ECE4634
Digital Communications
Fall 2007

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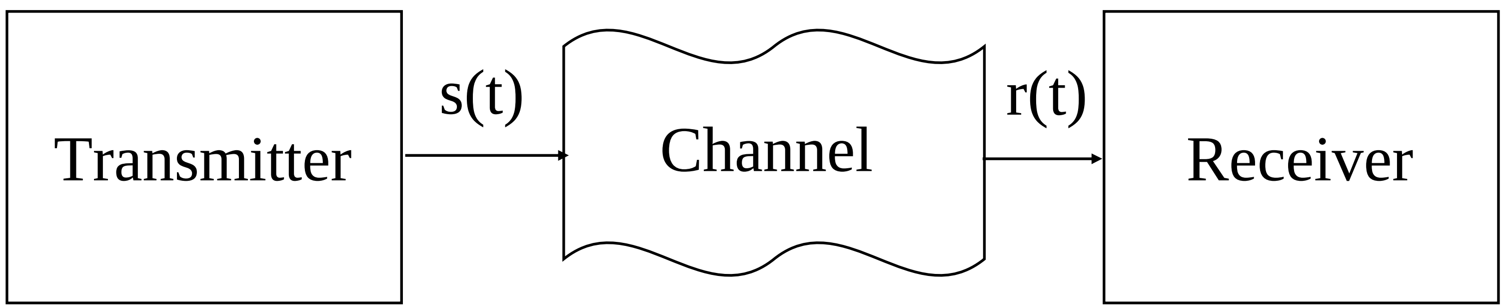
Lecture #31: Channels and Channel Models

Analog and Digital Communications

The Channel

- The channel is the medium through which the system communicates information
- In general $r(t) = s(t) \otimes h(t,\tau) + \nu(t)$ where $h(t,\tau)$ is the time-varying channel impulse response, $\nu(t)$ is thermal noise and $\otimes$ is the convolution operation.



- However, we will only consider $h(t,\tau)=\gamma(t)\,\delta(\tau-\tau_o)$ with $\tau_o=0$ where $\delta(\tau)$ is an impulse. Thus,

$$r(t) = \gamma(t)s(t) + n(t)$$

The Channel

- The received signal, channel, and bandpass noise can all be represented in complex baseband form as

$$
\begin{aligned}
r(t) &= \mathrm{Re}\left\{\tilde{r}(t)e^{j\omega_c t}\right\} \\
\gamma(t) &= \mathrm{Re}\left\{\tilde{\gamma}(t)e^{j\omega_c t}\right\} \\
s(t) &= \mathrm{Re}\left\{\tilde{s}(t)e^{j\omega_c t}\right\} \\
n(t) &= \mathrm{Re}\left\{\tilde{n}(t)e^{j\omega_c t}\right\}
\end{aligned}
$$

The Channel

- Thus, the simplest channel is one which causes only *multiplicative distortion*
- Normally, we deal with the complex baseband of the transmit and receive signals. Fortunately, the above formula will apply to both RF and complex baseband. Thus, in complex baseband we have

$$\tilde{r}(t) = \tilde{\gamma}(t)\tilde{s}(t) + \tilde{n}(t)$$

Note: In the complex baseband there is a factor of ½ which we commonly ignore

Thermal Noise

- All objects with physical temperature T_p greater than 0 Kelvin generate electrical noise.
- This noise power is given by

$$P_n = kT_n B$$

Botzman's Constant 1.38 x 10^{-23} J/K

Noise temperature of device

Bandwidth of device

$$kT_n = \text{Noise Spectral Density}$$

AWGN

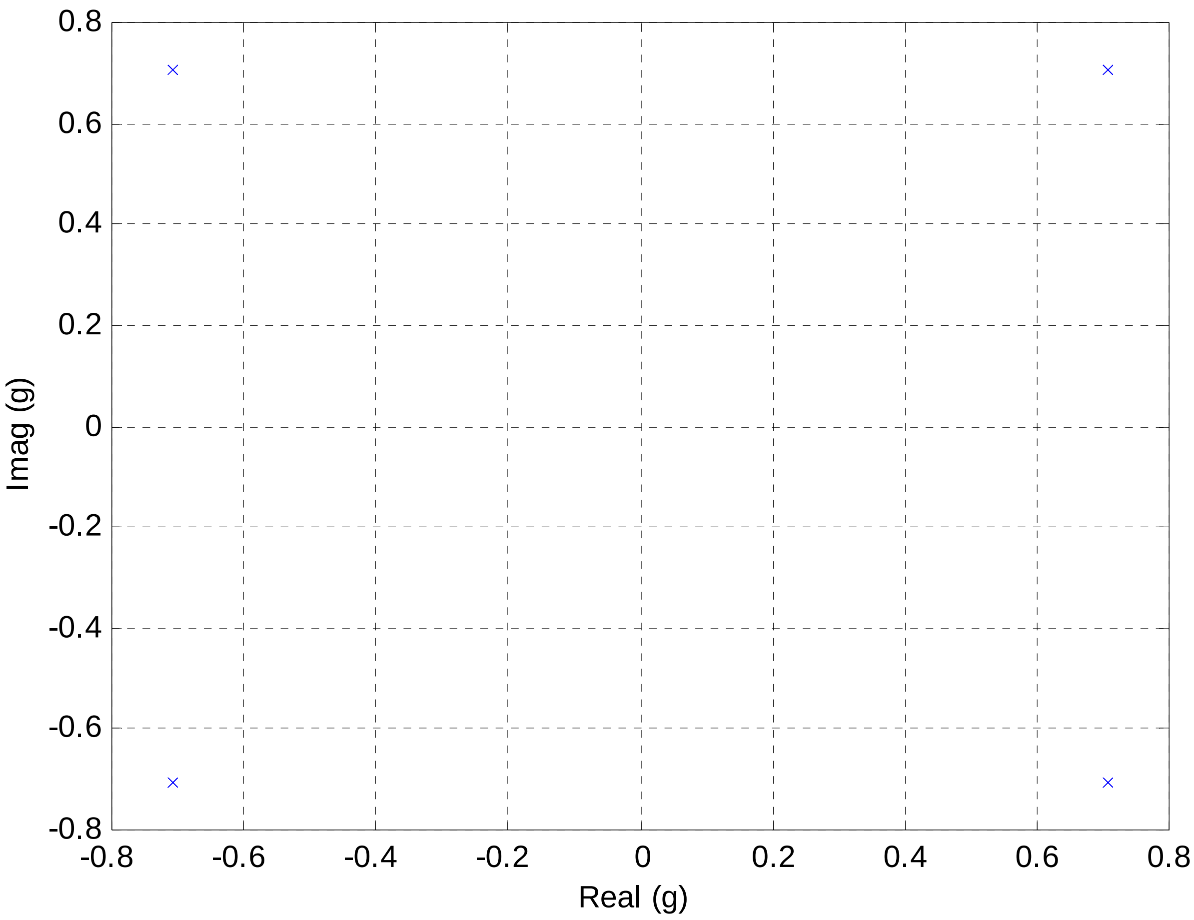
- Noise in the system contributes an additional voltage on top of the received signal. Thus, it is **additive**.
- The noise is uncorrelated from one sample to the next (i.e., $R_x(\tau)=\delta(\tau)$). A delta function in the correlation function means that the PSD is a constant, thus it is **white.**
- Noise in the system originates from components in the receiver. Since there are many discrete components all contributing some small amount, the sum tends to a Gaussian process (Central Limit Theorem). Thus, noise samples have a *Gaussian* probability distribution.
- **A**dditive **W**hite **G**aussian **N**oise (AWGN)

AWGN Channels

- The term ‘AWGN Channel’ is something of a misnomer.
- The channel doesn’t actually add noise. It attenuates the signal to such a degree that the internal noise of the receiver is comparable to that of the received signal.
- Usually, we assume that the received signal has normalized average received power, while the noise has some power σ^2 where σ is the standard deviation of the thermal noise.
- In “AWGN Channels” we assume that the only distortion to the signal is the AWGN. That is $\tilde{\gamma}(t) = 1$

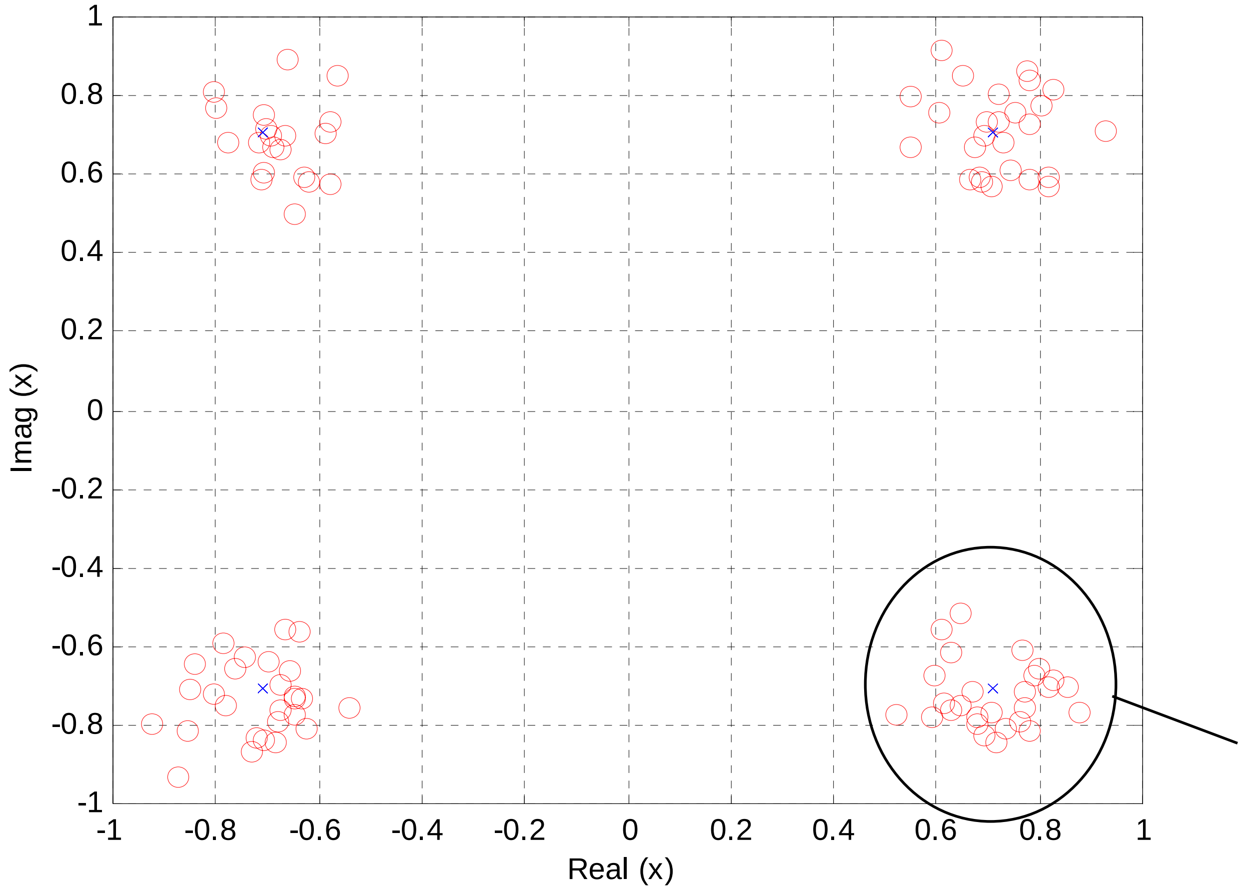
$$\tilde{x}(t) = \tilde{s}(t) + \tilde{n}(t)$$

AWGN Channels – Example



g(t) = QPSK signal

AWGN Channels – Example



'x' = QPSK signal
'o' = received signal

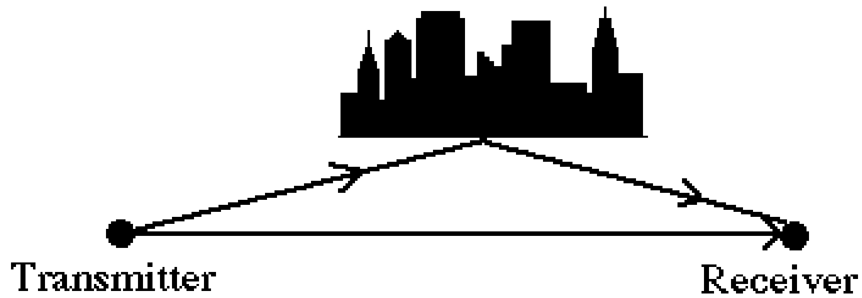
Thermal Noise moves points.
Since noise has zero mean, the average value is the same as the transmitted value

Multipath Fading

- Consider a Two-Path Constant Channel with BPSK:

$$r(t) = \underbrace{s(t)}_{\text{direct path}} + \underbrace{s(t-\tau_o)}_{\text{multipath}}$$

$$= b(t)\cos(\omega_c t) + b(t-\tau_o)\cos(\omega_c[t-\tau_o])$$



- Now, let us assume that $\tau_o << T_b$

$$r(t) \approx b(t)\{\cos(\omega_c t) + \cos(\omega_c t - \theta)\}$$

- Or in terms of the complex baseband

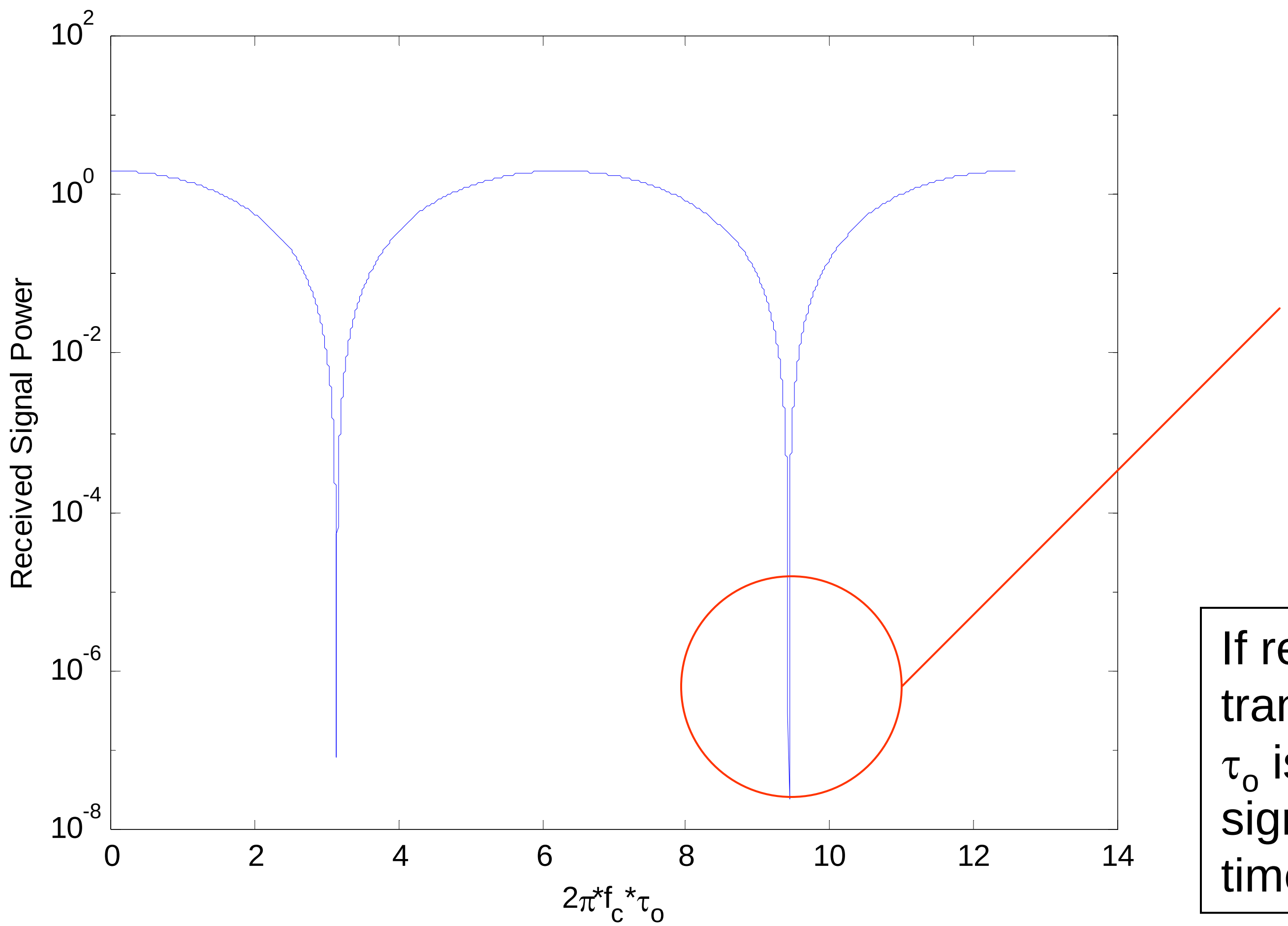
$$\tilde{r}(t) = \tilde{s}(t)\left[1 + e^{j\theta_o}\right]$$

$$= b(t)\left[1 + e^{j\theta_o}\right]$$

Example: a typical multipath delay is 100ns. If the bit rate is 50kbps T_b = 20μs. A typical transmit frequency is 2GHz (one cycle is 500ps). Thus, we can ignore the delay with respect to the symbol, but *not* with respect to the carrier.

Envelope Fading

- The envelope of the received signal |r(t)| clearly depends on the values of f_c and τ_o.



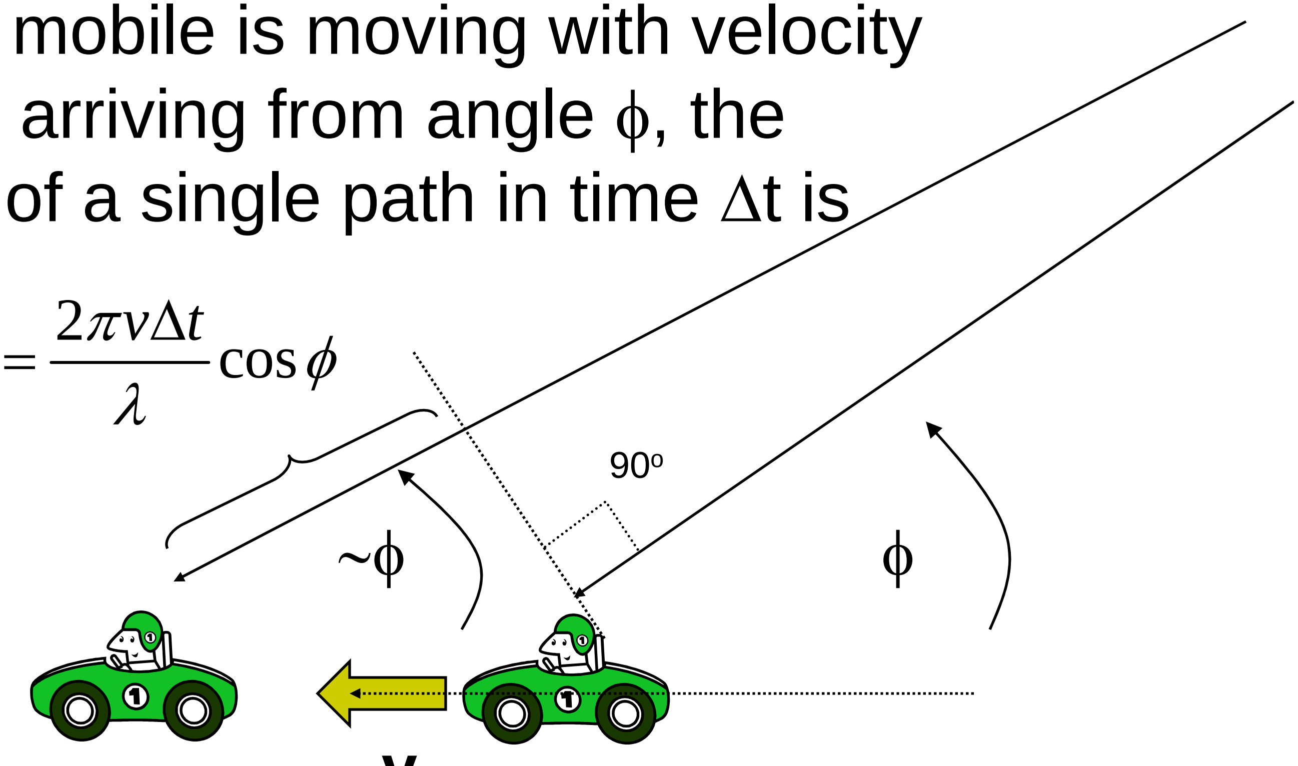
Signal power reduced drastically due to destructive phase combining at certain values of $f_c\tau_o$ (This is termed a “fade”)

If receiver or transmitter is moving, τ_o is changing and the signal envelope is time-varying

Doppler Shift

- Assume that a mobile is moving with velocity v and a path is arriving from angle ϕ, the phase change of a single path in time Δt is

$$\Delta\theta = \frac{2\pi}{\lambda} d\cos\phi = \frac{2\pi v \Delta t}{\lambda}\cos\phi$$



Thus, the mobile experiences a shift in frequency called the *Doppler shift* on each multipath component

$$f_d = \frac{1}{2\pi}\frac{\Delta\theta}{\Delta t} = \frac{v}{\lambda}\cos\phi$$

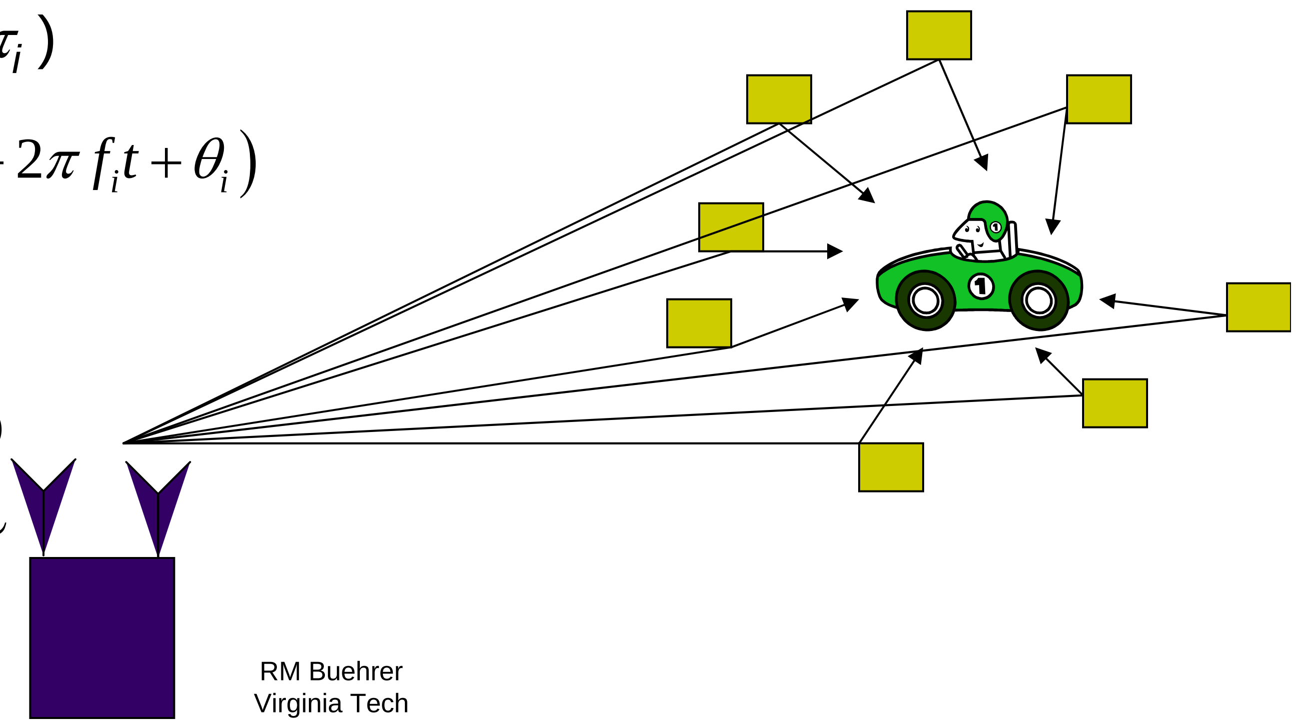
Time-Varying Fading

- Now consider a large number of multipath components which arrive at the receiver from 360°
- Further, if the receiver is moving each reflection experiences a Doppler shift of $f_i = \frac{v}{\lambda}\cos\Phi_i$ where λ is the wavelength of the carrier, v is the velocity of the receiver, and Φ_i is the angle of arrival of the path relative to the direction of the receiver (still assuming $T_b >> \tau_i$)

$$r(t) \approx \sum_{i=1}^{N} b(t)\cos\left(\omega_c t + 2\pi f_i t + \theta_i\right)$$

In complex baseband:

$$\tilde{r}(t) \approx b(t)\underbrace{\frac{1}{\sqrt{N}}\sum_{i=1}^{N} e^{j(2\pi f_i t + \theta_i)}}_{\text{time varying fading}}$$



Rayleigh Fading Channels

Analog and Digital Communications

- One mulitpath channel is what is termed a *Rayleigh fading* channel. This occurs in *mobile multipath* environments. In Rayleigh fading we have amplitude and phase distortion of the transmitted signal as well as attenuation and thermal noise:

$$\tilde{r}(t) = \tilde{\gamma}(t)\tilde{s}(t) + \tilde{n}(t)$$

$$\tilde{\gamma}(t) = A(t)e^{j\theta(t)}$$

Phase = uniform random variable

- where

Amplitude = Rayleigh random variable

$$\tilde{\gamma}(t) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N} e^{j(2\pi f_i t + \theta_i)}$$

Rayleigh Fading

- Further, the real and imaginary components of $\tilde{\gamma}(t)$ are complex Gaussian processes

$$\tilde{\gamma}(t) = a(t) + j\,b(t)$$

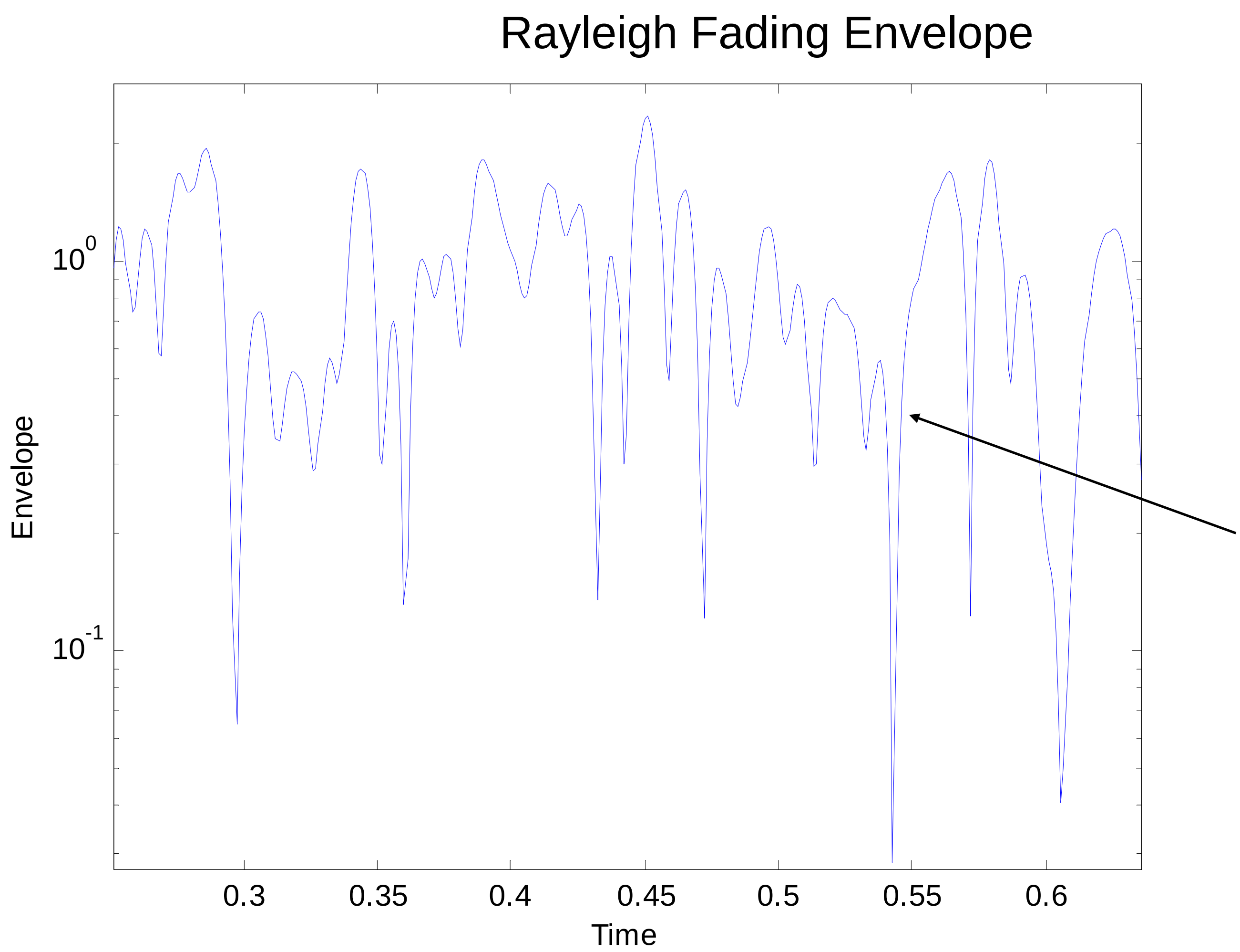
$a(t)$ ← Gaussian distributed

$b(t)$ ← Gaussian distributed

- The temporal correlation is related to the *maximum Doppler frequency.* This is in turn related to the mobile velocity.

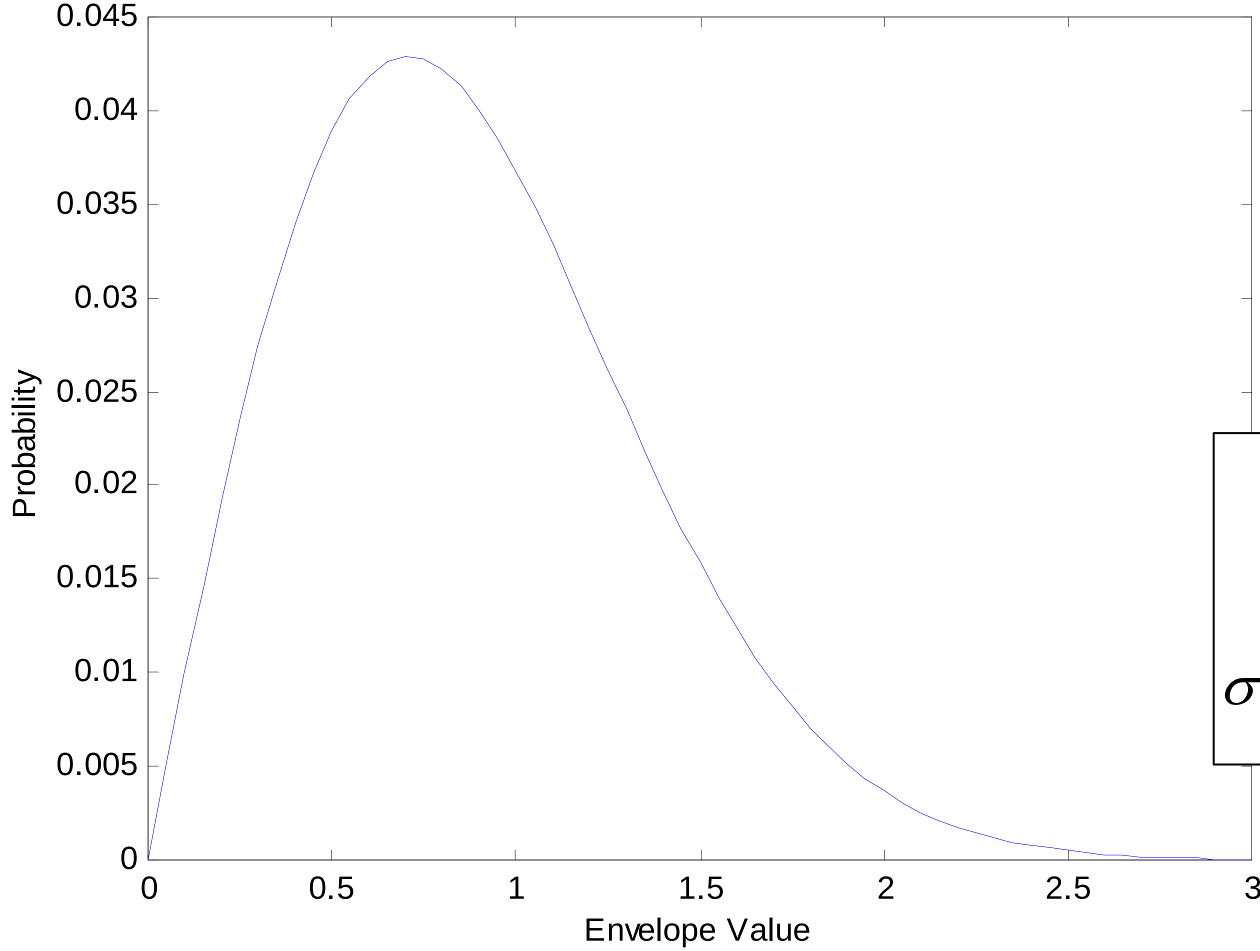
$$\max\{f_i\} = \max\left\{\frac{v}{\lambda}\cos\Phi_i\right\} = \frac{v}{\lambda}$$

Rayleigh Fading Channels



The temporal correlation (i.e., the rate of fading) is related to the *maximum Doppler frequency* which is due to mobile movement.

Rayleigh Fading Channel



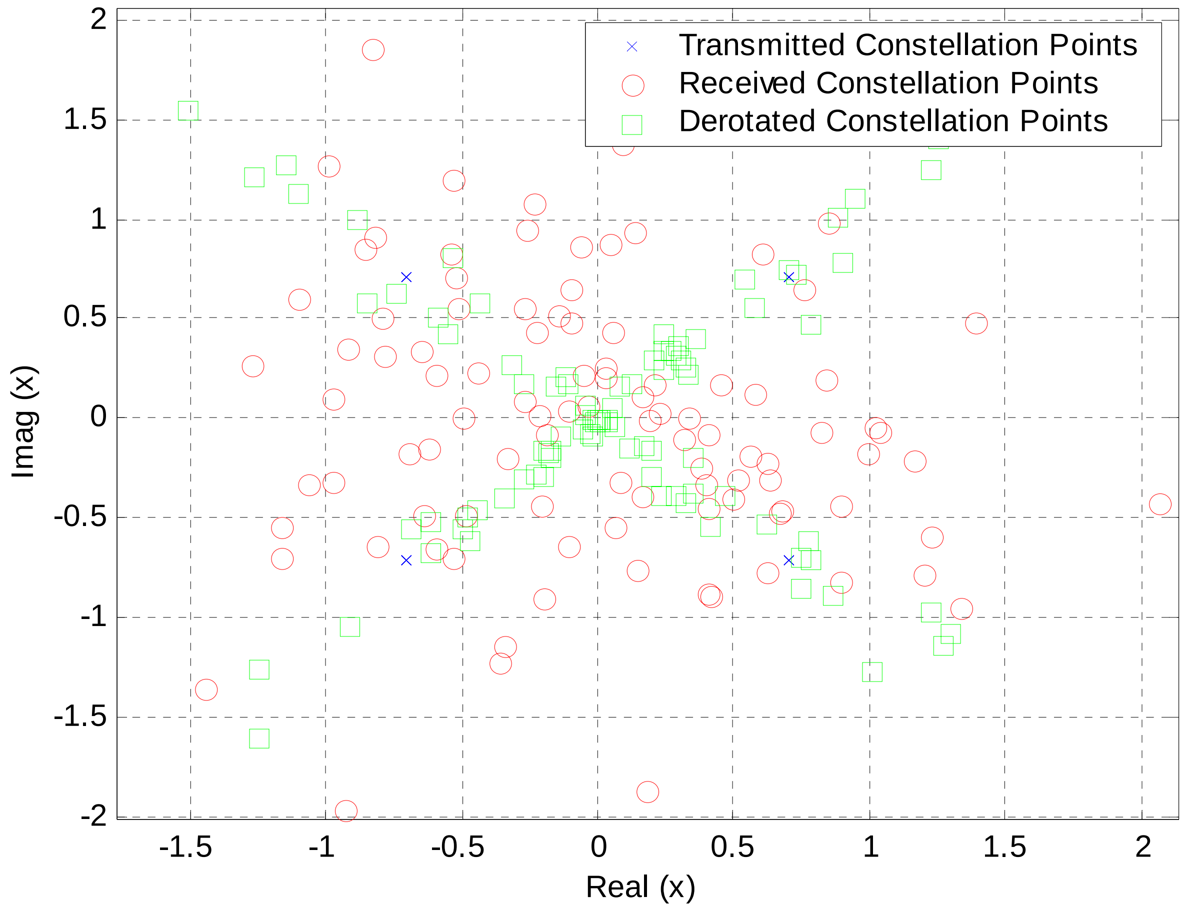
Envelope or Amplitude Distribution

$$p_A(x) = \frac{2}{a} e^{-\frac{x^2}{a}} \qquad x \geq 0$$

$$\sigma^2 = \frac{a(4-\pi)}{4}$$

Usually we assume $\sigma^2 = 1$;

Rayleigh Fading Channel – Ex.



'x' = QPSK signal
'o' = received signal

- Rayleigh fading and noise move the signal points all over the graph
- Channel compensation "rotates" the points back to approximately the right angle leaving only amplitude distortion.

Simulation

- We normally simulate a Rayleigh fading channel by using a summation of complex sinusoids:

$$\tilde{\gamma}(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} e^{j(2\pi f_k t + \theta_k)}$$

- For large *N* (>30) this will tend to a complex Gaussian random process.
- By letting $f_k = f_d \cos\left(\frac{2\pi}{N} k\right)$ where the maximum Doppler frequency is determined by the velocity of the mobile and and wavelength of the carrier $f_d = \frac{v}{\lambda}$ we obtain the proper time behavior (correct Doppler spectrum). The phase term θ_k is a uniform random variable over $[0,2\pi)$.

Ricean Fading Channels

- Ricean fading occurs when there is a single direct line-of-sight component in addition to many multipath components
- In Ricean fading we also have amplitude and phase distortion of the transmitted signal as well as thermal noise, but now the amplitude is a Ricean random variable

$$\tilde{r}(t) = \tilde{\gamma}(t)\tilde{s}(t) + \tilde{n}(t)$$

$$\tilde{\gamma}(t) = A(t)e^{j\theta(t)}$$

Phase = uniform random variable

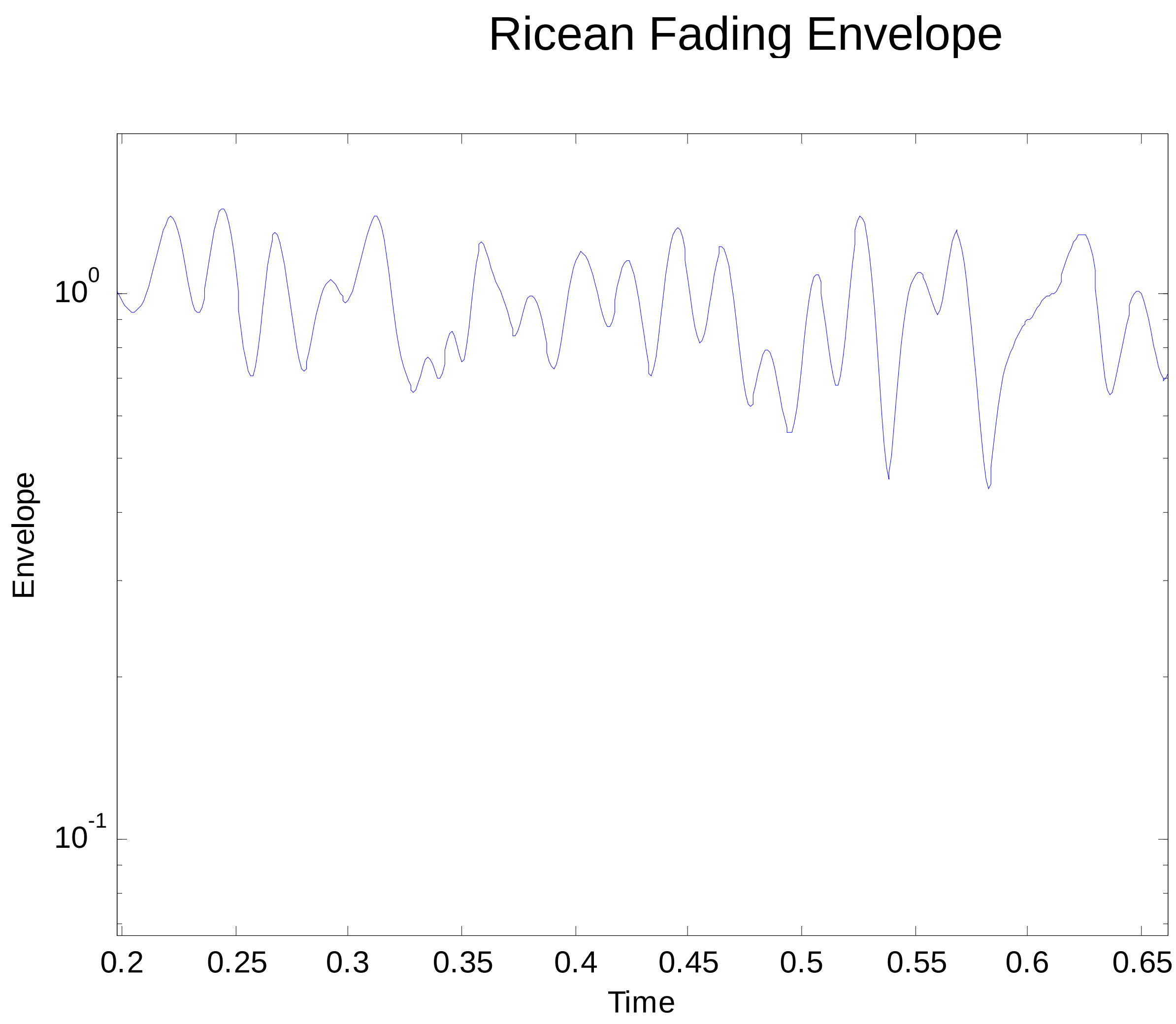
Amplitude = Ricean random variable

Ricean Fading – *K* factor

- Ricean channels are classified according to something called the *K* factor.
- *K* is the amount of power in the LOS component relative to the multipath terms
- The large the value of *K*, the less fading
- *K=0* is a Rayleigh fading channel
- *K* = infinity is an AWGN channel (no fading)

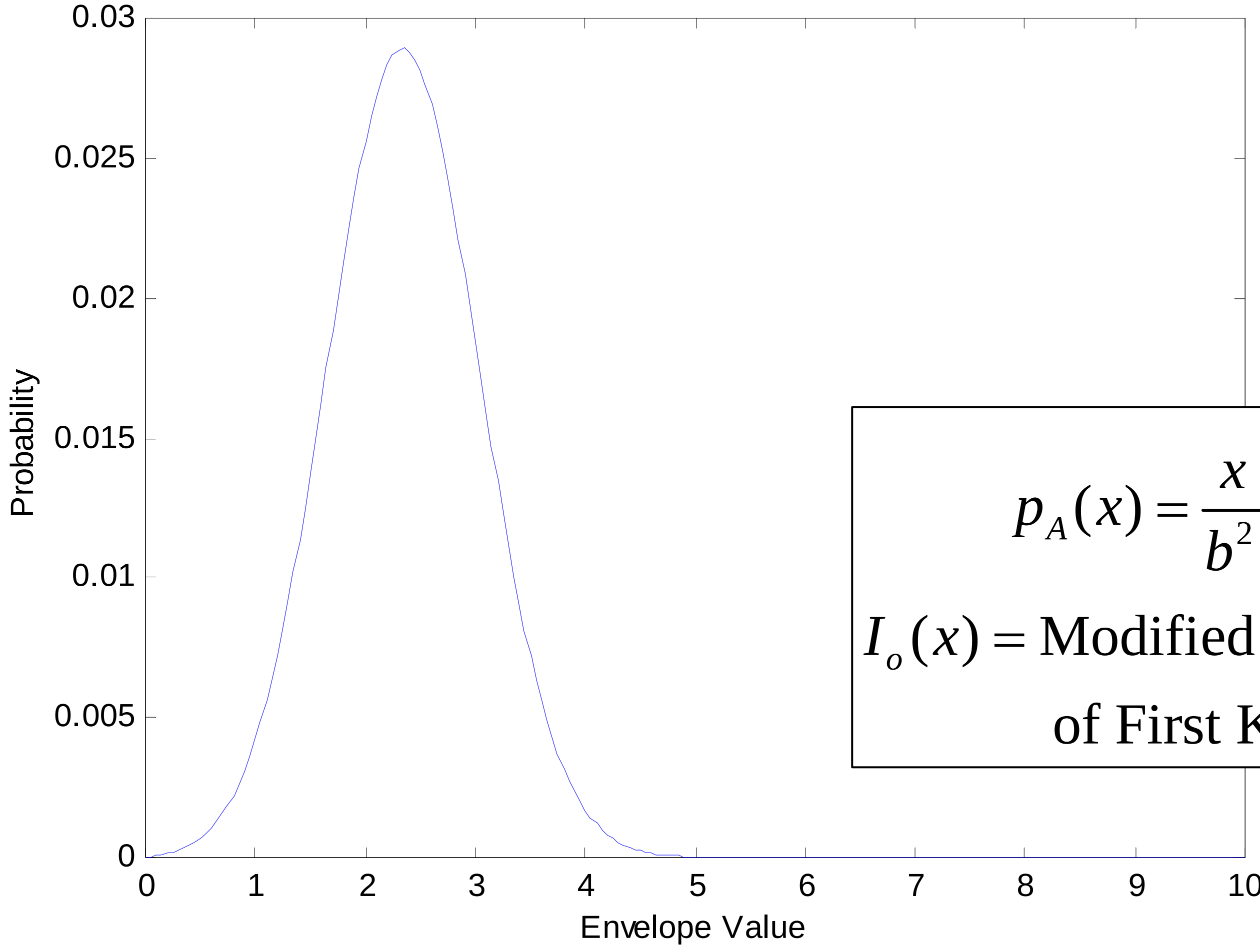
$$\tilde{\gamma}(t) = \underbrace{\sqrt{\frac{K}{K+1}}}_{\text{LOS}} + \underbrace{\sqrt{\frac{1}{K+1}}\frac{1}{\sqrt{N}}\sum_{i=1}^{N} e^{j(2\pi f_i t+\theta_i)}}_{\text{multipath}}$$

Ricean Fading Channels



Fading is less severe than Rayleigh fading

Ricean Fading Channel



Envelope or Amplitude Distribution

$$p_A(x) = \frac{x}{b^2} e^{-\frac{(a^2+x^2)}{2b^2}} I_o\left(\frac{ax}{b^2}\right) \quad x \geq 0$$

$$I_o(x) = \text{Modified Bessel Function of First Kind (order 0)}$$

Usually we assume $\sigma^2 = 1$;

Summary

- Channels distort the received signal as well as the add noise to the signal due to the receiver.
- AWGN channels cause no distortion, but do add thermal noise.
- Rayleigh fading channels add noise, but also introduce amplitude and phase distortion.
- Ricean fading channels also distort the amplitude but to a lesser degree.