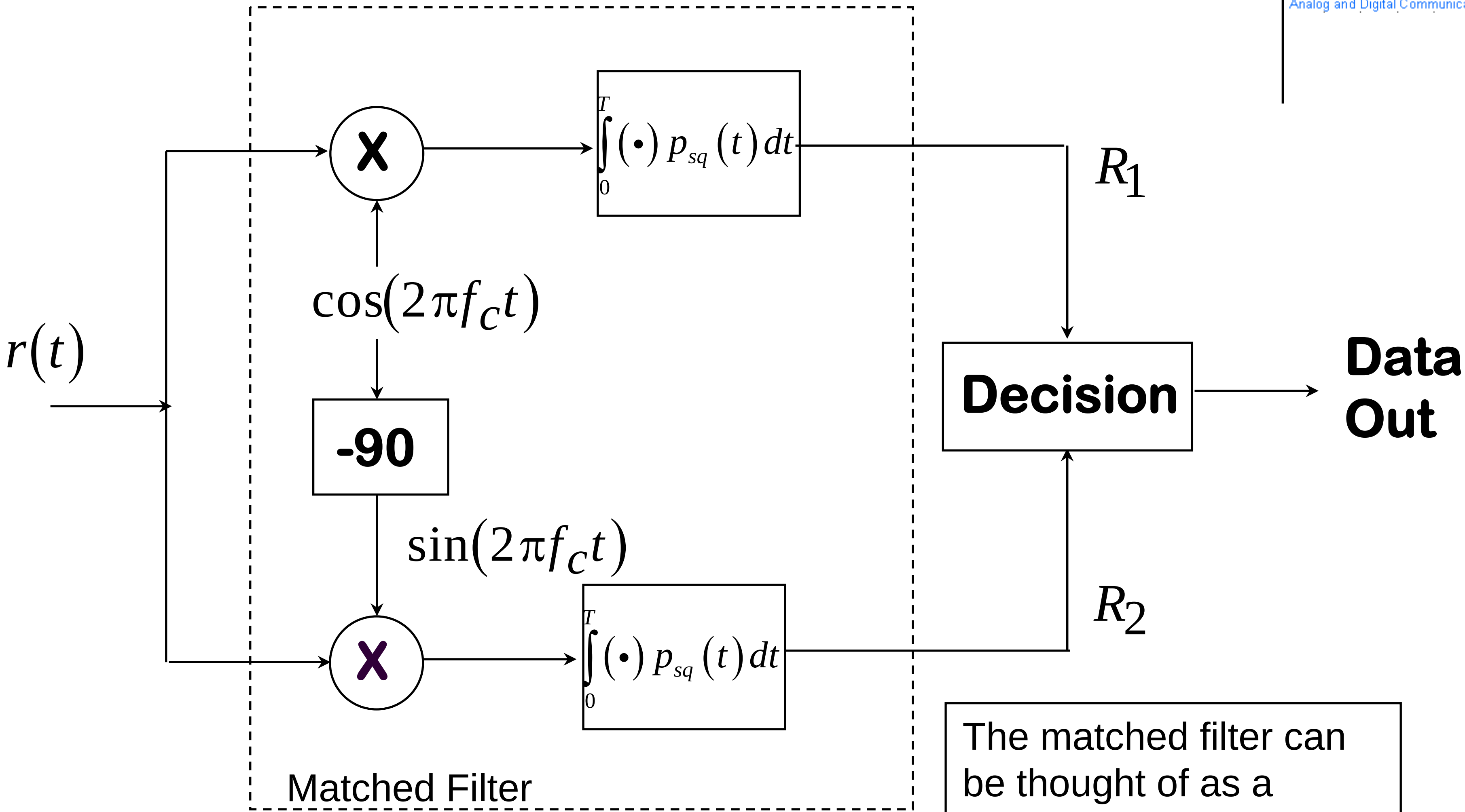
$$x(t)=\sum_{n=-\infty}^{\infty} A_n p_{sq}\left(t-nT_s\right)$$

$$A_n=\frac{1}{\sqrt{E_{avg}}}\left\{-\sqrt{M}+1,-\sqrt{M}+3,\ldots\sqrt{M}-3,\sqrt{M}-1\right\}$$

$$y(t)=\sum_{n=-\infty}^{\infty} B_n p_{sq}\left(t-nT_s\right)$$

$$B_n=\frac{1}{\sqrt{E_{avg}}}\left\{-\sqrt{M}+1,-\sqrt{M}+3,\ldots\sqrt{M}-3,\sqrt{M}-1\right\}$$

Receiver for M-ary PSK

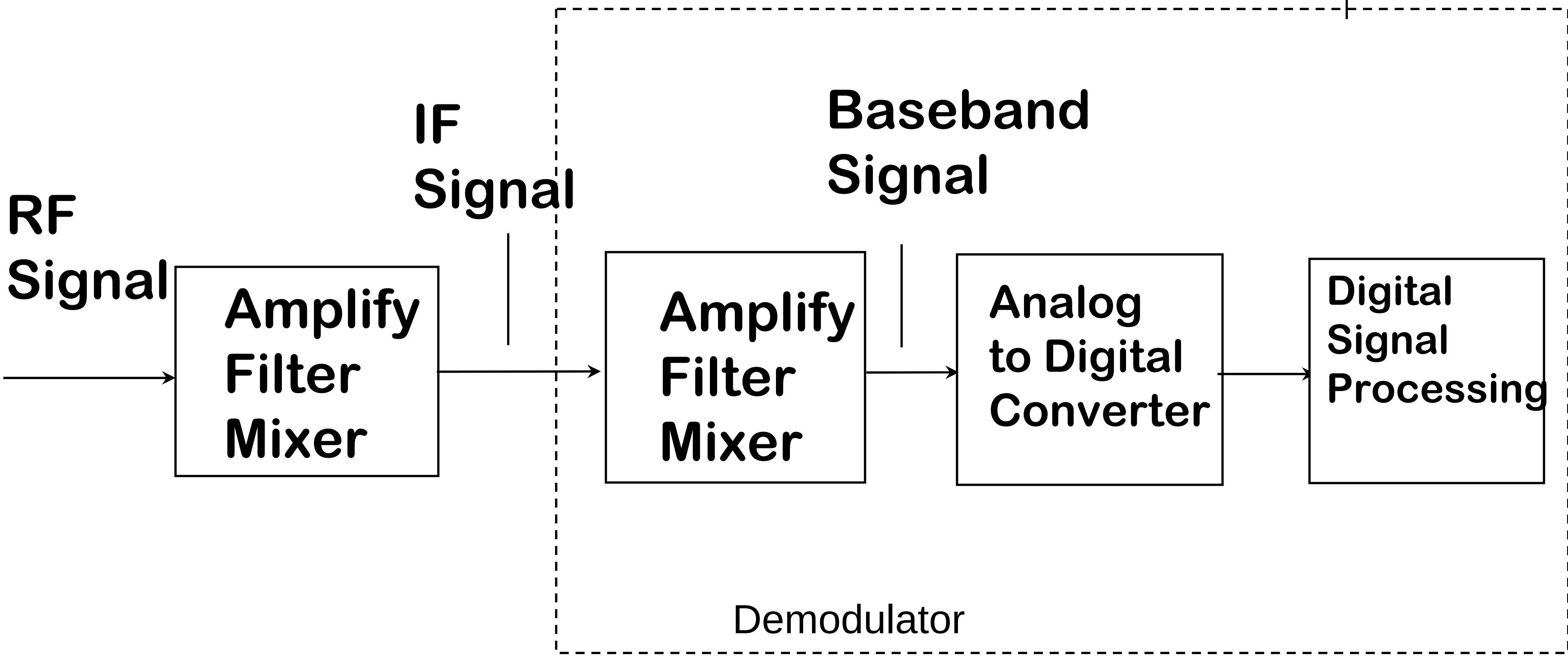


The matched filter can be thought of as a demodulator followed by a baseband matched filter.

Superheterodyne Receivers

- Edwin Armstrong (1918)
- Demodulation is accomplished in two stages
- It is very difficult to demodulate a signal exactly to zero frequency from the radio frequency (RF)
  - Typical RF values: 100 kHz - 30 GHz
- An intermediate frequency (IF) stage is introduced
  - Typical IF values: 50 kHz - 100 MHz (70MHz very common)

Superheterodyne Receiver



Summary

- The receiver design can be directly determined from the signal space representation
- The receiver must correlate the received signal with each of the basis functions (i.e., matched filters)
- The resulting values form a received point in the *K*-dimensional signal space which must be mapped to the nearest symbol
- This operation can also be thought of as demodulation followed by low-pass filtering
- Pulse shaping is readily incorporated into this framework as we have seen