ECE4634
Digital Communications
Fall 2007

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Lecture #9: Digital Pulse Modulation: Line Codes

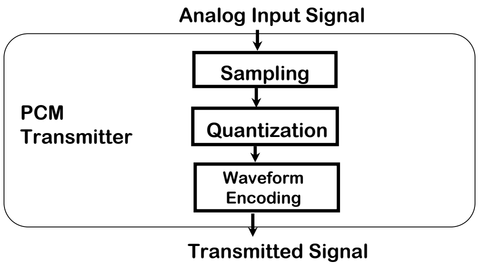
Lecture Objectives

- The objectives of this lecture are
  - to introduce ways of mapping data bits (1's and 0's) to waveforms which is termed a *line code*
  - to derive the Power Spectral Density of common line codes
  - to present properties of common line codes

Overview

- Digital communication systems transmit a finite number of messages (typically a train of pulses) from a transmitter to the receiver.
- These waveforms represent the bit stream.
- The bits can either represent an analog signal (after analog-to-digital conversion) or can be binary data.
- The data bits are mapped to waveforms (i.e., the messages) for transmission.
- Previously we assumed that bits were mapped to rectangular pulses
- Today we will study several different potential waveforms (sometimes termed "line codes") and their properties.
- What to read – Section 5.9

Structure of Digital Communications Transmitter


Analog Input Signal

PCM Transmitter

Sampling

Quantization

Waveform Encoding

Transmitted Signal

Digital Signaling and Line Codes

- We learned that PCM converts an analog signal into a serial stream of 1's and 0's to encode pulses (or waveforms) for transmission.
- We have not yet discussed the properties of these pulses.
- The mapping of the bits to waveforms or pulses is the essence of digital signaling.
- Today we look at several different mappings from {0,1} to signaling formats called *line codes*.

Binary Signaling

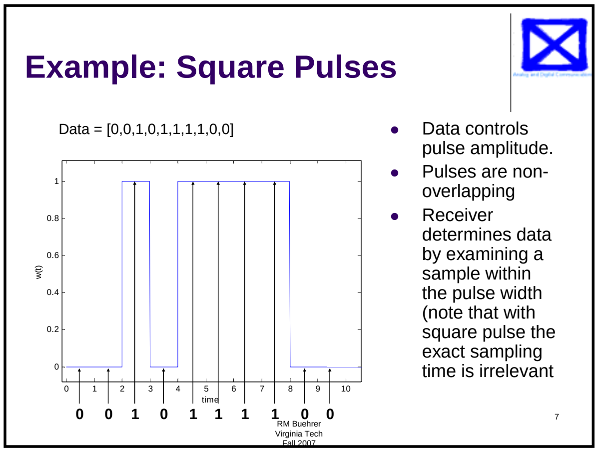
- The simplest method of digital signaling is *binary signaling*
- For binary data, $b_k = \{0,1\}$

Note $b_k = \{-1, +1\}$ is also possible

$$w(t) = \sum_{k=-\infty}^{\infty} b_k \, p(t - kT)$$

  - where *p(t)* is a pulse and *T* is the pulse duration.
- One simple pulse is a square pulse*:*

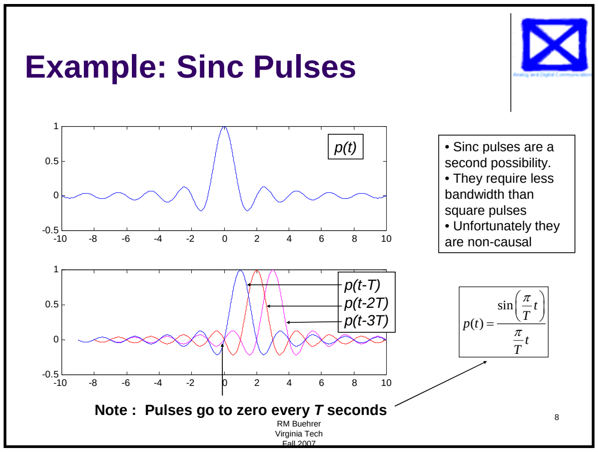
$$p(t) = \begin{cases} \frac{1}{\sqrt{T}} & T/2 < t < T/2 \\ 0 & \text{else} \end{cases}$$



Example: Square Pulses

Data = [0,0,1,0,1,1,1,1,0,0]

- Data controls pulse amplitude.
- Pulses are non-overlapping
- Receiver determines data by examining a sample within the pulse width (note that with square pulse the exact sampling time is irrelevant

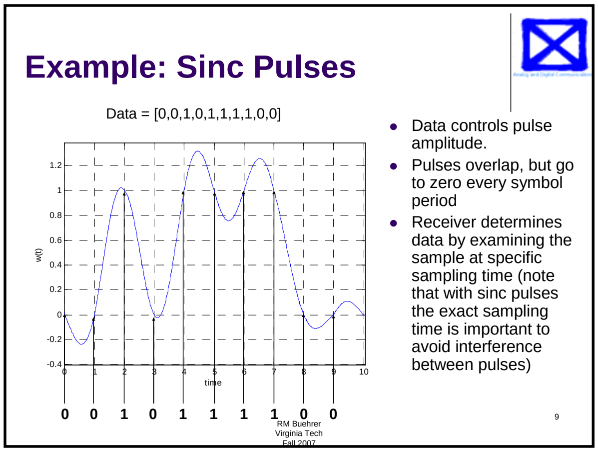


Example: Sinc Pulses

- Sinc pulses are a second possibility.
- They require less bandwidth than square pulses
- Unfortunately they are non-causal

$$p(t) = \frac{\sin\left(\frac{\pi}{T}t\right)}{\frac{\pi}{T}t}$$

Note : Pulses go to zero every *T* seconds



Example: Sinc Pulses

Data = [0,0,1,0,1,1,1,1,0,0]

- Data controls pulse amplitude.
- Pulses overlap, but go to zero every symbol period
- Receiver determines data by examining the sample at specific sampling time (note that with sinc pulses the exact sampling time is important to avoid interference between pulses)

Multi-level Signaling

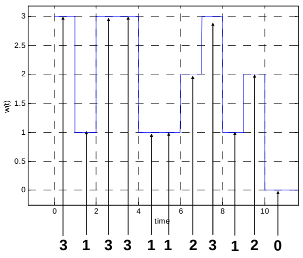
- With binary signaling each pulse is modulated by a single bit. Thus, the symbol rate is equal to the bit rate.
- One method of reducing bandwidth is to map more than one bit to each pulse.
- This can be accomplished by allowing the amplitude to take on $L > 2$ values where L is a power of 2
  - Groups of n bits are mapped into one of $L=2^n$ levels.
  - Ex: 4-ary signaling, 8-ary signaling

Multi-level Signaling (cont.)

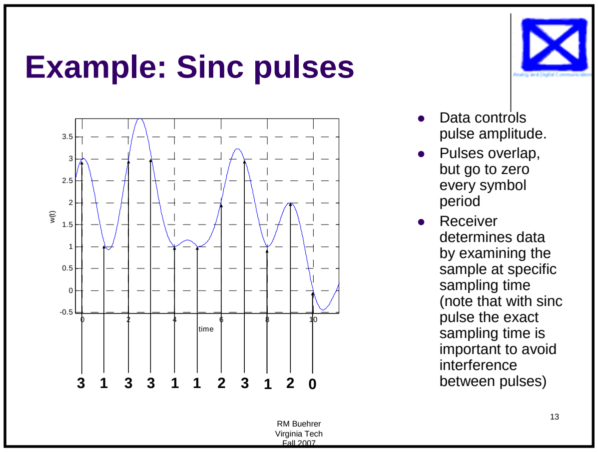
$$w(t)=\sum_{k=-\infty}^{\infty} w_k\, p\left(t-kT\right)$$

- Ex: 4-ary signaling → $w_k = \{0,1,2,3\}$
  - 00 → 0
  - 01 → 1
  - 10 → 2
  - 11 → 3

Example: Square pulses



- Data controls pulse amplitude.
- Pulses are non-overlapping
- Receiver determines data by examining a sample within the pulse width (note that with square pulse the exact sampling time is irrelevant

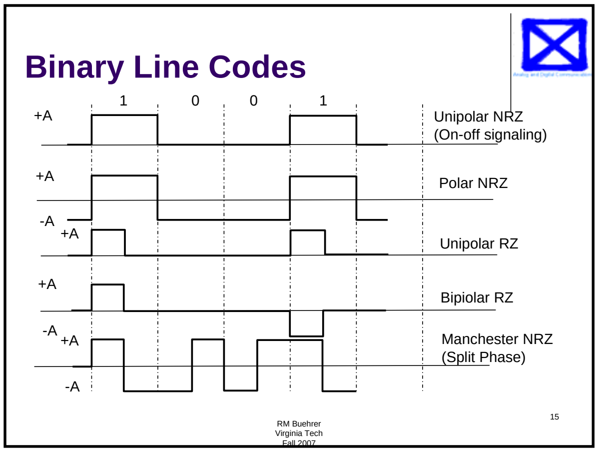


Example: Sinc pulses

- Data controls pulse amplitude.
- Pulses overlap, but go to zero every symbol period
- Receiver determines data by examining the sample at specific sampling time (note that with sinc pulse the exact sampling time is important to avoid interference between pulses)

Binary Line Codes

- When binary signaling is used, there are various ways to create waveforms with a square pulse shape (so far we have assumed *unipolar NRZ signaling*).
- These various signaling formats are called *line codes*.
- There are two major categories of line codes
  - Return to zero codes (RZ)
    - The signal returns to zero for ½ (or other fraction) of the pulse
  - Non-return to zero codes (NRZ)
- Codes have different
  - Spectral properties
  - Synchronization capabilities
  - Error performance



Binary Line Codes

Power Spectral Densities (Deterministic)

- Deterministic and Periodic Power Signals

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_o t}$$

  - where c_n are the Fourier Series Coefficients

- Deterministic power signals have Power Spectral Density (PSD):

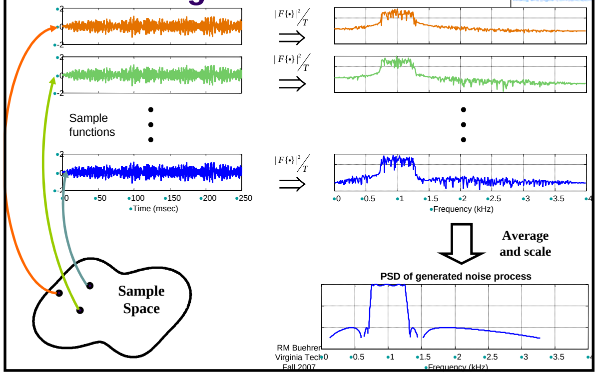
$$S_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \; \delta(f - nf_0)$$

Power Spectral Densities (Random)

- It is easiest to talk about random signals that extend for all time (i.e., power signals). Hence almost every sample function (or realization of a stochastic process) is a power signal with a power spectral density. In fact any *stationary* random process will have infinite energy. The PSD of the random process is just the ensemble average of all sample function power spectral densities.

$$\boxed{S_X(f) = E\left[\lim_{T\to\infty} \frac{|X_T(f)|^2}{T}\right] = \lim_{T\to\infty} \frac{E\left[|X_T(f)|^2\right]}{T}}$$

Visualizing PSD Definition



Power Spectral Density

- $S_x(f)$ (PSD) tells us how much power is at each frequency
- For deterministic signals, the function description *x(t)* tells us how the value changes with time, thus its Fourier transform *X(f)* gives us the spectral properties. However, random processes have a random function description
- Wiener-Khinchine Theorem
  - For a WSS process

$$S_x(f) = F\{R_x(\tau)\} = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

  - The autocorrelation function $R(\tau) = \int_{-\infty}^{\infty} w(t)w(t+\tau)dt$ tells us how the value is *expected* to change with time rather than the exact change.
  - Power spectral density and autocorrelation are a Fourier Transform pair

Power Spectral Density for Digital Modulated Pulse Train

- Consider a digitally modulated pulse train:

$$x(t) = \sum_{n=-\infty}^{\infty} a_n f(t - nT_s)$$

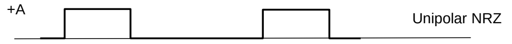
where f(t) is the pulse shape and a_n are the digital values.

- We can show (see Appendix) that the PSD of this signal is:

$$S_x(f) = \underbrace{\frac{\sigma_a^2}{T_s}|F(f)|^2}_{continuous} + \underbrace{\frac{m_a^2}{T_s^2}\sum_{n=-\infty}^{\infty}\left|F\left(\frac{n}{T_s}\right)\right|^2 \delta\left(f - \frac{n}{T_s}\right)}_{discrete}$$

- PSD has continuous portion which is dependent on the pulse shape *f(t)* and a discrete portion which is also dependent on the data mean value and variance.
- This will become very important when we examine the bandwidth requirements of digitally modulated signals

Unipolar NRZ Signaling

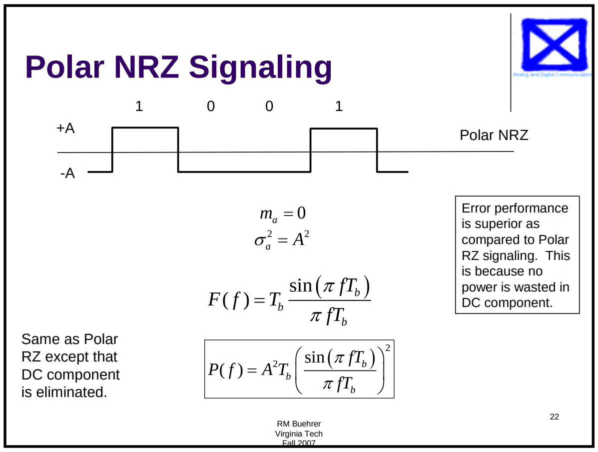


$$m_a = \frac{A}{2}$$

$$\sigma_a^2 = \frac{A^2}{4}$$

$$F(f) = T_b \frac{\sin(\pi f T_b)}{\pi f T_b}$$

$$\boxed{P(f) = \frac{A^2 T_b}{4} \underbrace{\left(\frac{\sin(\pi f T_b)}{\pi f T_b}\right)^2}_{\text{due to pulse shape}} \left[1 + \underbrace{\frac{1}{T_b}\delta(f)}_{\text{due to DC component}}\right]}$$



Polar NRZ Signaling

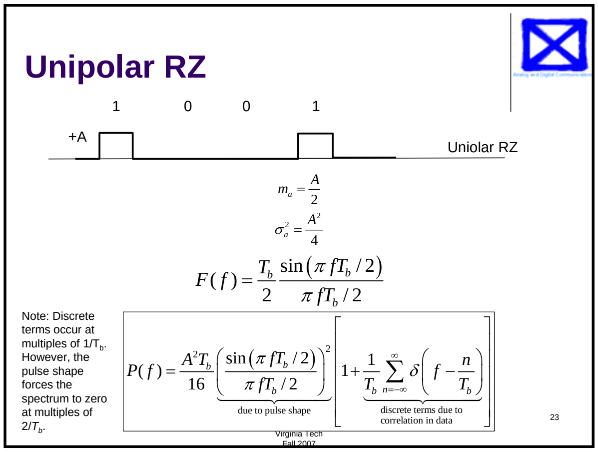
$$m_a = 0$$
$$\sigma_a^2 = A^2$$

$$F(f) = T_b \frac{\sin(\pi f T_b)}{\pi f T_b}$$

$$\boxed{P(f) = A^2 T_b \left(\frac{\sin(\pi f T_b)}{\pi f T_b}\right)^2}$$

Error performance is superior as compared to Polar RZ signaling. This is because no power is wasted in DC component.

Same as Polar RZ except that DC component is eliminated.



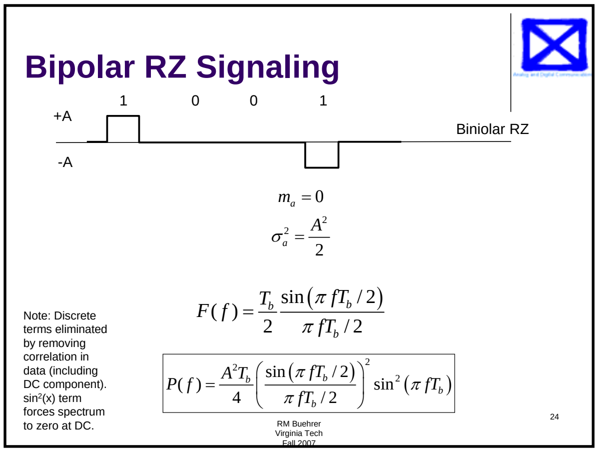
Unipolar RZ

$$m_a = \frac{A}{2}$$
$$\sigma_a^2 = \frac{A^2}{4}$$

$$F(f) = \frac{T_b}{2} \frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2}$$

$$\boxed{P(f) = \frac{A^2 T_b}{16} \underbrace{\left(\frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2}\right)^2}_{\text{due to pulse shape}} \left[1 + \underbrace{\frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)}_{\text{discrete terms due to correlation in data}} \right]}$$

Note: Discrete terms occur at multiples of $1/T_b$. However, the pulse shape forces the spectrum to zero at multiples of $2/T_b$.



Bipolar RZ Signaling

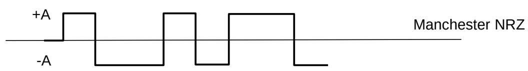
$$m_a = 0$$
$$\sigma_a^2 = \frac{A^2}{2}$$

$$F(f) = \frac{T_b}{2} \frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2}$$

$$\boxed{P(f) = \frac{A^2 T_b}{4} \left(\frac{\sin(\pi f T_b / 2)}{\pi f T_b / 2}\right)^2 \sin^2(\pi f T_b)}$$

Note: Discrete terms eliminated by removing correlation in data (including DC component). $\sin^2(x)$ term forces spectrum to zero at DC.

Manchester NRZ Signaling



$$m_a = 0$$
$$\sigma_a^2 = A^2$$

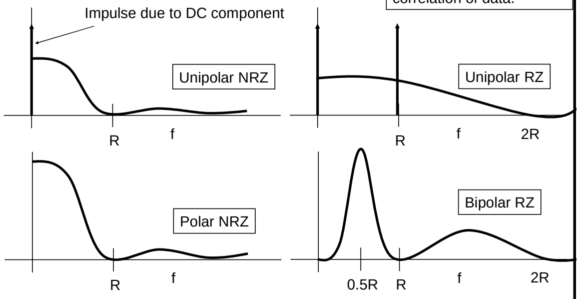
$$F(f) = jT_b \frac{\sin\left(\pi f T_b / 2\right)}{\pi f T_b / 2} \sin\left(\frac{2\pi f T_b}{4}\right)$$

$$\boxed{P(f) = A^2 T_b \left(\frac{\sin\left(\pi f T_b / 2\right)}{\pi f T_b / 2}\right)^2 \sin^2\left(\pi f T_b / 2\right)}$$

$\sin^2(x)$ term forces spectrum to zero at DC. First null BW twice that of bipolar RZ

Spectra of Line Codes

Note: Spectra are dependent on pulse shape and auto-correlation of data.



Properties of Line Codes

- Self-synchronization
  - Timing information is built into the code to allow clock recovery.
  - Strings of 1's and 0's do not cause timing problems
- Spectrum
  - The spectrum must be appropriate for the channel being used.
  - Bandwidth should be as small as possible
- Error-probability
  - The code should provide low probability of bit error
  - Code should allow easy implementation of error corrections coding/decoding

Properties of Line Codes

- Unipolar codes
  - Have the advantage of requiring only a single positive voltage power supply.
  - Have the disadvantage of having a DC value which is less power efficient and requires channels that have DC response.
- Bipolar Codes
  - Have zero DC level provided 0's and 1's occur at the same frequency and there are not long strings of 0's or 1's.
- Manchester Codes
  - Have no DC value regardless of the number of consecutive 1's or 0's.
  - Twice the bandwidth of NRZ codes
- RZ codes
  - Have self-synchronization properties due to spectral lines (or ability to produce spectral lines) at $f = R$.
  - Have twice the bandwidth of NRZ signals

Summary

- PCM is a common method of converting an analog signal to a digital waveform
- Sampling and quantization converts analog waveforms into a series of bits
- There are several methods of mapping the bits to digital waveforms
  - Binary vs. multi-level signaling
  - Various pulse shapes
  - Various line codes
- Next class we will look more closely at the effect of pulse shape

Appendix

Derivation of the PSD for a digitally modulated pulse train

Power Spectral Density for Digitally Modulated Pulse Train

Let us define a general digitally modulated pulse train as $x(t)=\sum_{n=-\infty}^{\infty} a_n f\left(t-nT_s\right)$

where a_n is the sequence of data values, *f(t)* is the pulse shape used and T_s is the symbol duration. This is sometimes called a modulated pulse train.

Now the Power Spectral Density can be found as $P_x(f)=\lim_{T\to\infty}\left(\frac{\overline{\left|X_T(f)\right|^2}}{T}\right)$

Where $X_T(f)=\int_{-T/2}^{T/2} x(t)e^{-j2\pi ft}dt$ and $\overline{X(f)}=E[X(f)]=\text{ensemble average}$

From our signal definition we have $X_T(f)=\int_{-T/2}^{T/2}\sum_{n=-\infty}^{\infty} a_n f\left(t-nT_s\right)e^{-j2\pi ft}dt$

$$=\sum_{n=-N}^{N} a_n \int_{-\infty}^{\infty} f\left(t-nT_s\right)e^{-j2\pi ft}dt \qquad \boxed{T=(2N+1)T_s}$$

Power Spectral Density for Digital Signals (cont.)

Continuing from the previous slide $X_T(f)=\sum_{n=-N}^{N} a_n \int_{-\infty}^{\infty} f\left(t-nT_s\right)e^{-j2\pi ft}dt$

$$=\sum_{n=-N}^{N} a_n F(f)e^{-j2\pi fnT_s}$$

$$=F(f)\sum_{n=-N}^{N} a_n e^{-j2\pi fnT_s}$$

Returning to our definition for PSD $P_x(f)=\lim_{T\to\infty}\left(\frac{\overline{\left|X_T(f)\right|^2}}{T}\right)$

$$=\lim_{T\to\infty}\left(\frac{\overline{\left|F(f)\sum_{n=-N}^{N} a_n e^{-j2\pi fnT_s}\right|^2}}{T}\right)$$

$$=\left|F(f)\right|^2\lim_{N\to\infty}\left(\frac{1}{(2N+1)T_s}\sum_{n=-N}^{N}\sum_{m=-N}^{N}\overline{a_n a_m}e^{-j2\pi f(n-m)T_s}\right)$$

Power Spectral Density for Digital Signals (cont.)

Defining $R(k)=\overline{a_n a_{n+k}}$ and $m = n+k$

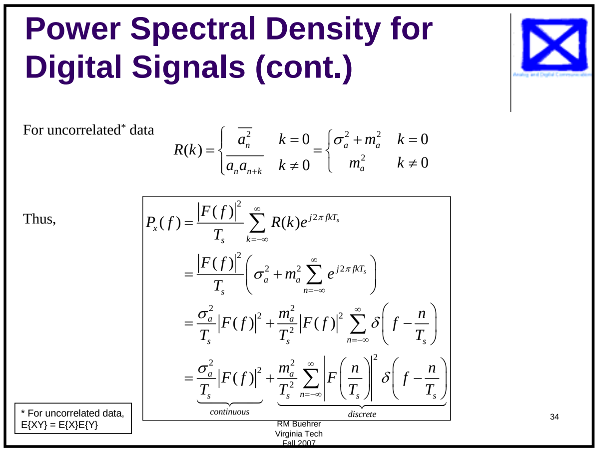
$$P_x(f)=\left|F(f)\right|^2\lim_{N\to\infty}\left(\frac{1}{(2N+1)T_s}\sum_{n=-N}^{N}\sum_{m=-N}^{N}\overline{a_n a_m}e^{-j2\pi f(n-m)T_s}\right)$$

$$=\left|F(f)\right|^2\lim_{N\to\infty}\left(\frac{1}{(2N+1)T_s}\sum_{n=-N}^{N}\sum_{k=-N-n}^{N-n} R(k)e^{-j2\pi fkT_s}\right)$$

$$=\frac{\left|F(f)\right|^2}{T_s}\lim_{N\to\infty}\left(\frac{(2N+1)}{(2N+1)}\sum_{k=-N}^{N} R(k)e^{-j2\pi fkT_s}\right)$$

$$=\frac{\left|F(f)\right|^2}{T_s}\sum_{k=-\infty}^{\infty} R(k)e^{-j2\pi fkT_s}$$

$$=\frac{\left|F(f)\right|^2}{T_s^2}\sum_{k=-\infty}^{\infty} R(k)\delta\left(f-\frac{k}{T_s}\right) \qquad \boxed{\sum_{k=-\infty}^{\infty} e^{-j2\pi fkT_s}=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T_s}\right)}$$



Power Spectral Density for Digital Signals (cont.)

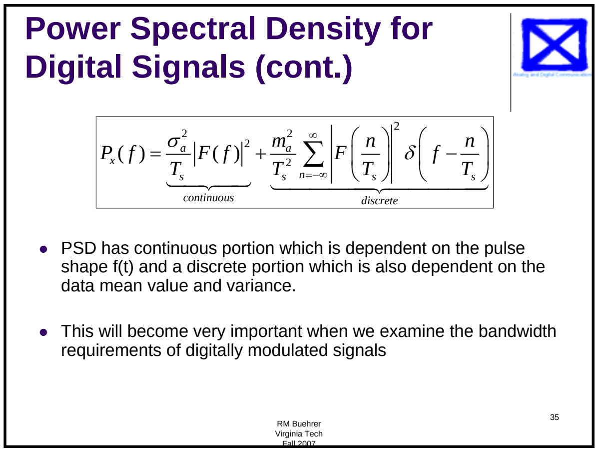
For uncorrelated* data

$$R(k)=\begin{cases}\overline{a_n^2} & k=0\\ \overline{a_n a_{n+k}} & k\neq 0\end{cases}=\begin{cases}\sigma_a^2+m_a^2 & k=0\\ m_a^2 & k\neq 0\end{cases}$$

Thus,

$$\boxed{\begin{aligned}P_x(f)&=\frac{|F(f)|^2}{T_s}\sum_{k=-\infty}^{\infty}R(k)e^{j2\pi fkT_s}\\ &=\frac{|F(f)|^2}{T_s}\left(\sigma_a^2+m_a^2\sum_{n=-\infty}^{\infty}e^{j2\pi fkT_s}\right)\\ &=\frac{\sigma_a^2}{T_s}|F(f)|^2+\frac{m_a^2}{T_s^2}|F(f)|^2\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_s}\right)\\ &=\underbrace{\frac{\sigma_a^2}{T_s}|F(f)|^2}_{continuous}+\underbrace{\frac{m_a^2}{T_s^2}\sum_{n=-\infty}^{\infty}\left|F\left(\frac{n}{T_s}\right)\right|^2\delta\left(f-\frac{n}{T_s}\right)}_{discrete}\end{aligned}}$$

* For uncorrelated data, E{XY} = E{X}E{Y}



Power Spectral Density for Digital Signals (cont.)

$$\boxed{P_x(f)=\underbrace{\frac{\sigma_a^2}{T_s}|F(f)|^2}_{continuous}+\underbrace{\frac{m_a^2}{T_s^2}\sum_{n=-\infty}^{\infty}\left|F\left(\frac{n}{T_s}\right)\right|^2\delta\left(f-\frac{n}{T_s}\right)}_{discrete}}$$

- PSD has continuous portion which is dependent on the pulse shape f(t) and a discrete portion which is also dependent on the data mean value and variance.
- This will become very important when we examine the bandwidth requirements of digitally modulated signals