ECE4634
Digital Communications
Fall 2007

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Lecture #32: BERs for

Non-coherent Demodulation

and Rayleigh Fading

Overview

- In the previous classes we examined the bit error rate performance of PSK, ASK, and FSK using a *coherent* receiver in the presence of AWGN
- Now we would like to look at other important cases including
  - *Non-coherent* receivers in AWGN
  - Coherent receivers in Rayleigh fading

Probability of Error for Noncoherent Reception

- Differentially coherent receivers can detect changes in phase from one symbol to the next
  - Can be used with BPSK, MPSK, and QAM signal constellations
- Noncoherent receivers can only detect the energy of signals in certain frequency bands
  - Used for binary and M-ary FSK receivers

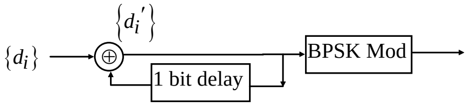
Differential Encoding of Data

- Consider BPSK modulation:

$$0 \Rightarrow s(t) = \sqrt{2P}\cos(2\pi f_c t)\Big|_0^T$$

$$1 \Rightarrow s(t) = \sqrt{2P}\cos(2\pi f_c t + \pi)\Big|_0^T = -\sqrt{2P}\cos(2\pi f_c t)\Big|_0^T$$

- Differential Encoding Transforms Raw Data:



$\{d_i'\}$

$\{d_i\}$

$\oplus$

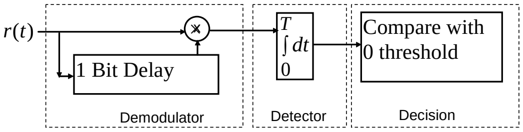
BPSK Mod

1 bit delay

- Rule: “Change the phase if input data is a 1”

Differential Reception

- We can think of differential reception as using the noisy version of the received signal as its phase reference for the correlation operation:



$r(t)$

$\times$

1 Bit Delay

$\int_0^T dt$

Compare with
0 threshold

Demodulator

Detector

Decision

Probability of Error for Differential Receivers

- Standard BPSK: $P_e = P_b = Q\left\{\sqrt{\frac{2E_b}{N_0}}\right\}$
  - symbol error probability and bit error probability the same for binary case
- Standard QPSK

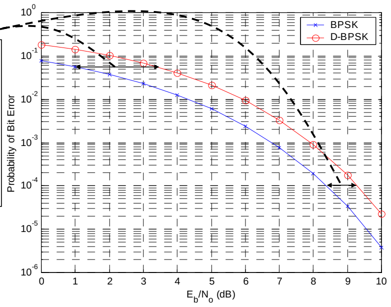
$$P_b = Q\left\{\sqrt{\frac{2E_b}{N_0}}\right\}$$

- DBPSK with Differential Demodulation:

$$P_e = P_b = \frac{1}{2}e^{-E_b/N_0}$$

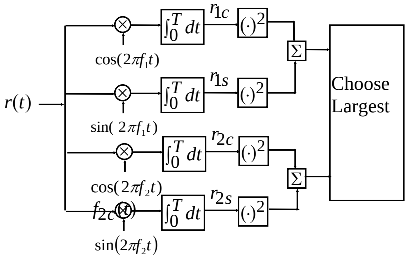
Performance Comparison of BPSK and DPSK

DPSK gives about 1.5-2 dB loss from coherent reception.



The difference is more pronounced at lower BER values: 2.5dB at 5% BER; 1dB at 10^{-4} BER

Structure of Optimum Noncoherent Receiver for Binary FSK



Probability of Error for Noncoherent FSK

- Probability of error for Binary FSK with coherent reception:

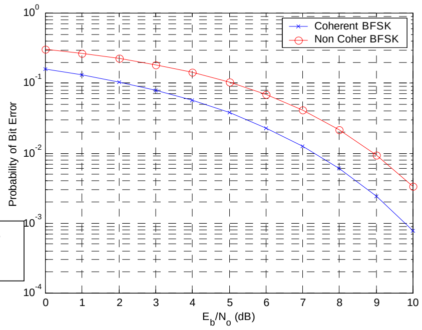
$$P_e = Q\left\{\sqrt{\frac{E_b}{N_0}}\right\}$$

- Probability of error for Binary FSK with noncoherent reception:

$$P_e = \frac{1}{2}e^{-E_b/2N_0}$$

- This results in about a 2 dB loss from coherent reception

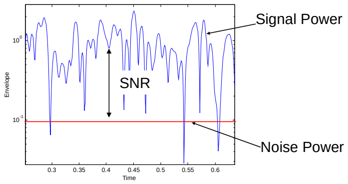
Comparison of Coherent and Noncoherent Binary FSK



~ 1.5-3 dB loss from coherent reception

Impact of Rayleigh Fading

- In a Rayleigh fading scenario the desired signal power fluctuates due to movement while the noise power remains constant
- As a result the SNR (and thus the performance) fluctuates



Performance of BPSK in Rayleigh Fading

- Consider the received baseband signal

$$r(t) = \gamma s(t) + n(t)$$

where γ is a constant complex Gaussian value

- If we follow through the same derivation as in standard BPSK (keeping γ as a constant) we arrive at a probability of error given as

$$P_e = Q\left(\sqrt{\frac{2E_b|\gamma|^2}{N_o}}\right)$$

Performance depends on the amplitude of γ. When in a deep fade $|\gamma|$ can be very small and thus performance very bad.

Performance

- For a given channel realization the performance is then

$$P_e = Q\left(\sqrt{\frac{2E_b|\gamma|^2}{N_o}}\right)$$
$$= Q\left(\sqrt{2\beta}\right)$$

$$\boxed{\beta = \frac{E_b}{N_o}|\gamma|^2}$$

- However, we desire the performance averaged over all channel realizations. Thus, we require the distribution of the random signal-to-noise ratio β
- Since the γ is a complex Gaussian random variable, β is a central Chi-Square random variable with two degrees of freedom. The underlying GRV has variance equal to the average signal-to-noise ratio of the channel $\bar{\beta}$

$$p(\beta) = \frac{1}{\bar{\beta}}e^{-\frac{\beta}{\bar{\beta}}} \quad \beta \geq 0$$

Performance (cont.)

- Substituting

$$P_e = \int_0^\infty p(\beta)Q\left(\sqrt{2\beta}\right)d\beta$$
$$= \int_0^\infty \frac{1}{\bar{\beta}}e^{-\frac{\beta}{\bar{\beta}}}Q\left(\sqrt{2\beta}\right)d\beta$$
$$= \frac{1}{2}\left(1-\sqrt{\frac{\bar{\beta}}{1+\bar{\beta}}}\right)$$

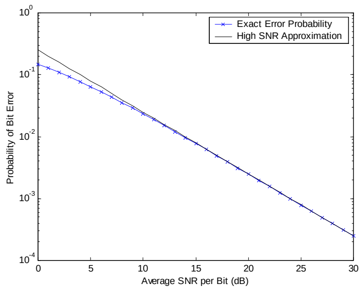
- In high SNR situations $\left(\bar{\beta} >> 1\right)$

$$P_e = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\beta}}{1+\bar{\beta}}}\right) = \frac{1}{2}\left(1-\frac{1}{\sqrt{1+\frac{1}{\bar{\beta}}}}\right)$$
$$\approx \frac{1}{2}\left(1-\left(1-\frac{1}{2\bar{\beta}}\right)\right)$$
$$= \frac{1}{4\bar{\beta}}$$

Recall the binomial series

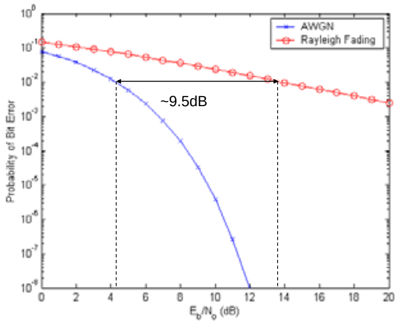
$$(1+x)^\alpha = 1+\alpha x+\frac{\alpha(\alpha-1)x^2}{2!}+\ldots$$
$$\frac{\alpha(\alpha-1)(\alpha-2)x^3}{3!}+\ldots$$

BPSK Performance in Rayleigh Fading



- Rayleigh fading with perfect channel knowledge (we needed to know γ)
- This assumes a *coherent receiver* for BPSK
- Note that the performance is significantly worse than simple AWGN

Rayleigh Fading vs. AWGN

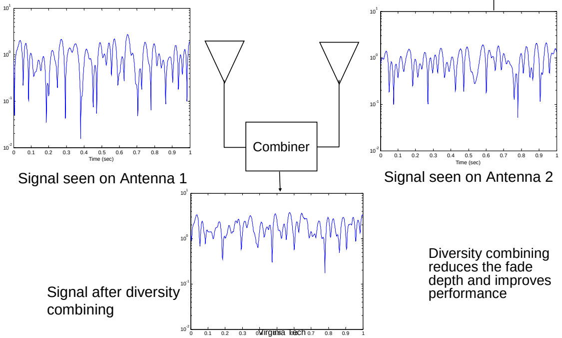


- Rayleigh fading results in extremely large performance degradation
- This is particularly true at low BER values

Combating Rayleigh Fading

- The standard method of improving performance in the presence of Rayleigh fading is *diversity*
- Diversity involves having multiple *independent* copies of the received signal
  - If one copy is in a fade, hopefully the other is not
- Diversity can be accomplished using
  - Space – multiple antennas [most common]
  - Time – error correction coding (requires the channel to change in time)
  - Frequency - sending over multiple frequency bands

Two Antenna Receive Diversity



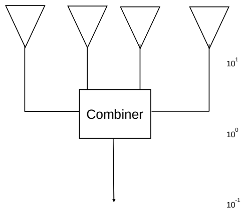
Signal seen on Antenna 1

Signal seen on Antenna 2

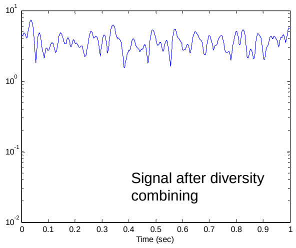
Signal after diversity combining

Diversity combining reduces the fade depth and improves performance

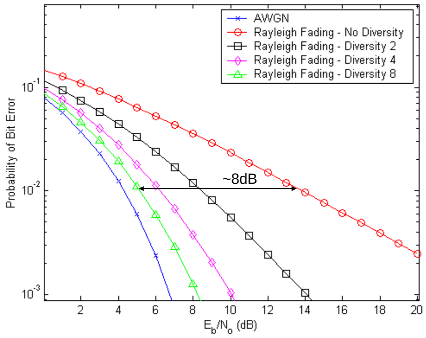
Four Antenna Receive Diversity



- Combining more antennas (diversity branches) decreases the fading severity



Diversity Improvement



•Large improvements achievable

•Law of diminishing returns applies

Summary

- We have investigated the performance non-coherent demodulation in an AWGN channel and the performance of coherent demodulation in a Rayleigh fading channel
- Non-coherent reception of BPSK and BFSK results in ~2dB loss over coherent reception
- Similar differences exist for non-coherent *M*-ary receivers
- Rayleigh fading results in substantial degradation in BER performance
- Diversity is the standard means of improving performance in Rayleigh fading
- Diversity obeys the law of diminishing returns and can be attained through space (i.e., antennas), frequency or time.