Digital Communications
Midterm Exam II
November 3, 2006

I pledge that I have neither given nor received any assistance on this exam.

____
(signed)

____________________________ ____________________
Name (print) Student Number

1. (20 points) Multiple Choice – Choose the answer that best completes the sentence.

(a) [5 points] Differential PSK modulation

(a) requires a non-coherent receiver
(b) requires a coherent receiver
(c) can use either a coherent or a non-coherent receiver
(d) All of the above
(e) None of the above

(b) [5 points] Non-coherent demodulation of M-FSK

(a) is possible since there is no information in the phase
(b) performs worse than coherent demodulation of M-FSK
(c) requires $2M$ correlator branches in the receiver
(d) All of the above
(e) None of the above

(c) [5 points] An eye diagram

(a) shows the amount of bandwidth being used by a pulse shape
(b) shows the amount of ISI introduced by a pulse shape
(c) shows the impact of phase error in a coherent receiver
(d) All of the above
(e) None of the above

(d) [5 points] Which pulse shape causes the least degradation due to timing error?

(a) square pulse
(b) raised cosine pulse
(c) sinc pulse
(d) All of the above would show equal degradation
(e) None of the above

2. (20 points) Bandpass Representation

Consider the following modulation scheme where each symbol is defined as:

$$s(t) = Ap(t)\cos(2\pi f_c t) + Bp(t)\sin(2\pi f_c t)$$

where

$$A \in \{-3, -1, 1, 3\}$$
$$B \in \{-4, -2, 2, 4\}$$

(i.e., 16 possible symbols) and *p(t)* is an arbitrary pulse shape.

(a) [5 points] Write the in-phase and quadrature representation of this signal.

(b) [5 points] Write the complex baseband version of this signal

(c) [5 points] Write the amplitude/phase representation of this signal.

(d) [5 points] What is the bandwidth efficiency (in bps/Hz) of this modulation scheme if *p(t)* has a bandwidth of $1.25\ R_s$ Hz where R_s is the pulse rate ?

3. (25 points) Binary Bandpass Modulation / Pulse Shaping

Consider the following modulation scheme:

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \, p(t - kT) \cos(2\pi f_c t)$$

where $A_k \in \{0,1\}$

$$p(t) = \text{sinc}(t/T)$$

(a) (10 points) Plot the power spectral density of this modulation scheme. Carefully label all axes.

(b (5 points) Do you see any practical difficulties with this scheme? If so, suggest a fix (please be specific!)

(c) (10 points) One of you colleagues suggests reducing the bandwidth of this system. Specifically, he suggests passing the baseband signal through a low-pass filter with a bandwidth of 1/4T Hz where $1/T$ is the symbol rate. Would this reduce the bandwidth of the transmit signal? Even if it did reduce the bandwidth of the signal, does it introduce any difficulties at the receiver?

4. (35 points) M-ary Modulation / Signal Space Representation

Consider the following modulation scheme

$$
\begin{aligned}
x_1(t) &= \sqrt{2}A\cos(2\pi f_1 t) \\
x_2(t) &= 0 \\
x_3(t) &= A\cos(2\pi f_2 t) \\
x_4(t) &= -A\cos(2\pi f_2 t)
\end{aligned}
$$

where $f_2 = f_1 + R_s$ and R_s is the symbol rate.

(a) [15 points] If this modulation scheme requires two basis functions, suggest two appropriate functions and show that they are indeed an orthonormal basis.

(b) [10 points] Draw the signal space diagram in terms of energy per symbol. Be careful to label all axes and points.

(c) [5 points] What is the minimum distance between signal points in terms of energy per bit?

(d) [5 points] Is this scheme more or less bandwidth efficient than QPSK? Support your answer.

Rectangular Pulse	$\text{rect}\left(\frac{t}{T}\right)$	$T\left[\text{sinc}(fT)\right]$
Triangular Pulse	$\text{tri}\left(\frac{t}{T}\right)$	$T\left[\text{sinc}(fT)\right]^2$
Unit Step	$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
exponential	$e^{-at}u(t)$	$\frac{1}{a+j2\pi f}$
Constant	1	$\delta(f)$
Impulse at t_o	$\delta(t-t_o)$	$e^{-j2\pi f t_o}$
Sinc	$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
Phasor	$e^{j\omega_o t+\varphi}$	$e^{j\varphi}\delta(f-f_o)$
Sinusoid	$\cos(2\pi ft+\varphi)$	$\frac{1}{2}e^{j\varphi}\delta(f-f_o)+\frac{1}{2}e^{-j\varphi}\delta(f+f_o)$
Gaussian	$e^{-\pi(t/t_o)^2}$	$t_o e^{-\pi(f\,t_o)^2}$

Property	
Conjugation	$x^*(t) \rightleftharpoons X^*(-f)$
Linearity	$\alpha x(t)+\beta y(t) \rightleftharpoons \alpha X(f)+\beta Y(f)$
Time-shifting	$x(t-t_o) \rightleftharpoons e^{-j2\pi f t_o} X(f)$
Frequency-shifting	$e^{j2\pi f_o t} x(t) \rightleftharpoons X(f-f_o)$
Time reversal	$x(-t) \rightleftharpoons X(-f)$
Time-differentiation	$\frac{d}{dt}\{x(t)\} \rightleftharpoons (j2\pi f) X(f)$
Time-integration	$\int_{-\infty}^{t} x(\tau)\,d\tau \overset{*}{\rightleftharpoons} \frac{1}{j2\pi f} X(f)$
Time/freq-scaling	$x(at) \rightleftharpoons \frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Multiplication	$x(t)\,y(t) \rightleftharpoons X(f) * Y(f)$
Convolution	$x(t) * y(t) \rightleftharpoons X(f)\,Y(f)$