

# ECE4634

# Digital Communications

# Fall 2007

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Lecture #6: Analog Pulse Modulation



Analog and Digital Communications



# Overview

- We are primarily interested in studying *digital* baseband and bandpass communication systems
  - Baseband systems typically modulate a pulse train
  - Bandpass systems typically modulate a sinusoid
- Before we study digital baseband systems, we first study baseband communication systems which modulate pulses with an analog message.
- These are termed *discrete baseband* communication systems and are a first step to digital systems
- What to read – Sections 5.2-5.3



# Lecture Objectives

- To review the difference between baseband and bandpass
- To review the various definitions of bandwidth
- To introduce three discrete baseband pulse modulation schemes
  - Pulse amplitude modulation
  - Pulse width modulation
  - Pulse position modulation

# Baseband vs. Bandpass



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- A baseband signal  $w(t)$  with bandwidth  $B$  is a signal for which  $W(f)$  is non-negligible for  $|f| \leq B$  and for which  $W(f) \approx 0$  for  $|f| > B$
- A bandpass signal  $w(t)$  with bandwidth  $B = f_2 - f_1$  is a signal for which  $W(f)$  is non-negligible for  $0 < f_1 \leq |f| \leq f_2$  and for which  $W(f) \approx 0$  otherwise
- There are many definitions of what  $W(f) \approx 0$  means

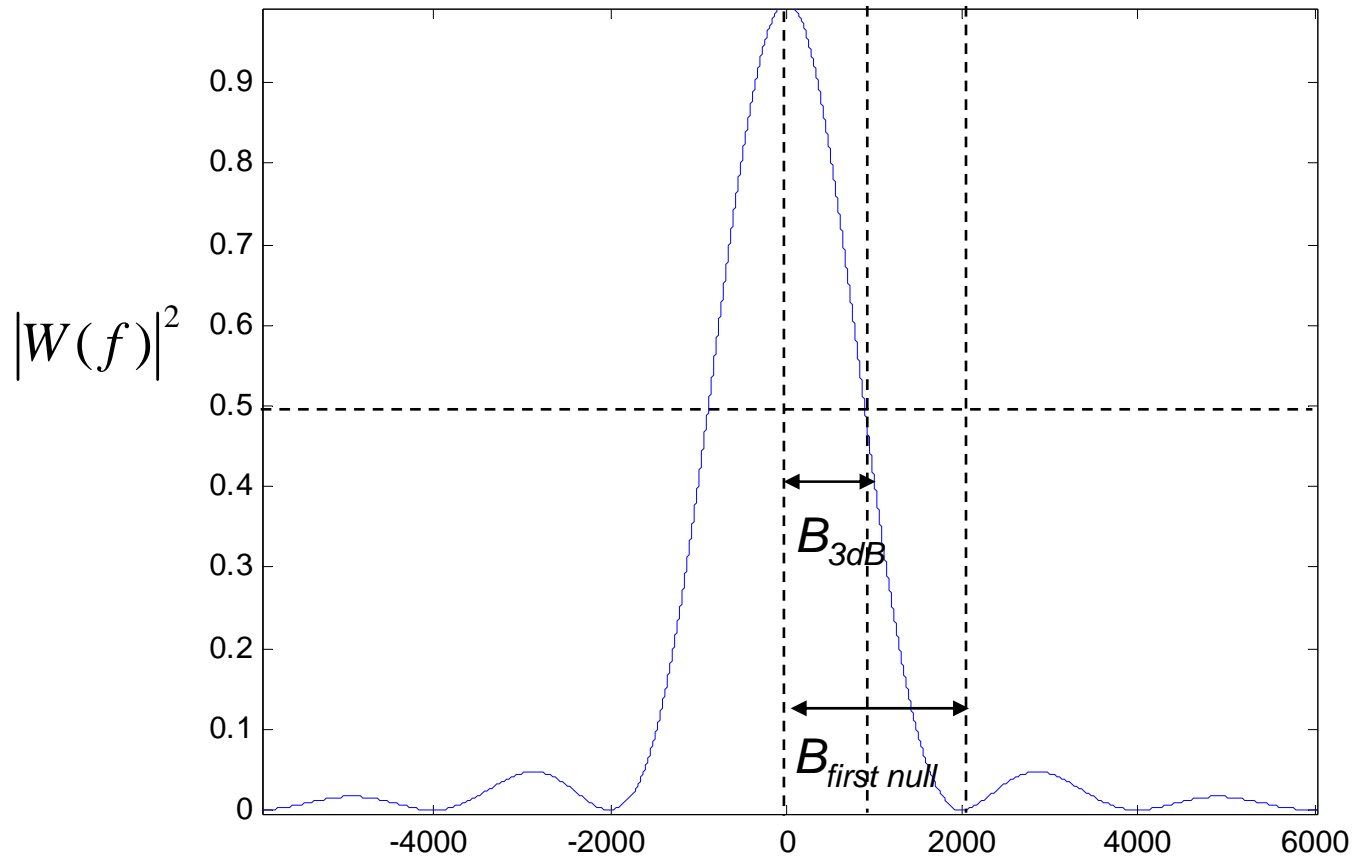
# Definitions of Bandwidth for Baseband Signals



- Bandwidth is a term used to describe a *positive* frequency range over which the signal has significant content. There are various definitions for bandwidth including:
  - Absolute Bandwidth ( $B$ )
    - Defined as  $B$  where  $W(f) = 0 \quad f > B$
  - 3-dB Bandwidth (half-power bandwidth - ( $B_{3dB}$ ))
    - Defined as  $B$  where  $|W(f)|^2 < \frac{|W(f)|_{\max}^2}{2} \quad f > B$
  - $X$ -dB Bandwidth
    - Defined as  $B$  where  $20 \log_{10} (|W(f)|) < \left[ 20 \log_{10} (|W(f)|_{\max}) - X \right] \quad f > B$
  - First Null Bandwidth ( $B_{\text{first null}}$ )
    - For baseband systems this is equal to the frequency of the first null in the spectrum



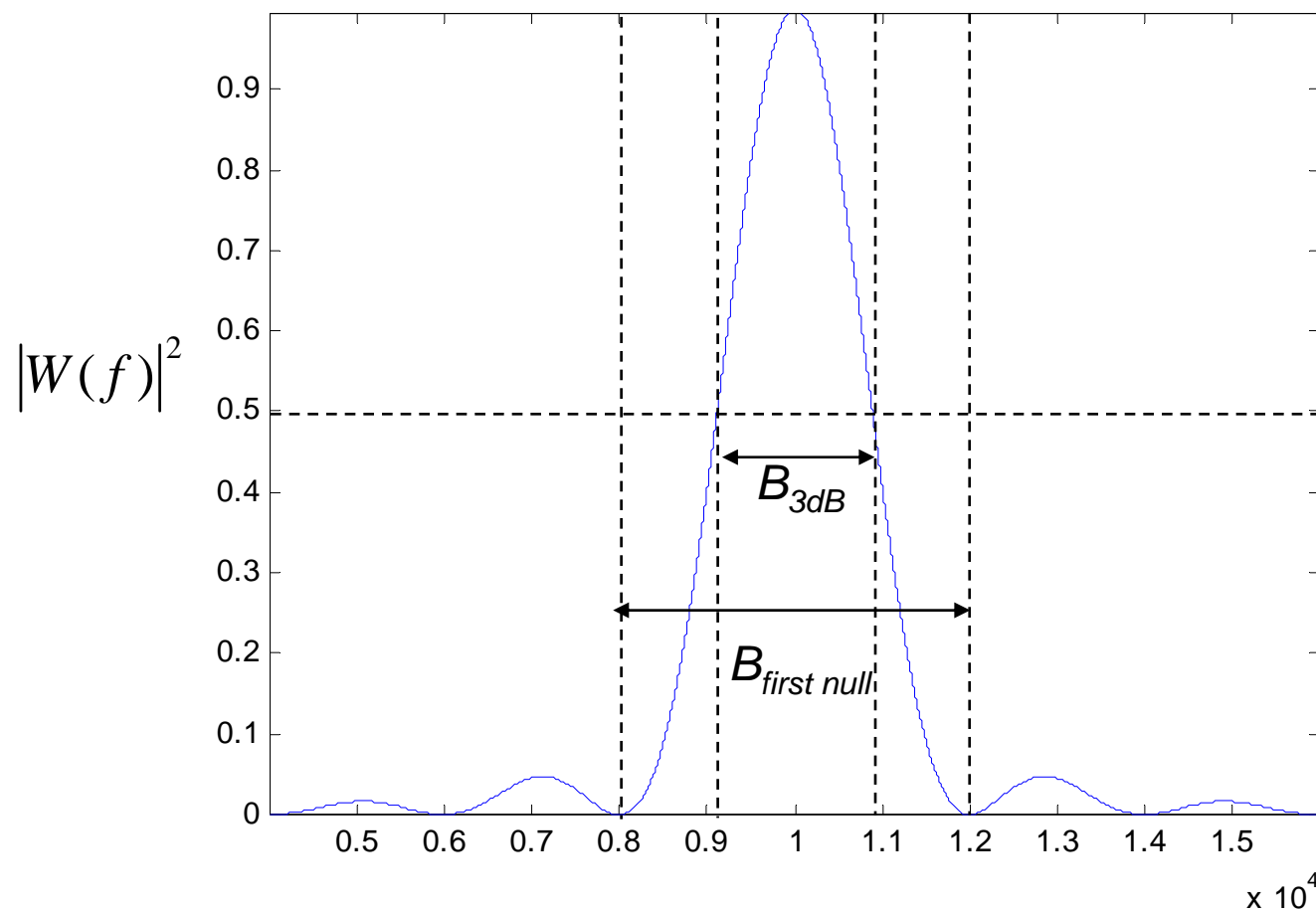
# Bandwidth - Baseband



$B_{3dB} = 900\text{Hz}$   
 $B_{first\ null} = 2\text{kHz}$



# Bandwidth - Bandpass



$$B_{3dB} = 1.8\text{kHz}$$

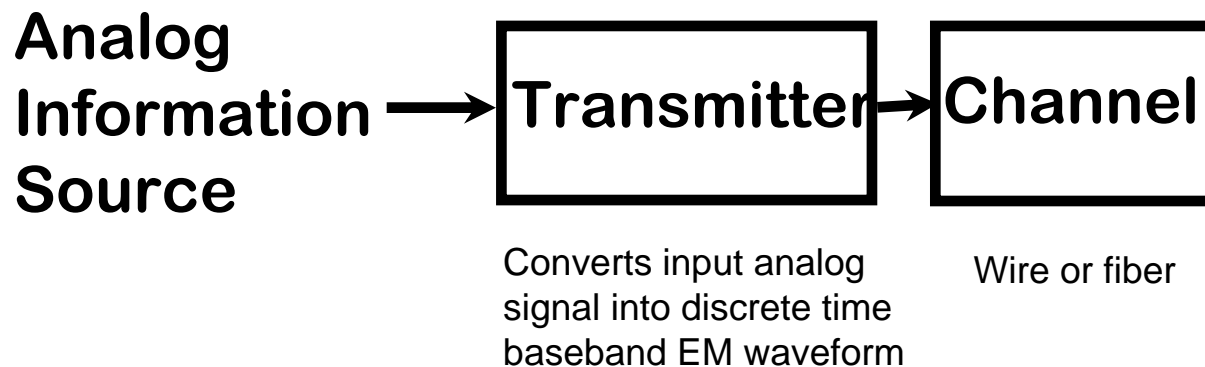
$$B_{first\ null} = 4\text{kHz}$$

# Discrete Baseband Communications System



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- Analog pulse modulation
- This system takes an analog message signal and converts it to a message which is discrete in time (but continuous in amplitude or some other parameter) and uses the values to modulate a pulse stream
- Examples: Pulse Amplitude Modulation, Pulse Width Modulation, Pulse Position Modulation





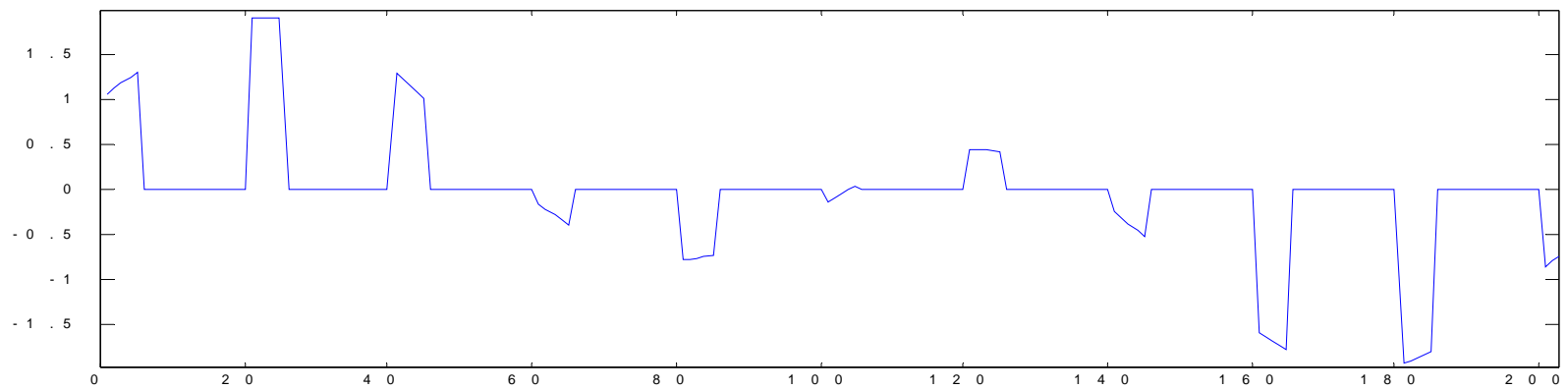
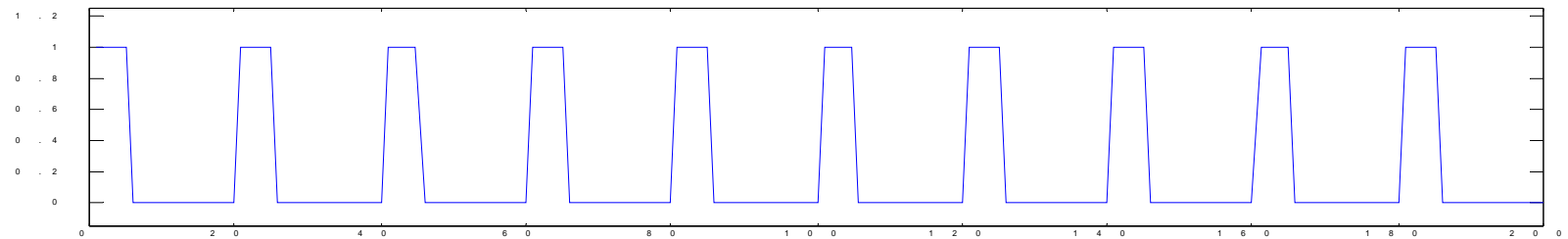
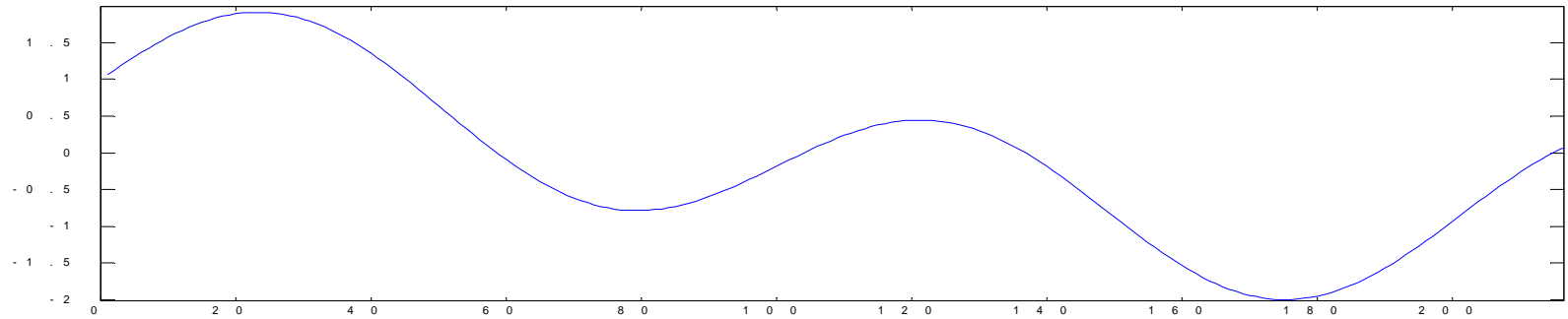
# Pulse Amplitude Modulation

- Pulse Amplitude Modulation (PAM) is a term used to describe the conversion of analog signals to a pulse signal where the amplitudes of the pulses are related to the waveform values.
- Two general types of PAM:
  - PAM with natural sampling (gating)
  - PAM with instantaneous sampling (flat-top)
    - This is more useful for Pulse Code Modulation which we will discuss later

# Natural Sampling

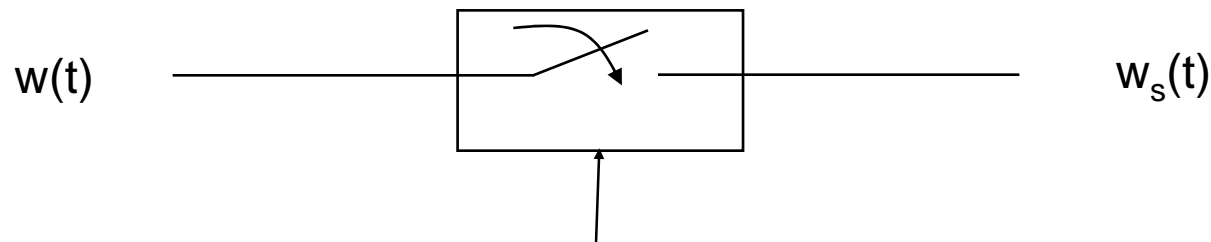


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# PAM: Natural Sampling

Natural Sampling is simply gating:



Clock at rate  $f_s$   
Gate closes  
for  $\tau$  sec.

In time

In frequency

$$w_s(t) = w(t)s(t)$$

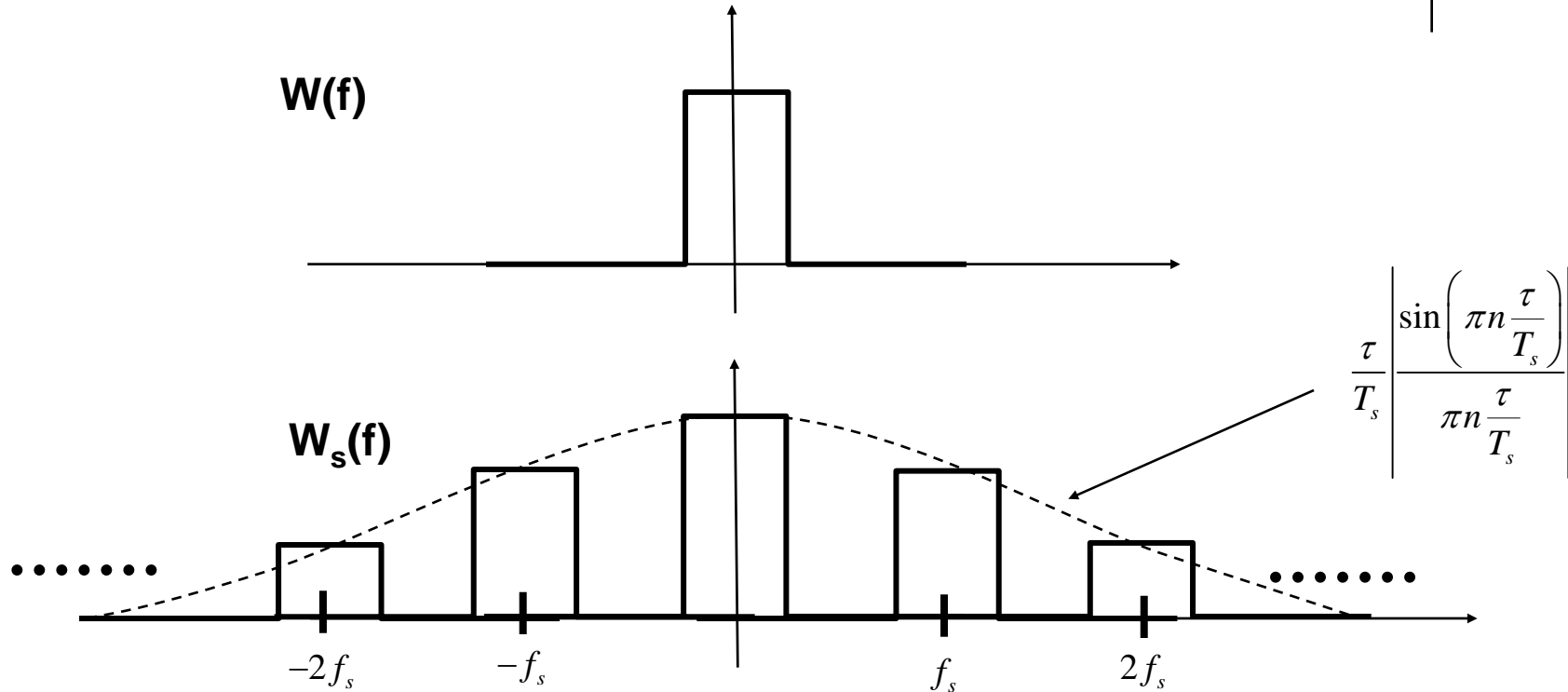
$$= w(t) \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t - kT_s}{\tau}\right)$$

$$W_s(f) = \mathbb{F}\{w_s(t)\}$$

$$= \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \frac{\sin\left(\pi n \frac{\tau}{T_s}\right)}{\pi n \frac{\tau}{T_s}} W(f - nf_s)$$



# Natural Sampling



Spectrum goes to zero when  $n \tau / T_s$  is an integer.

What is the relationship with “impulse sampling?”<sup>12</sup>



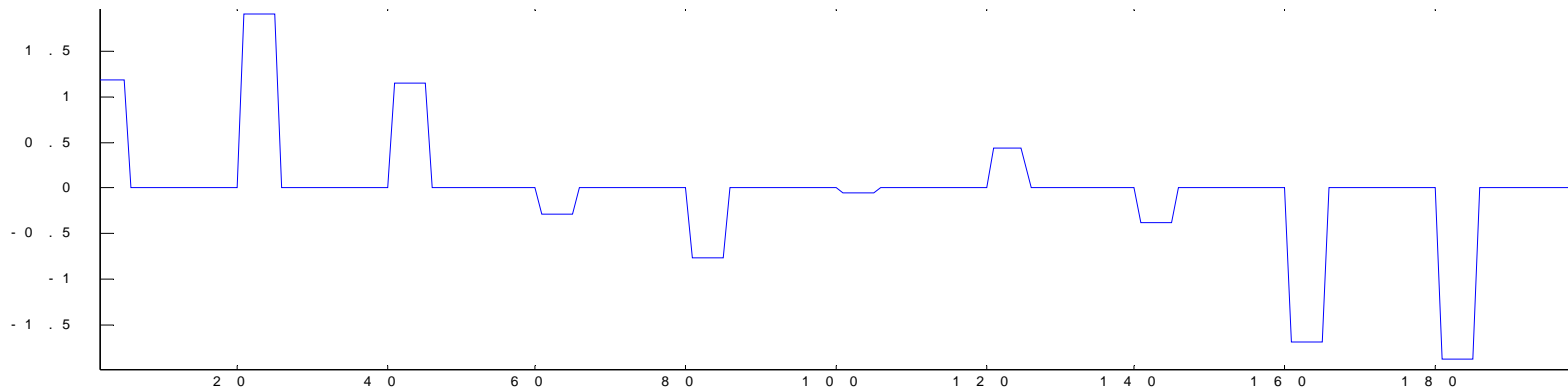
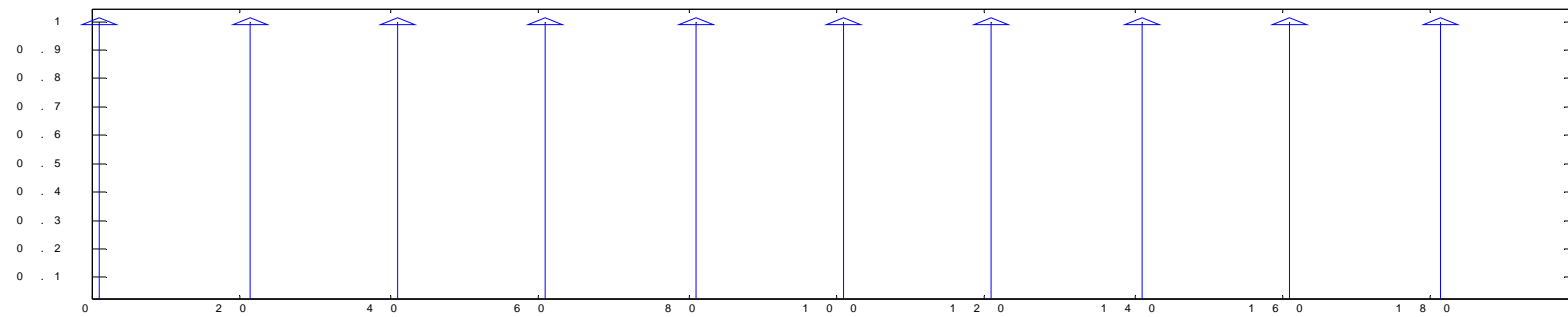
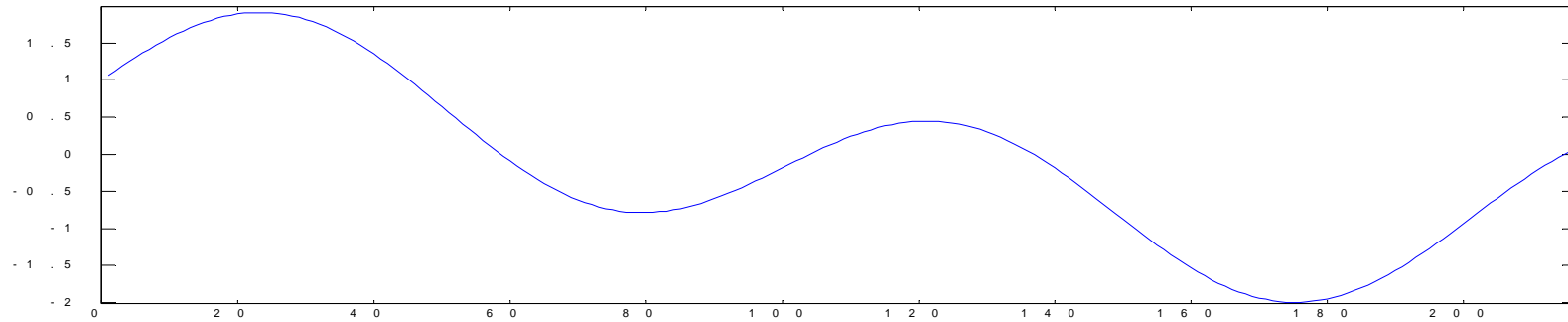
# Natural Sampling

- As the width of the pulse decreases ( $\tau/T_s$  or duty cycle) we approach impulse sampling and the spectrum approaches simple replication of the original spectrum
  - For smaller values of  $\tau/T_s$  larger values of  $n$  are required for  $n\tau/T_s$  to be an integer
- The original signal can be retrieved with a simple low-pass filter provided  $f_s > 2B$ .

# PAM: Flat-Top Sampling



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# PAM: Flat-Top Sampling

This is the type of PAM presented in the text.

Flat-top Sampling is instantaneous sampling which modulates a pulse train:

In time

$$\begin{aligned}w_s(t) &= w(t)s(t) \\ &= \sum_{k=-\infty}^{\infty} w(kT_s)h(t - kT_s) \\ &= \sum_{k=-\infty}^{\infty} w(kT_s)\Pi\left(\frac{t - kT_s}{\tau}\right)\end{aligned}$$

In frequency

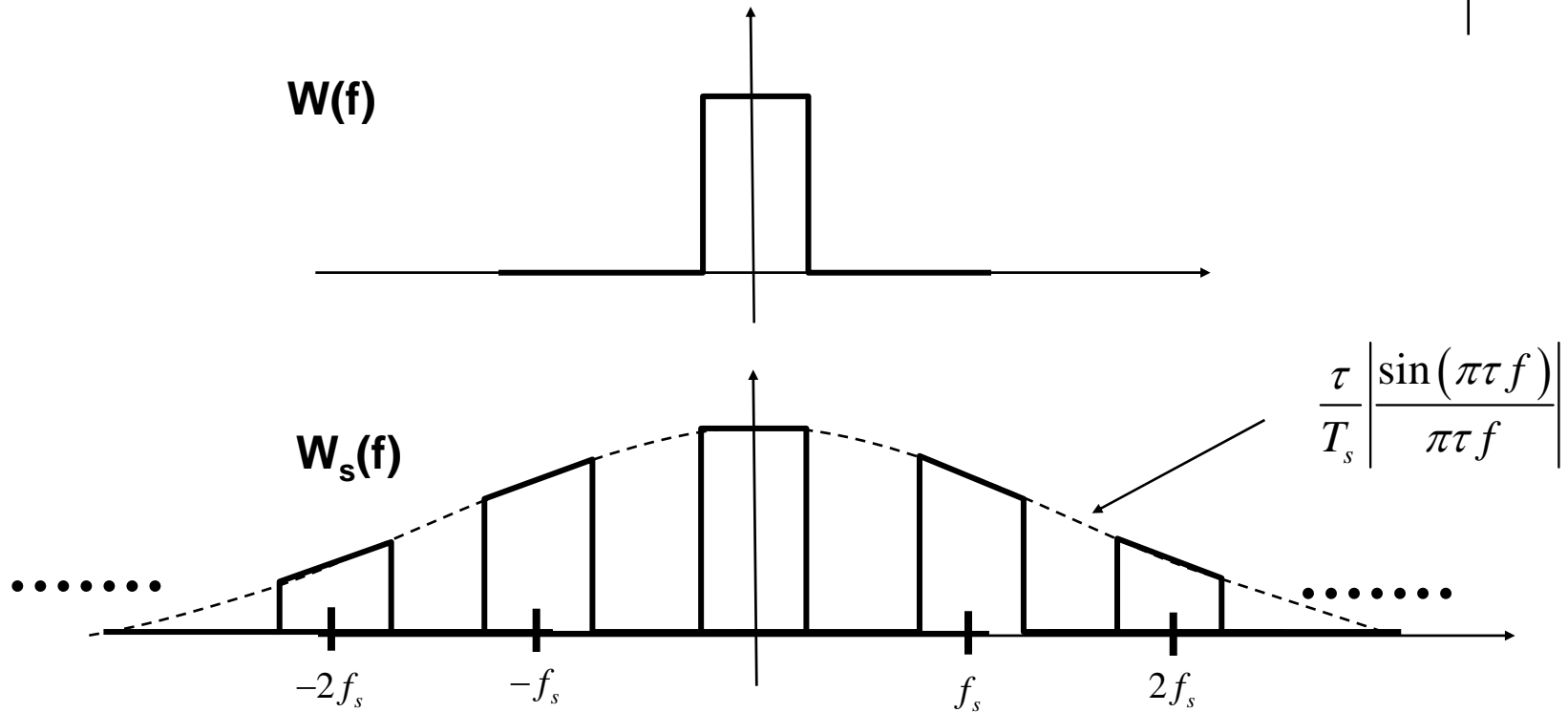
$$\begin{aligned}W_s(f) &= \mathbb{F}\{w_s(t)\} \\ &= \frac{\tau}{T_s} \frac{\sin(\pi\tau f)}{\pi\tau f} \sum_{n=-\infty}^{\infty} W(f - nf_s)\end{aligned}$$

\* - See appendix for proof

# Flat-Top Sampling



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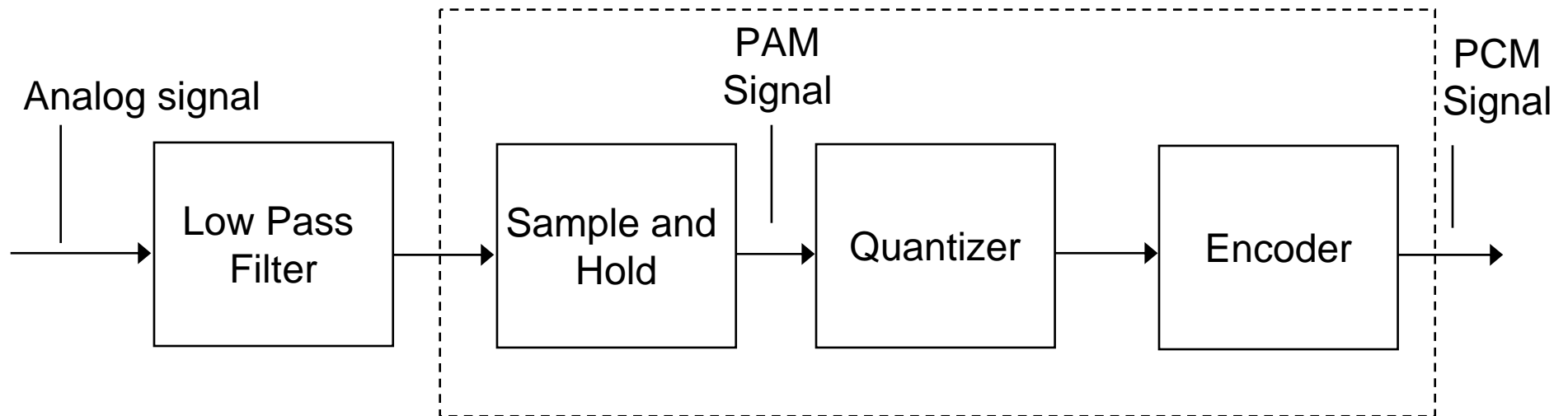
# Flat-Top Sampling

- Note that due to the flat-top pulses, the spectrum of the sampled signal is distorted.
- The narrower the pulse width, the less distortion.
- The original signal may be obtained by using a low-pass filter with a characteristic which inverts the distortion.

# Baseband Digital Transmitter



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Flat-top PAM is the first step in creating a PCM signal (i.e., full analog-to-digital conversion)



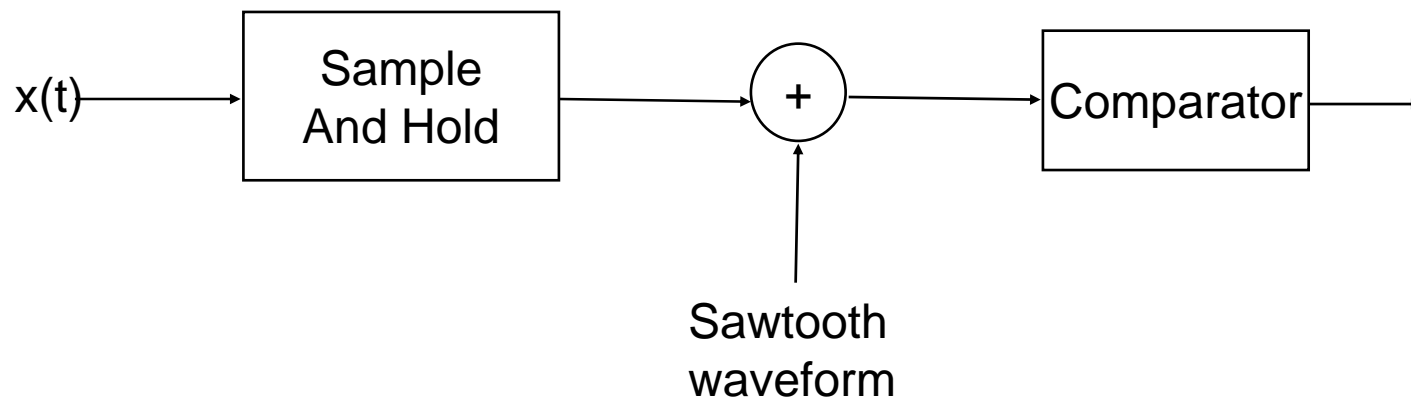
# PAM: Summary

- The transmission of PAM requires much more bandwidth than the original signal due to the narrow pulses used.
- The noise performance is equal to or worse than using analog transmission.
- PAM is good for time-multiplexing multiple signals onto a single channel
- PAM is an intermediate step in producing a Pulse-Code Modulated (PCM) signal
- It is this last point that makes PAM important to our class.



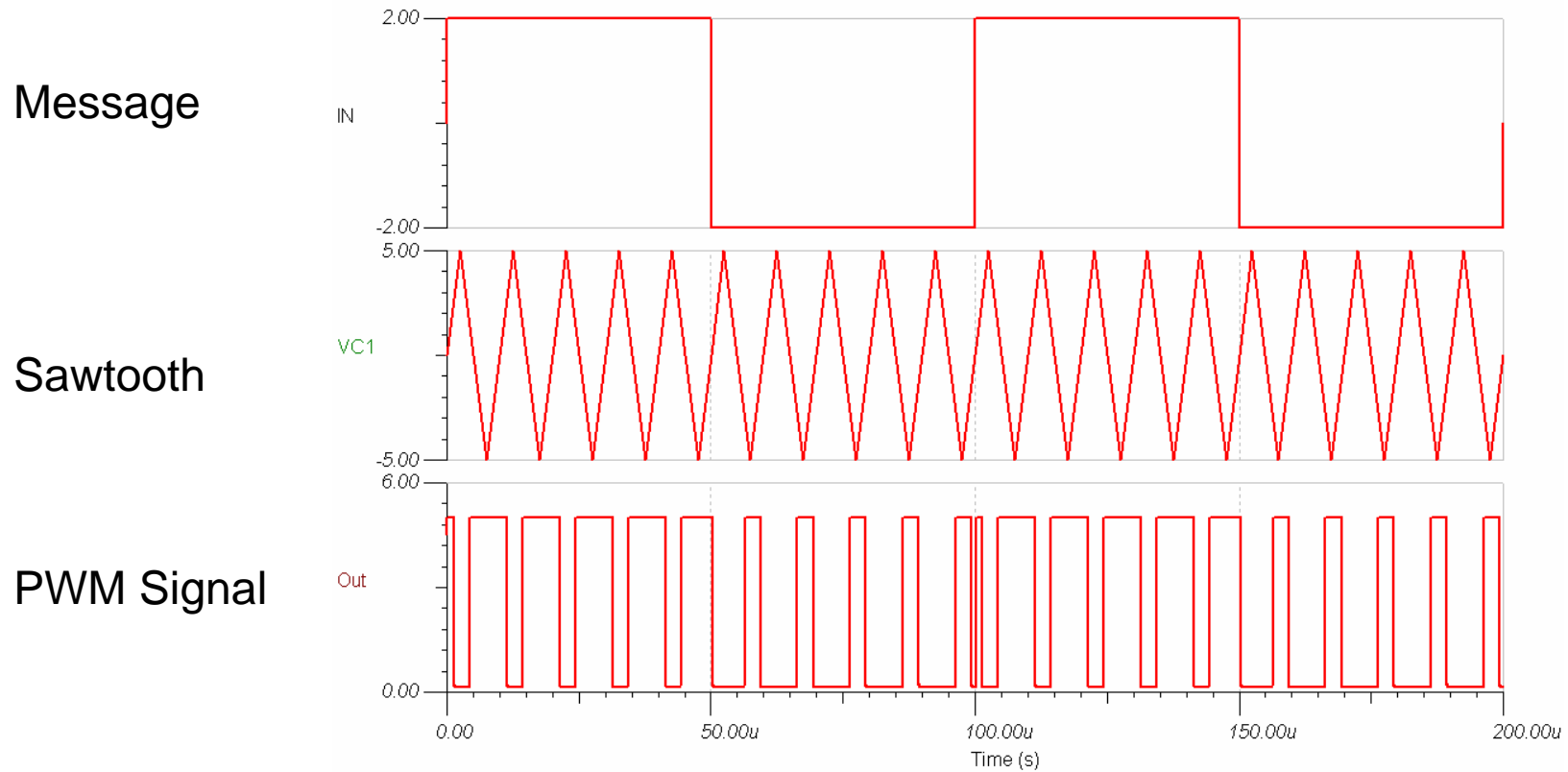
# Pulse Width Modulation

- Pulses are sent periodically (i.e., pulse train) as in PAM
- Pulse width is varied based on message signal.
- Signal is discrete in time, but analog.
- Non-linear form of modulation



# Pulse Width Modulation

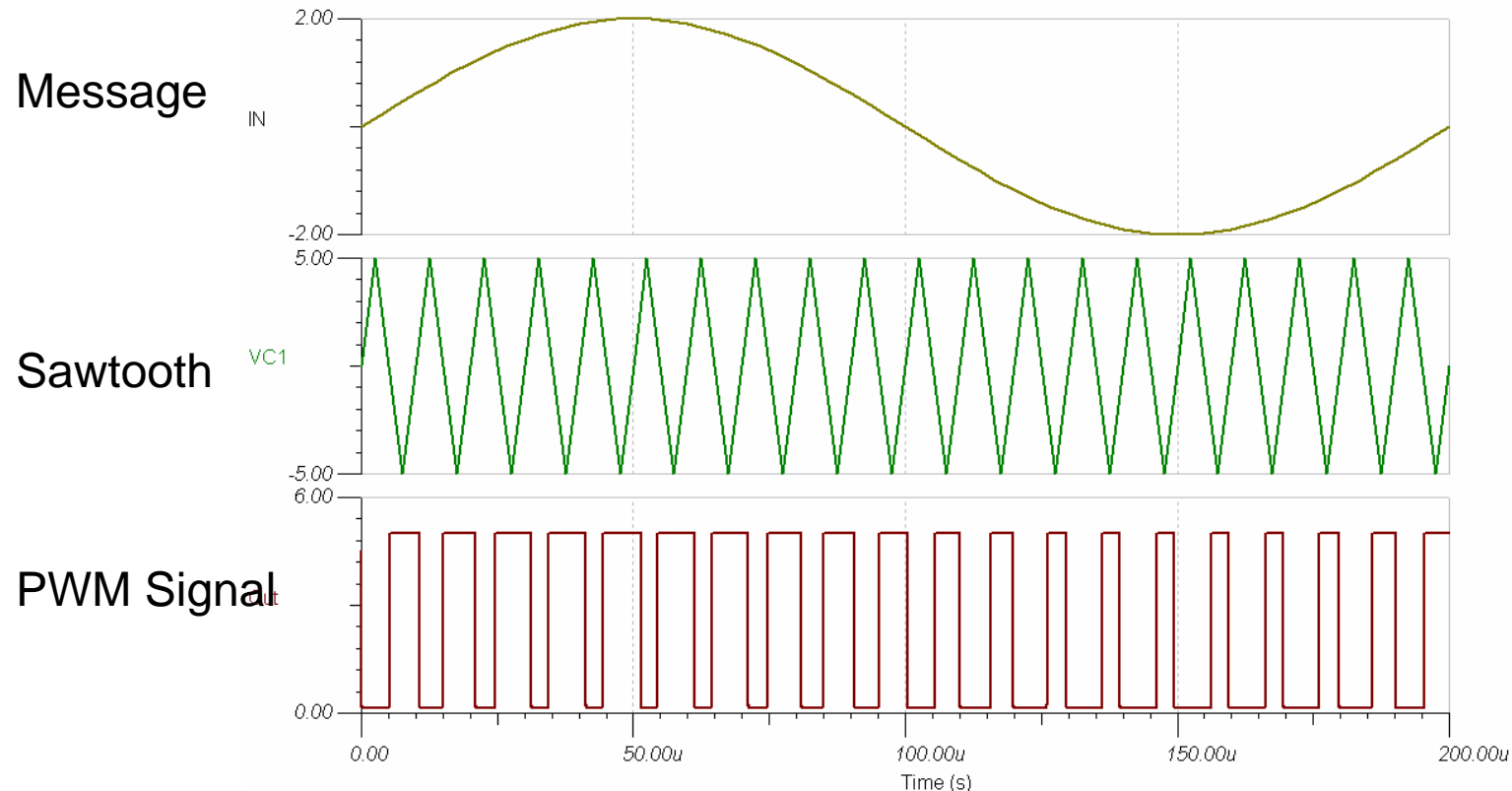
- Example:





# Pulse Width Modulation

- Example:

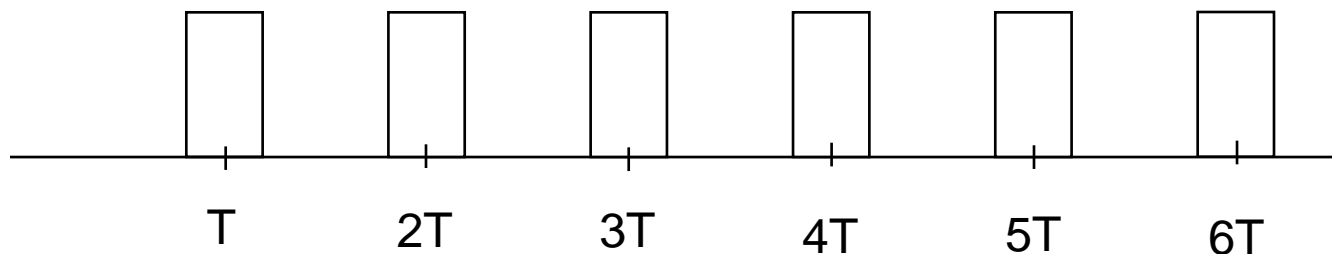




# Pulse Position Modulation

- Pulse train no longer transmitted at regular intervals.
- Instead the pulse is transmitted slightly before or slightly after the scheduled symbol time.

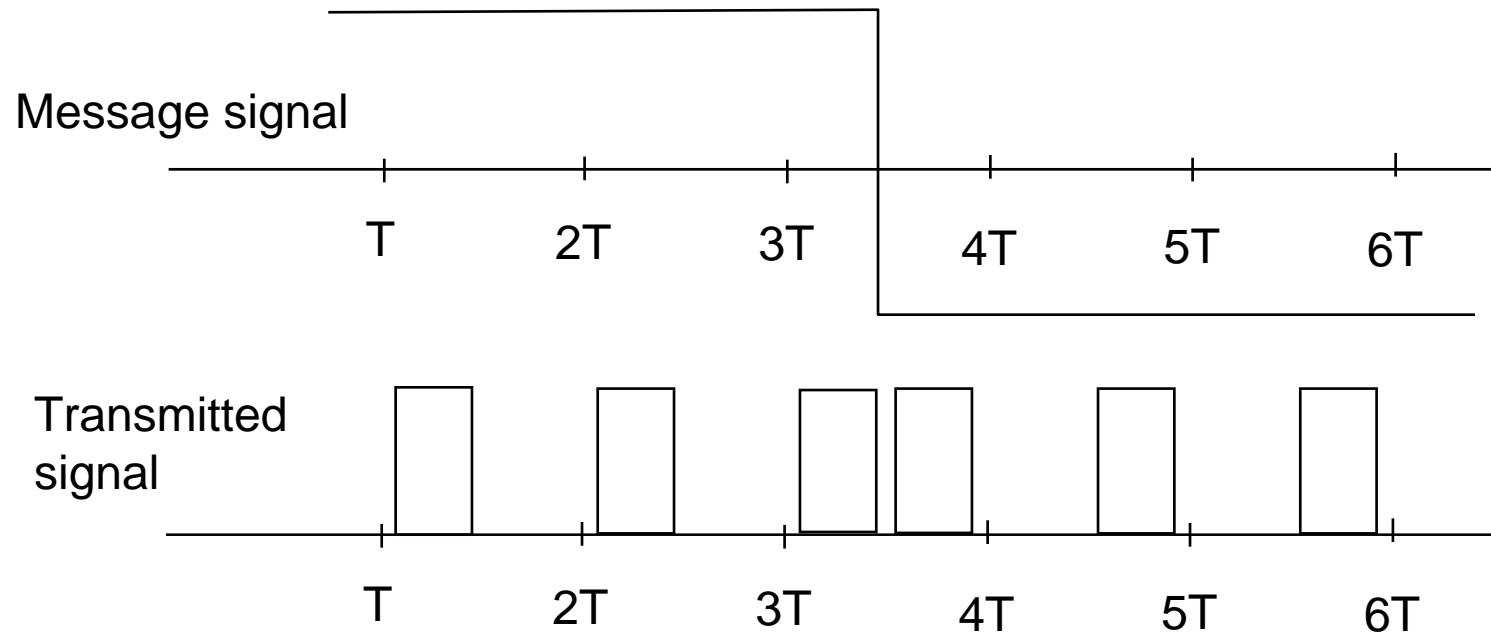
Unmodulated signal





# Pulse Position Modulation

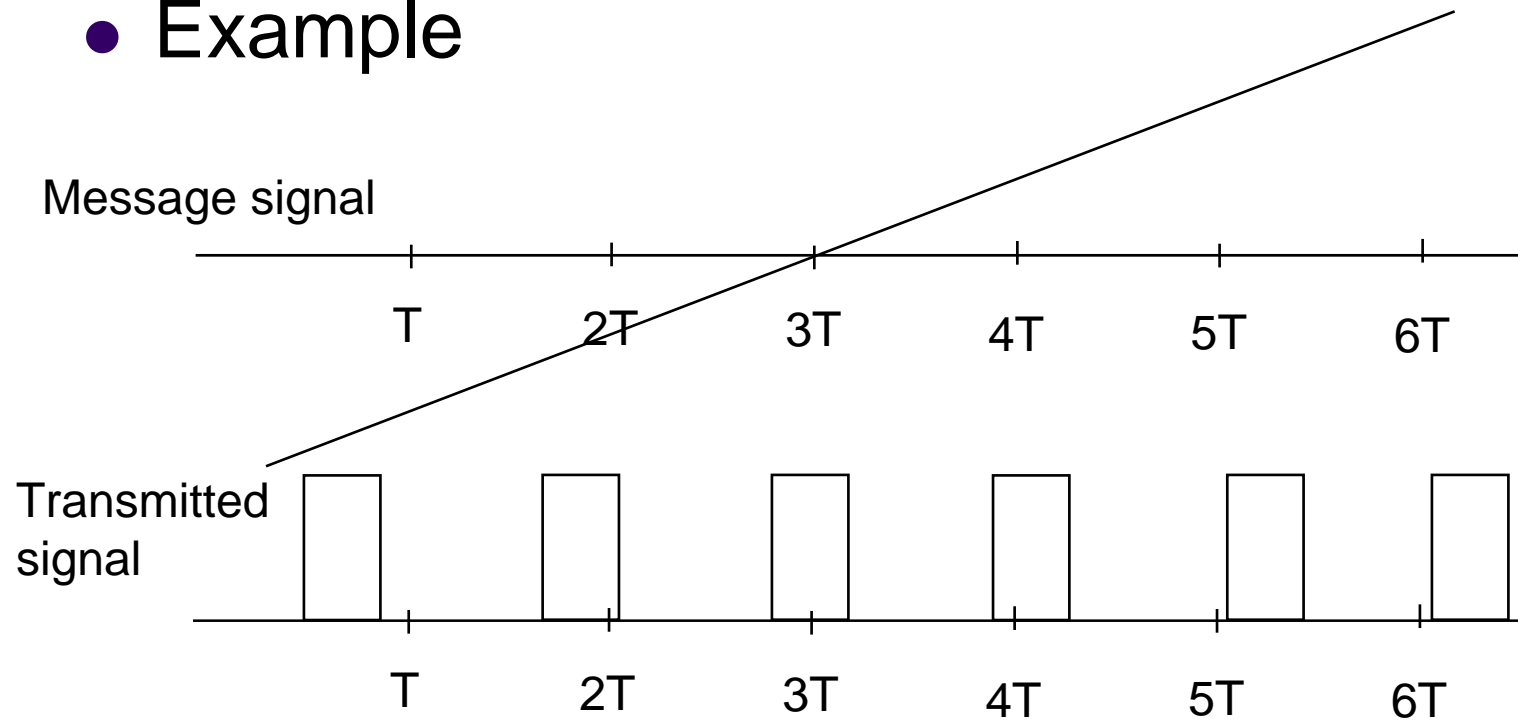
- Example





# Pulse Position Modulation

- Example



# Summary



- In our study of baseband communication systems we start with *discrete* baseband communications
  - The message signal is continuous but is sampled to modulate a pulse train
  - These types of systems are not particularly common but are useful for instructive purposes – can be useful in multiplexing multiple data streams
- We will next study a more important form of baseband communications, *digital communications*.
  - In digital systems there are a finite number of possible messages.
  - PCM is the most common form, but there are others

# Appendix

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## Proofs for Natural and Flat-top Sampling

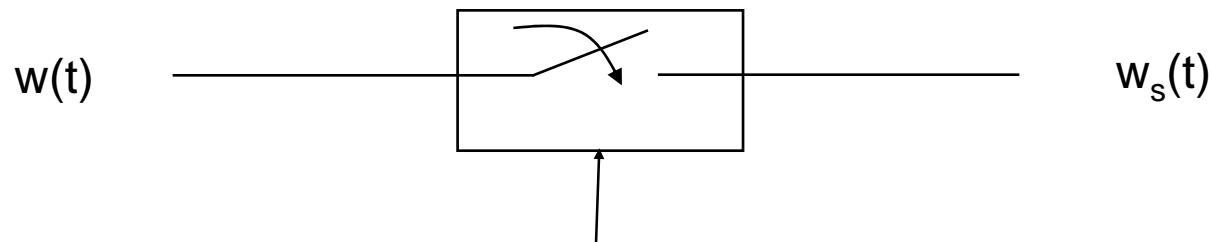


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$$= w(t) \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t - kT_s}{\tau}\right)$$

$$W_s(f) = \mathbb{F}\{w_s(t)\}$$
$$= \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \frac{\sin\left(\pi n \frac{\tau}{T_s}\right)}{\pi n \frac{\tau}{T_s}} W(f - nf_s)$$

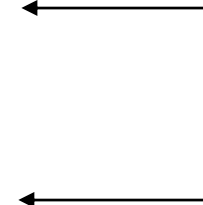
# Proof



$$W_s(f) = W(f) * S(f)$$

Where  $S(f)$  is the Fourier Transform of the sampling function  $s(t)$

$$\begin{aligned} s(t) &= \sum_{n=-\infty}^{\infty} \Pi\left(\frac{t - nT_s}{\tau}\right) \\ &= \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \end{aligned}$$



Fourier Series  
Representation

and

$$c_n = \frac{\tau}{T_s} \frac{\sin\left(n\pi \frac{\tau}{T_s}\right)}{n\pi \frac{\tau}{T_s}}$$



# Proof (cont.)

$$\begin{aligned} S(f) &= \mathfrak{T}\{s(t)\} \\ &= \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_s) \end{aligned}$$

Returning to the sampled signal:

$$\begin{aligned} W_s(f) &= W(f) * S(f) \\ &= W(f) * \left\{ \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_s) \right\} \\ &= \sum_{n=-\infty}^{\infty} c_n W(f - nf_s) \\ &= \sum_{n=-\infty}^{\infty} \frac{\sin\left(n\pi \frac{\tau}{T_s}\right)}{n\pi} W(f - nf_s) \end{aligned}$$

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In time

$$\begin{aligned}
 w_s(t) &= w(t)s(t) \\
 &= \sum_{k=-\infty}^{\infty} w(kT_s)h(t - kT_s) \\
 &= \sum_{k=-\infty}^{\infty} w(kT_s)\Pi\left(\frac{t - kT_s}{\tau}\right)
 \end{aligned}$$

In frequency

$$\begin{aligned}
 W_s(f) &= \mathbb{F}\{w_s(t)\} \\
 &= \frac{\tau}{T_s} \frac{\sin(\pi\tau f)}{\pi\tau f} \sum_{n=-\infty}^{\infty} W(f - nf_s)
 \end{aligned}$$

# Proof

$$\begin{aligned}W_s(f) &= \mathfrak{F}\{w_s(t)\} \\&= \mathfrak{F}\{w(t)s(t)\} \\&= \mathfrak{F}\left\{\sum_{k=-\infty}^{\infty} w(kT_s)(h(t) * \delta(t - kT_s))\right\} \\&= \mathfrak{F}\left\{h(t) * \sum_{k=-\infty}^{\infty} w(kT_s)\delta(t - kT_s)\right\} \\&= \mathfrak{F}\left\{h(t) * \left[w(t) \sum_{k=-\infty}^{\infty} \delta(t - kT_s)\right]\right\} \\&= \frac{H(f)}{T_s} \sum_{k=-\infty}^{\infty} W(f - kf_s) \\&= \frac{\tau}{T_s} \frac{\sin(\tau\pi f)}{\tau\pi f} \sum_{k=-\infty}^{\infty} W(f - kf_s)\end{aligned}$$



# Flat-Top Sampling



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