ECE4634
Digital Communications
Fall 2007

Instructor: R. Michael Buehrer

Lecture #6: Analog Pulse Modulation

Overview

- We are primarily interested in studying *digital* baseband and bandpass communication systems
  - Baseband systems typically modulate a pulse train
  - Bandpass systems typically modulate a sinusoid
- Before we study digital baseband systems, we first study baseband communication systems which modulate pulses with an analog message.
- These are termed *discrete baseband* communication systems and are a first step to digital systems
- What to read – Sections 5.2-5.3

Lecture Objectives

- To review the difference between baseband and bandpass
- To review the various definitions of bandwidth
- To introduce three discrete baseband pulse modulation schemes
  - Pulse amplitude modulation
  - Pulse width modulation
  - Pulse position modulation

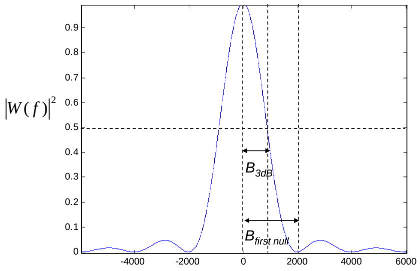
Baseband vs. Bandpass

- A baseband signal *w(t)* with bandwidth *B* is a signal for which $W(f)$ is non-negligible for $|f| \leq B$ and for which $W(f) \approx 0$ for $|f| > B$
- A bandpass signal *w(t)* with bandwidth $B = f_2 - f_1$ is a signal for which $W(f)$ in non-negligible for $0 < f_1 \leq |f| \leq f_2$ and for which $W(f) \approx 0$ otherwise
- There are many definitions of what $W(f) \approx 0$ means

Definitions of Bandwidth for Baseband Signals

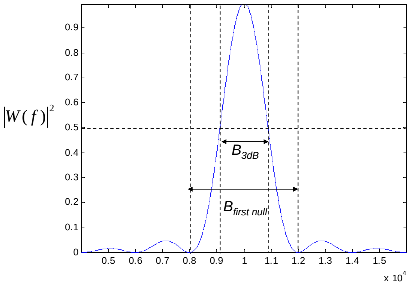
- Bandwidth is a term used to describe a *positive* frequency range over which the signal has significant content. There are various definitions for bandwidth including:
  - Absolute Bandwidth (*B*)
    - Defined as ***B*** where $W(f) = 0 \quad f > B$
  - *3*-dB Bandwidth (half-power bandwidth - (B_{3dB}))
    - Defined as ***B*** where $|W(f)|^2 < \frac{|W(f)|^2_{\max}}{2} \quad f > B$
  - *X-dB Bandwidth*
    - *Defined as **B** where* $20\log_{10}(|W(f)|) < \left[20\log_{10}(|W(f)|_{\max}) - X\right] \quad f > B$
  - First Null Bandwidth ($B_{first\ null}$)
    - For baseband systems this is equal to the frequency of the first null in the spectrum

Bandwidth - Baseband



B_{3dB} = 900Hz
$B_{first\ null}$ = 2kHz

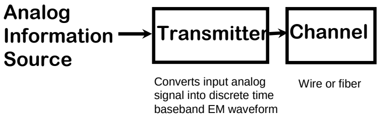
Bandwidth - Bandpass



B_{3dB} = 1.8kHz
$B_{first\ null}$ = 4kHz

Discrete Baseband Communications System

- Analog pulse modulation
- This system takes an analog message signal and converts it to a message which is discrete in time (but continuous in amplitude or some other parameter) and uses the values to modulate a pulse stream
- Examples: Pulse Amplitude Modulation, Pulse Width Modulation, Pulse Position Modulation



Analog Information Source → **Transmitter** → **Channel**

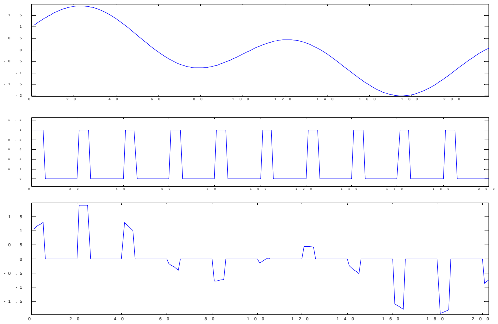
Converts input analog signal into discrete time baseband EM waveform

Wire or fiber

Pulse Amplitude Modulation

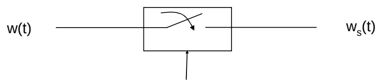
- Pulse Amplitude Modulation (PAM) is a term used to describe the conversion of analog signals to a pulse signal where the amplitudes of the pulses are related to the waveform values.
- Two general types of PAM:
  - PAM with natural sampling (gating)
  - PAM with instantaneous sampling (flat-top)
    - This is more useful for Pulse Code Modulation which we will discuss later

Natural Sampling



PAM: Natural Sampling

Natural Sampling is simply gating:



w(t)

$w_s(t)$

Clock at rate f_s
Gate closes
for τ sec.

In time

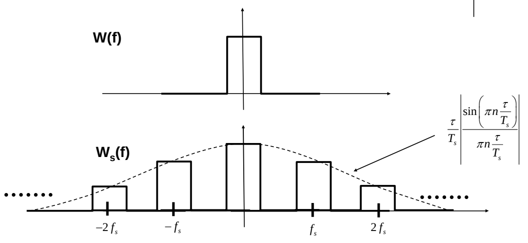
$$w_s(t) = w(t)s(t) = w(t)\sum_{k=-\infty}^{\infty}\Pi\left(\frac{t-kT_s}{\tau}\right)$$

In frequency

$$W_s(f) = \mathbb{F}\{w_s(t)\} = \frac{\tau}{T_s}\sum_{n=-\infty}^{\infty}\frac{\sin\left(\pi n\frac{\tau}{T_s}\right)}{\pi n\frac{\tau}{T_s}}W(f-nf_s)$$

* - See appendix for proof

Natural Sampling



$$\frac{\tau}{T_s}\left|\frac{\sin\left(\pi n\frac{\tau}{T_s}\right)}{\pi n\frac{\tau}{T_s}}\right|$$

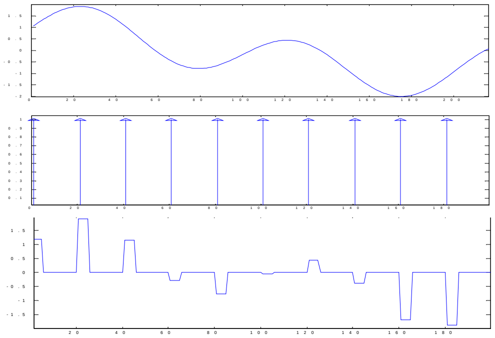
Spectrum goes to zero when $n\,\tau / T_s$ is an integer.

What is the relationship with “impulse sampling?

Natural Sampling

- As the width of the pulse decreases (τ/T_s or duty cycle) we approach impulse sampling and the spectrum approaches simple replication of the original spectrum
  - For smaller values of τ/T_s larger values of n are required for $n\tau/T_s$ to be an integer
- The original signal can be retrieved with a simple low-pass filter provided $f_s > 2B$.

PAM: Flat-Top Sampling



PAM: Flat-Top Sampling

This is the type of PAM presented in the text.

Flat-top Sampling is instantaneous sampling which modulates a pulse train:

In time

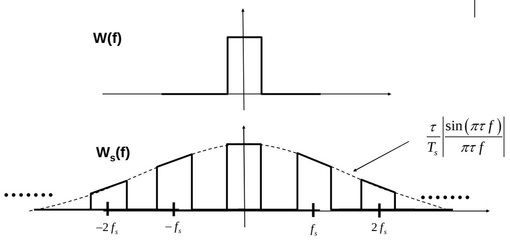
$$\begin{aligned} w_s(t) &= w(t)s(t) \\ &= \sum_{k=-\infty}^{\infty} w(kT_s)h(t-kT_s) \\ &= \sum_{k=-\infty}^{\infty} w(kT_s)\Pi\left(\frac{t-kT_s}{\tau}\right) \end{aligned}$$

In frequency

$$\begin{aligned} W_s(f) &= \mathbb{F}\left\{w_s(t)\right\} \\ &= \frac{\tau}{T_s}\frac{\sin\left(\pi\tau f\right)}{\pi\tau f}\sum_{n=-\infty}^{\infty} W(f-nf_s) \end{aligned}$$

* - See appendix for proof

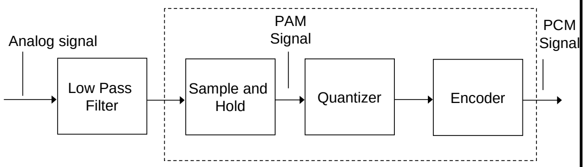
Flat-Top Sampling



Flat-Top Sampling

- Note that due to the flat-top pulses, the spectrum of the sampled signal is distorted.
- The narrower the pulse width, the less distortion.
- The original signal may be obtained by using a low-pass filter with a characteristic which inverts the distortion.

Baseband Digital Transmitter



Analog signal

Low Pass Filter

PAM Signal

Sample and Hold

Quantizer

Encoder

PCM Signal

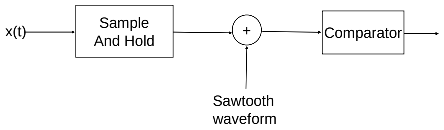
Flat-top PAM is the first step in creating a PCM signal (i.e., full analog-to-digital conversion)

PAM: Summary

- The transmission of PAM requires much more bandwidth than the original signal due to the narrow pulses used.
- The noise performance is equal to or worse than using analog transmission.
- PAM is good for time-multiplexing multiple signals onto a single channel
- PAM is an intermediate step in producing a Pulse-Code Modulated (PCM) signal
- It is this last point that makes PAM important to our class.

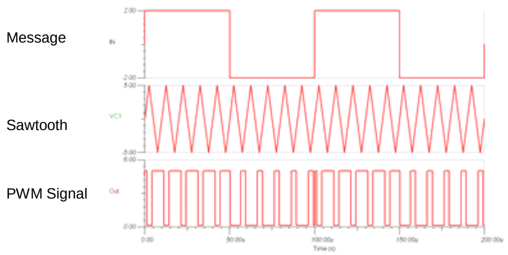
Pulse Width Modulation

- Pulses are sent periodically (i.e., pulse train) as in PAM
- Pulse width is varied based on message signal.
- Signal is discrete in time, but analog.
- Non-linear form of modulation


x(t)
Sample And Hold
+
Comparator
Sawtooth waveform

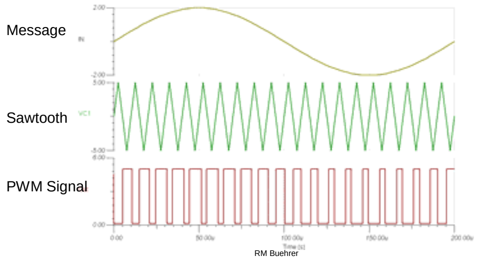
Pulse Width Modulation

- Example:



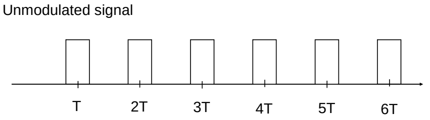
Pulse Width Modulation

- Example:



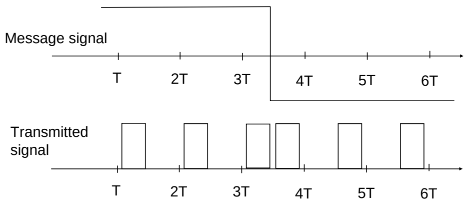
Pulse Position Modulation

- Pulse train no longer transmitted at regular intervals.
- Instead the pulse is transmitted slightly before or slightly after the scheduled symbol time.



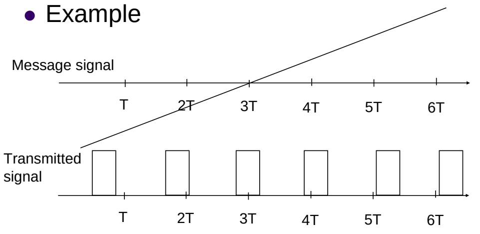
Pulse Position Modulation

- Example



Pulse Position Modulation

- Example



Summary

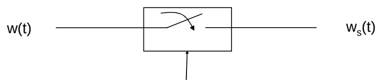
- In our study of baseband communication systems we start with *discrete* baseband communications
  - The message signal is continuous but is sampled to modulate a pulse train
  - These types of systems are not particularly common but are useful for instructive purposes – can be useful in multiplexing multiple data streams
- We will next study a more important form of baseband communications, *digital communications.*
  - In digital systems there are a finite number of possible messages.
  - PCM is the most common form, but there are others

Appendix

Proofs for Natural and Flat-top Sampling

PAM: Natural Sampling

Natural Sampling is simply gating:



Clock at rate f_s
Gate closes
for τ sec.

In time

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$$= w(t)\sum_{k=-\infty}^{\infty}\Pi\left(\frac{t-kT_s}{\tau}\right)$$

In frequency

$$W_s(f)=\mathbb{F}\{w_s(t)\}$$
$$=\frac{\tau}{T_s}\sum_{n=-\infty}^{\infty}\frac{\sin\left(\pi n\frac{\tau}{T_s}\right)}{\pi n\frac{\tau}{T_s}}W(f-nf_s)$$

RM Buehrer
Virginia Tech
Fall 20067

Proof

$$W_s(f)=W(f)*S(f)$$

Where *S(f)* is the Fourier Transform of the sampling function *s(t)*

$$s(t)=\sum_{n=-\infty}^{\infty}\Pi\left(\frac{t-nT_s}{\tau}\right)$$
$$=\sum_{n=-\infty}^{\infty}c_n e^{jn\omega_0 t}$$

Fourier Series Representation

and

$$c_n=\frac{\tau}{T_s}\frac{\sin\left(n\pi\frac{\tau}{T_s}\right)}{n\pi\frac{\tau}{T_s}}$$

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Proof (cont.)

$$S(f)=\Im\{s(t)\}$$
$$=\sum_{n=-\infty}^{\infty}c_n\delta(f-nf_s)$$

Returning to the sampled signal:

$$W_s(f)=W(f)*S(f)$$
$$=W(f)*\left\{\sum_{n=-\infty}^{\infty}c_n\delta(f-nf_s)\right\}$$
$$=\sum_{n=-\infty}^{\infty}c_n W(f-nf_s)$$
$$=\sum_{n=-\infty}^{\infty}\frac{\sin\left(n\pi\frac{\tau}{T_s}\right)}{n\pi}W(f-nf_s)$$

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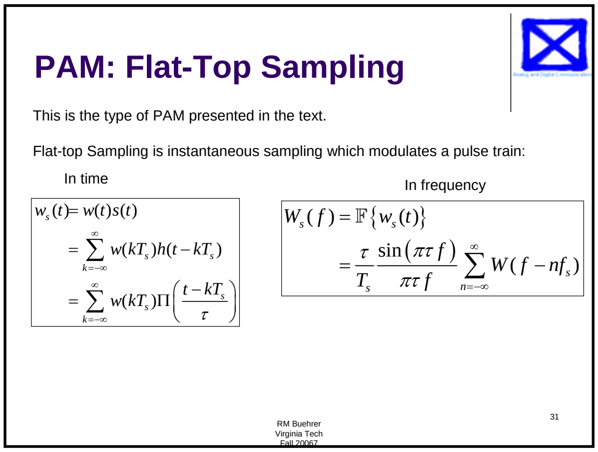

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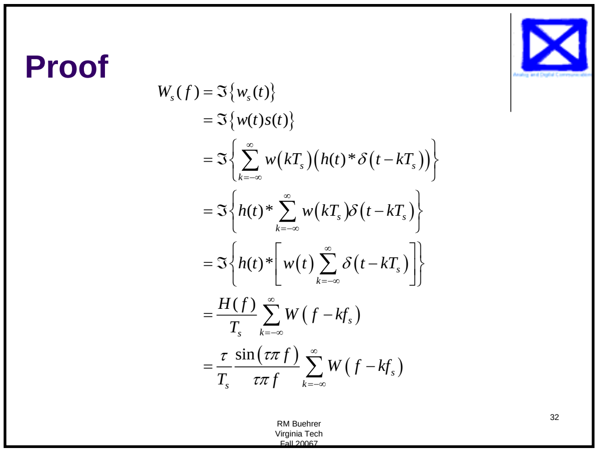
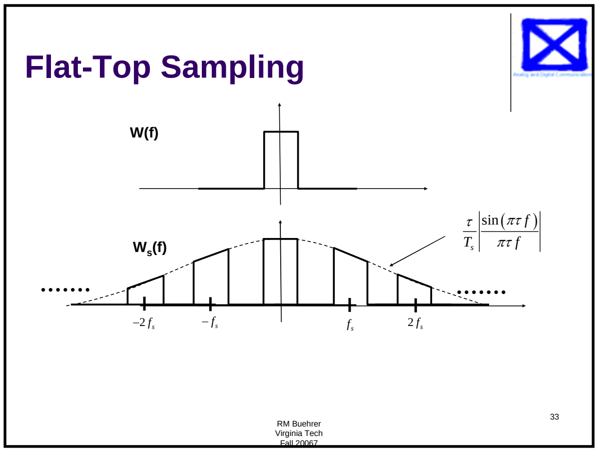


Proof

$$\begin{aligned} W_s(f) &= \Im\left\{w_s(t)\right\} \\ &= \Im\left\{w(t)s(t)\right\} \\ &= \Im\left\{\sum_{k=-\infty}^{\infty} w(kT_s)\left(h(t)*\delta(t-kT_s)\right)\right\} \\ &= \Im\left\{h(t)*\sum_{k=-\infty}^{\infty} w(kT_s)\delta(t-kT_s)\right\} \\ &= \Im\left\{h(t)*\left[w(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s)\right]\right\} \\ &= \frac{H(f)}{T_s}\sum_{k=-\infty}^{\infty} W(f-kf_s) \\ &= \frac{\tau}{T_s}\frac{\sin(\tau\pi f)}{\tau\pi f}\sum_{k=-\infty}^{\infty} W(f-kf_s) \end{aligned}$$



Flat-Top Sampling