

ECE4634 Digital Communications Fall 2007

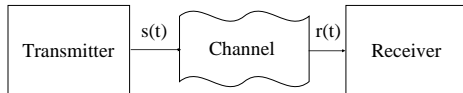
Instructor: R. Michael Buehrer
Lecture #31: Channels and
Channel Models



The Channel



- The channel is the medium through which the system communicates information
- In general $r(t) = s(t) \otimes h(t, \tau) + v(t)$ where $h(t, \tau)$ is the time-varying channel impulse response, $v(t)$ is thermal noise and \otimes is the convolution operation.



- However, we will only consider $h(t, \tau) = \gamma(t) \delta(\tau - \tau_0)$ with $\tau_0 = 0$ where $\delta(\tau)$ is an impulse. Thus,

$$r(t) = \gamma(t)s(t) + n(t)$$

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The Channel



- The received signal, channel, and bandpass noise can all be represented in complex baseband form as

$$\begin{aligned} r(t) &= \text{Re} \left\{ \tilde{r}(t) e^{j\omega_c t} \right\} \\ \gamma(t) &= \text{Re} \left\{ \tilde{\gamma}(t) e^{j\omega_c t} \right\} \\ s(t) &= \text{Re} \left\{ \tilde{s}(t) e^{j\omega_c t} \right\} \\ n(t) &= \text{Re} \left\{ \tilde{n}(t) e^{j\omega_c t} \right\} \end{aligned}$$

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The Channel



- Thus, the simplest channel is one which causes only *multiplicative distortion*
- Normally, we deal with the complex baseband of the transmit and receive signals. Fortunately, the above formula will apply to both RF and complex baseband. Thus, in complex baseband we have

Note: In the complex baseband there is a factor of $\frac{1}{2}$ which we commonly ignore

$$\tilde{r}(t) = \tilde{\gamma}(t)\tilde{s}(t) + \tilde{n}(t)$$

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Thermal Noise



- All objects with physical temperature T_p greater than 0 Kelvin generate electrical noise.
- This noise power is given by

$$P_n = kT_n B$$

Botzman's Constant 1.38×10^{-23} J/K
Noise temperature of device
Bandwidth of device

$$kT_n = \text{Noise Spectral Density}$$

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AWGN



- Noise in the system contributes an additional voltage on top of the received signal. Thus, it is **additive**.
- The noise is uncorrelated from one sample to the next (i.e., $R_x(\tau) = \delta(\tau)$). A delta function in the correlation function means that the PSD is a constant, thus it is **white**.
- Noise in the system originates from components in the receiver. Since there are many discrete components all contributing some small amount, the sum tends to a Gaussian process (Central Limit Theorem). Thus, noise samples have a *Gaussian* probability distribution.
- **Additive White Gaussian Noise (AWGN)**

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AWGN Channels

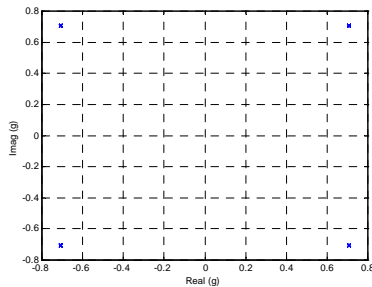


- The term 'AWGN Channel' is something of a misnomer.
- The channel doesn't actually add noise. It attenuates the signal to such a degree that the internal noise of the receiver is comparable to that of the received signal.
- Usually, we assume that the received signal has normalized average received power, while the noise has some power σ^2 where σ is the standard deviation of the thermal noise.
- In "AWGN Channels" we assume that the only distortion to the signal is the AWGN. That is $\gamma(t) = 1$

$$\tilde{x}(t) = \tilde{s}(t) + \tilde{n}(t)$$

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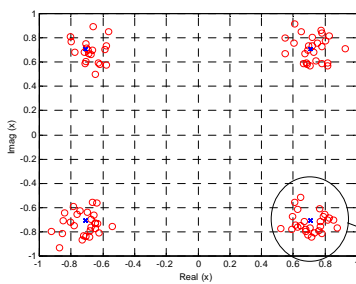
AWGN Channels – Example



$g(t)$ = QPSK signal

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AWGN Channels – Example



'x' = QPSK signal
'o' = received signal

Thermal Noise moves points.
Since noise has zero mean, the average value is the same as the transmitted value

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Multipath Fading



- Consider a Two-Path Constant Channel with BPSK:

$$r(t) = \underbrace{s(t)}_{\text{direct path}} + \underbrace{s(t-\tau_o)}_{\text{multipath}}$$

$$= b(t)\cos(\omega_c t) + b(t-\tau_o)\cos(\omega_c [t-\tau_o])$$

- Now, let us assume that $\tau_o \ll T_b$

$$r(t) \approx b(t)\{\cos(\omega_c t) + \cos(\omega_c t - \theta)\}$$

- Or in terms of the complex baseband

$$\tilde{r}(t) = \tilde{s}(t)[1 + e^{j\theta}]$$

$$= b(t)[1 + e^{j\theta}]$$

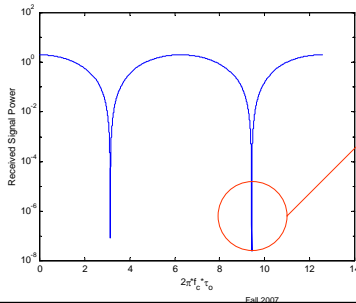
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Example: a typical multipath delay is 100ns. If the bit rate is 50kbps $T_b = 20\mu s$. A typical transmit frequency is 2GHz (one cycle is 500ps). Thus, we can ignore the delay with respect to the symbol, but *not* with respect to the carrier.

Envelope Fading



- The envelope of the received signal $|r(t)|$ clearly depends on the values of f_c and τ_o .



Signal power reduced drastically due to destructive phase combining at certain values of $f_c \tau_o$. (This is termed a "fade")

If receiver or transmitter is moving, τ_o is changing and the signal envelope is time-varying

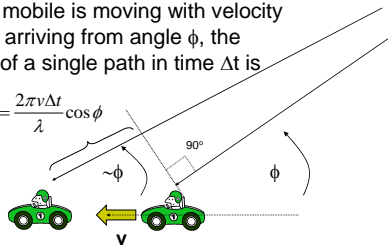
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Doppler Shift



- Assume that a mobile is moving with velocity v and a path is arriving from angle ϕ , the phase change of a single path in time Δt is

$$\Delta\theta = \frac{2\pi}{\lambda} d \cos\phi = \frac{2\pi v \Delta t}{\lambda} \cos\phi$$



Thus, the mobile experiences a shift in frequency called the *Doppler shift* on each multipath component

$$f_d = \frac{1}{2\pi} \frac{\Delta\theta}{\Delta t} = \frac{v}{\lambda} \cos\phi$$

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Time-Varying Fading



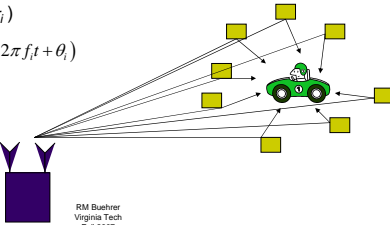
- Now consider a large number of multipath components which arrive at the receiver from 360°
- Further, if the receiver is moving each reflection experiences a Doppler shift of $f_i = \frac{v}{\lambda} \cos \Phi_i$, where λ is the wavelength of the carrier, v is the velocity of the receiver, and Φ_i is the angle of arrival of the path relative to the direction of the receiver (still assuming $T_b \gg \tau_i$)

$$r(t) \approx \sum_{i=1}^N b(t) \cos(\omega_c t + 2\pi f_i t + \theta_i)$$

In complex baseband:

$$\tilde{r}(t) \approx b(t) \frac{1}{\sqrt{N}} \sum_{i=1}^N e^{j(2\pi f_i t + \theta_i)}$$

time varying fading



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Rayleigh Fading Channels



- One multipath channel is what is termed a *Rayleigh fading* channel. This occurs in *mobile multipath* environments. In Rayleigh fading we have amplitude and phase distortion of the transmitted signal as well as attenuation and thermal noise:

$$\tilde{r}(t) = \tilde{\gamma}(t)\tilde{s}(t) + \tilde{n}(t)$$

$$\tilde{\gamma}(t) = A(t)e^{j\theta(t)}$$

Phase = uniform random variable

- where

Amplitude = Rayleigh random variable

$$\tilde{\gamma}(t) = \frac{1}{\sqrt{N}} \sum_{i=1}^N e^{j(2\pi f_i t + \theta_i)}$$

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Rayleigh Fading



- Further, the real and imaginary components of $\tilde{\gamma}(t)$ are complex Gaussian processes

$$\tilde{\gamma}(t) = a(t) + j b(t)$$

Gaussian distributed

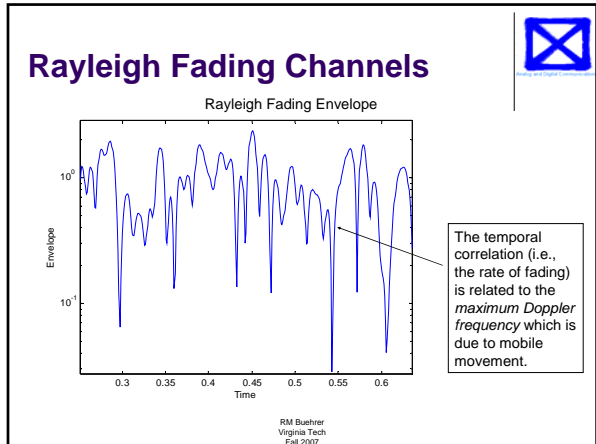
Gaussian distributed

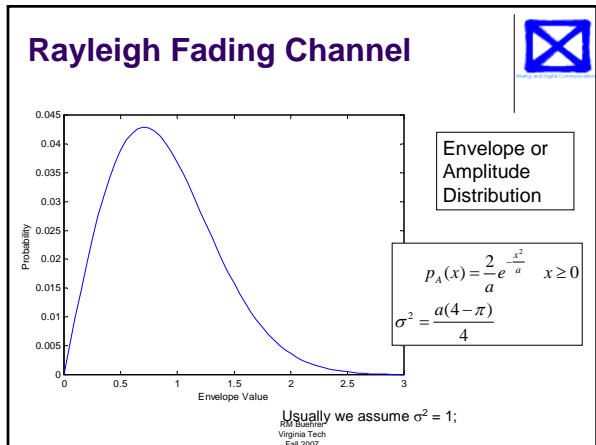
- The temporal correlation is related to the *maximum Doppler frequency*. This is in turn related to the mobile velocity.

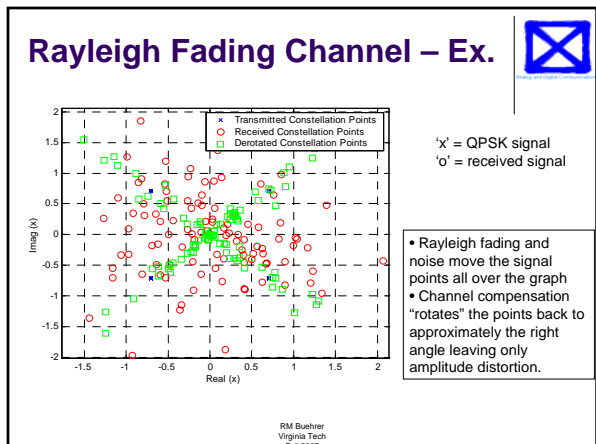
$$\max\{f_i\} = \max\left\{\frac{v}{\lambda} \cos \Phi_i\right\}$$

$$= \frac{v}{\lambda}$$

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Simulation



- We normally simulate a Rayleigh fading channel by using a summation of complex sinusoids:

$$\tilde{\gamma}(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^N e^{j(2\pi f_k t + \theta_k)}$$

- For large $N (>30)$ this will tend to a complex Gaussian random process.
- By letting $f_k = f_d \cos\left(\frac{2\pi k}{N}\right)$ where the maximum Doppler frequency is determined by the velocity of the mobile and wavelength of the carrier $f_d = \frac{v}{\lambda}$ we obtain the proper time behavior (correct Doppler spectrum). The phase term θ_k is a uniform random variable over $[0, 2\pi)$.

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Ricean Fading Channels



- Ricean fading occurs when there is a single direct line-of-sight component in addition to many multipath components
- In Ricean fading we also have amplitude and phase distortion of the transmitted signal as well as thermal noise, but now the amplitude is a Ricean random variable

$$\tilde{r}(t) = \tilde{\gamma}(t)\tilde{s}(t) + \tilde{n}(t)$$

$$\tilde{\gamma}(t) = A(t)e^{j\theta(t)}$$

Phase = uniform random variable

Amplitude = Ricean random variable

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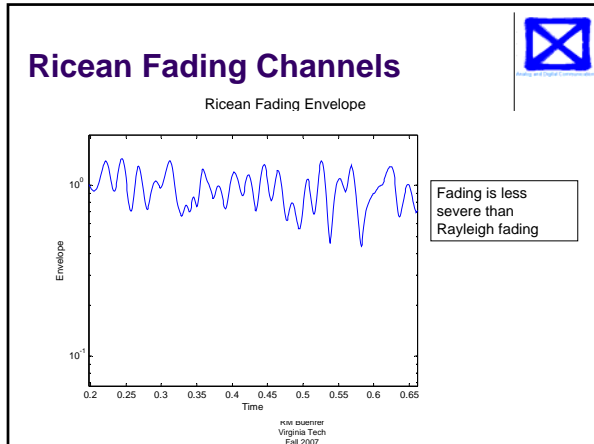
Ricean Fading – K factor

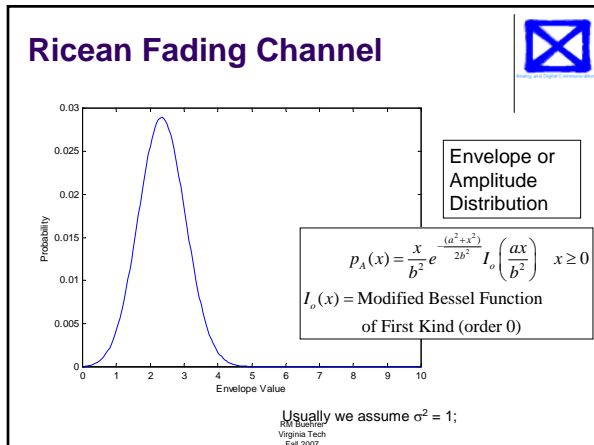


- Ricean channels are classified according to something called the K factor.
- K is the amount of power in the LOS component relative to the multipath terms
- The larger the value of K , the less fading
- $K=0$ is a Rayleigh fading channel
- $K = \infty$ is an AWGN channel (no fading)

$$\tilde{\gamma}(t) = \underbrace{\sqrt{\frac{K}{K+1}}}_{\text{LOS}} + \underbrace{\sqrt{\frac{1}{K+1}} \frac{1}{\sqrt{N}} \sum_{i=1}^N e^{j(2\pi f_i t + \theta_i)}}_{\text{multipath}}$$

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- ### Summary
- Channels distort the received signal as well as the add noise to the signal due to the receiver.
 - AWGN channels cause no distortion, but do add thermal noise.
 - Rayleigh fading channels add noise, but also introduce amplitude and phase distortion.
 - Ricean fading channels also distort the amplitude but to a lesser degree.
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