Digital Communications
Midterm Exam I
September 21, 2007

SOLUTION

I pledge that I have neither given nor received any assistance on this exam.

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(signed)

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Name (print) Student Number

1. (20 points) Short Answer/Multiple Choice

(1.1) [5 points] The signal $x(t) = \sum_{n=-\infty}^{\infty} b_n rect\left(\frac{t-nT}{T}\right)$ where b_n is a binary random variable equal to +1 or -1 with equal probability is a

(a) periodic signal
(b) power signal
(c) energy signal
(d) unipolar RZ line code
(e) None of the above
(f) All of the above

(1.2) [5 points] A unipolar RZ line code has the advantage that

(a) it has a smaller bandwidth than other line codes
(b) it has no DC offset
(c) there is no power wasted in an unmodulated component
(d) it has self-synchronizing capabilities
(e) None of the above
(f) All of the above

(1.3) [5 points] The step size Δ in a delta modulation scheme

(a) trades between slope overload and granular noise
(b) trades between bandwidth and performance
(c) trades between bandwidth and granular noise
(d) trades between data rate and bandwidth
(e) None of the above
(f) All of the above

(1.4) [5 points] Intersymbol interference can be caused by

(a) the channel
(b) improper pulse design
(c) poor filter design at the transmitter
(d) poor filter design at the receiver
(e) None of the above
(f) All of the above

2. (25 points) The Sampling Theorem

Consider the following signal:

$$x(t) = \text{sinc}^2(10t)\cos(200\pi t)$$

The signal is sampled at a rate of 40Hz. Plot the spectrum of the sampled signal

Define $M(f) = \mathcal{F}\{\text{sinc}^2(10t)\}$

Modulation property

$$X(f) = \tfrac{1}{2}M(f-f_0) + \tfrac{1}{2}M(f+f_0) \qquad f_0 = 100\text{Hz}$$

$M(f) = ?$ From table we know: $\text{tri}(t/T) \rightleftharpoons T\,\text{sinc}^2(fT)$

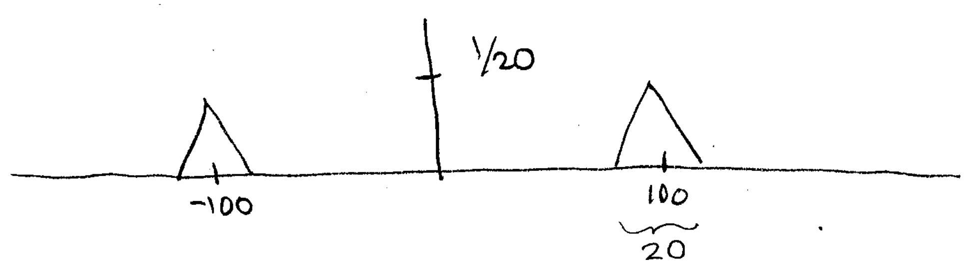
Duality: If $g(t) \rightleftharpoons G(f)$

then $G(t) \rightleftharpoons g(-f)$

Thus $W\,\text{sinc}^2(Wt) \rightleftharpoons \text{tri}(-f/W) = \text{tri}(f/W)$

Thus $M(f) = \frac{1}{10}\cdot\text{tri}(f/10)$ 5

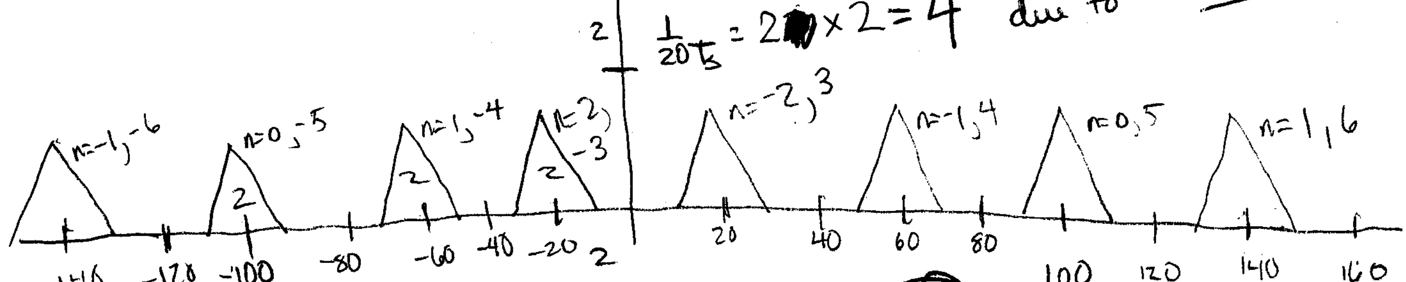
$X(f) = \frac{1}{20}\text{tri}\left(\frac{f-100}{10}\right) + \frac{1}{20}\text{tri}\left(\frac{f+100}{10}\right)$ 5



$$X_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(f - nf_s) \qquad f_s = 40$$

5

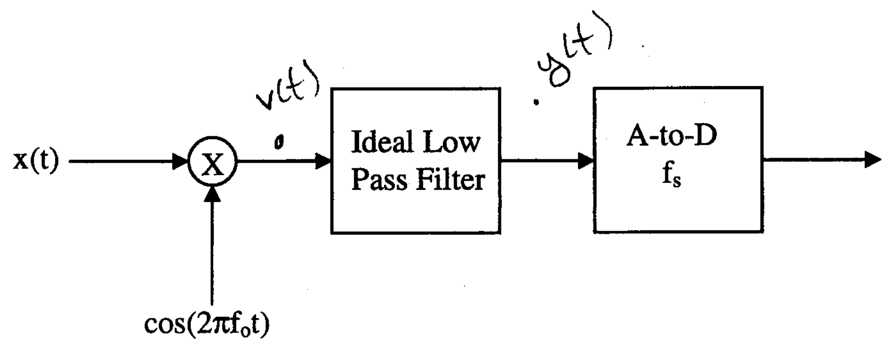
$\frac{1}{20T_s} = 2 \times 2 = 4$ due to 2 images



10

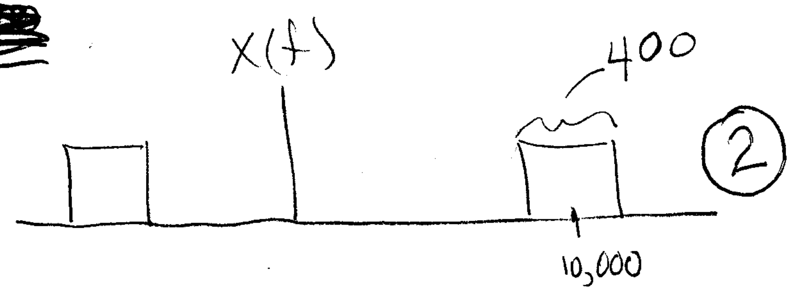
3. (10 points) Sampling and Filtering

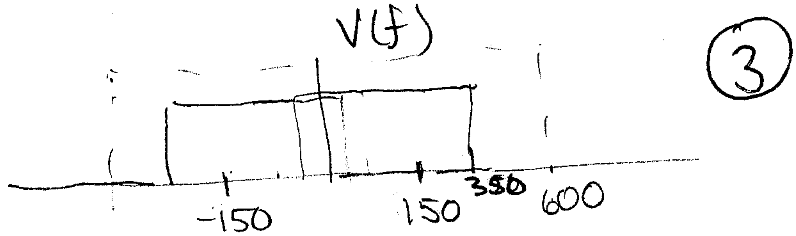
Consider the signal $x(t) = m(t)\cos(20000\pi t)$ which is input into the following system.



If the bandwidth of the ideal low pass filter is $3B$ where B=200Hz is the bandwidth of $m(t)$ and f_o = ~~19700,~~ determine the minimum sampling rate f_s to avoid aliasing.

$\boxed{f_o = 9{,}850}$





$f_s \geq 2 \cdot 350 = \underline{700\text{Hz}}$ ⑤

4. (10 points) Power Spectral Density and Energy Spectral Density

If *x(t) = exp(-at) u(t)* what is $R_x(\tau)$?

$$R_x(\tau) = \mathcal{F}^{-1}\{\Psi_x(f)\} \quad (2)$$

$$\Psi_x(f) = |X(f)|^2 \quad (2)$$

$$X(f) = \frac{1}{a + j2\pi f} \quad (2)$$

$$|X(f)|^2 = \frac{1}{a^2 + (2\pi f)^2}$$

$$= \frac{1}{2a} \cdot \mathcal{F}\{\exp(-a|t|)$$

$$R_x(\tau) = \frac{1}{2a} \exp(-a|t|) \quad (4)$$

5. (35 points) PCM

(a) [15 points] A signal is to be sampled, quantized and sent across a channel using binary polar NRZ line code with one of the pulse shapes shown below. We require the 30dB bandwidth of the transmitted digital signal to be less than or equal to 450kHz. Assuming PCM with no companding, what is the maximum quantization SNR achievable for an analog signal that has a bandwidth of 15kHz and a peak-to-average voltage ratio of 25?

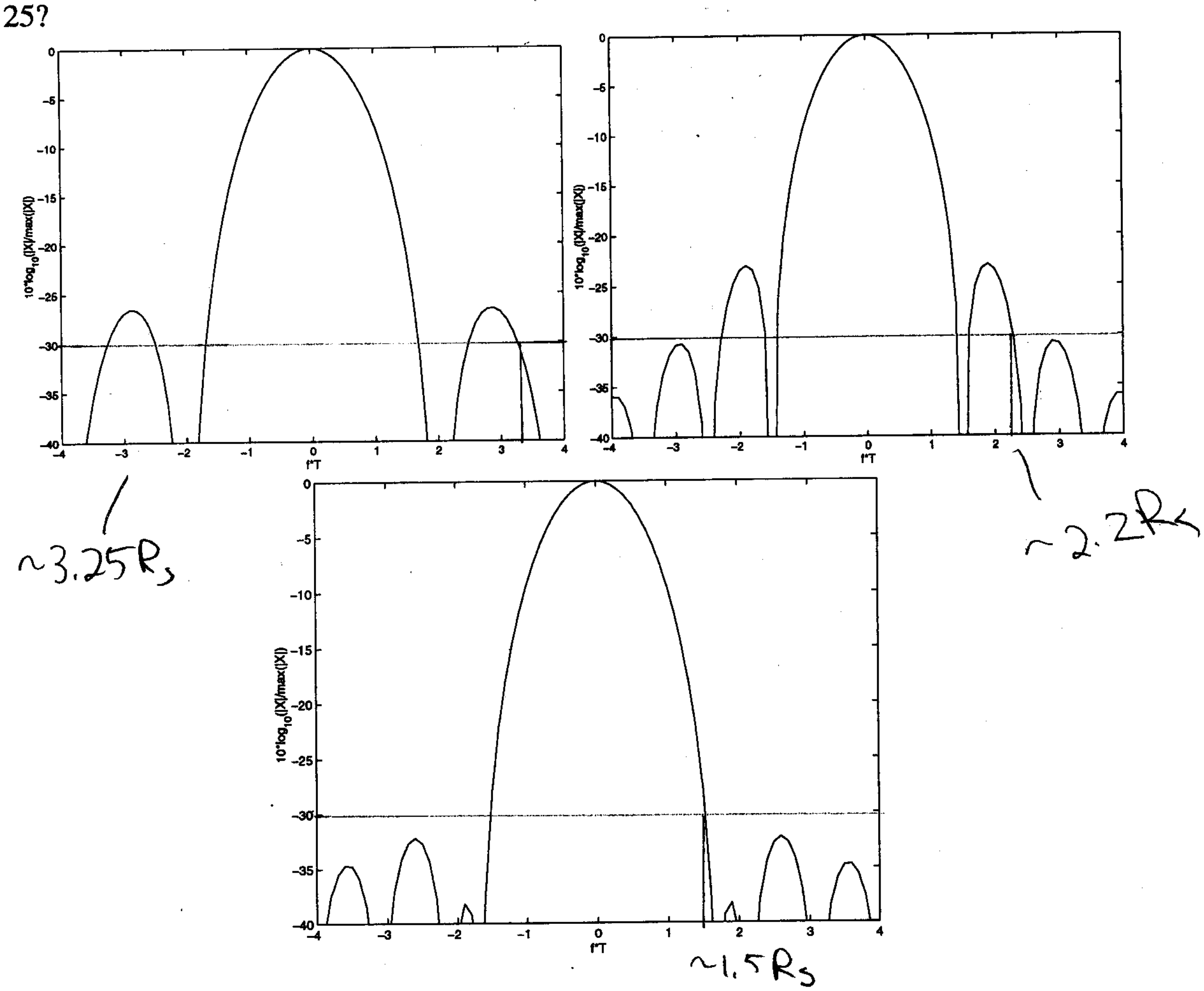


Figure 1: Fourier Transforms of the Pulse Shapes Used in Problem 4 – Clockwise from Top left: Triangular, Sinunsoidal and Gaussian
(NOTE: The y-axis is $20\log_{10}(|X(f)|)$ and the x-axis is fT where f is frequency and T is the pulse duration)

Best pulse is Gaussian

$B = 1.5R_s$

$B \leq 450\text{ KHz}$

$1.5\, R_s \leq 450\text{KHz}$ 3

binary $\longrightarrow$ $R_b = R_s$ 2

$R_b \leq \frac{450\text{KHz}}{1.5} = 300\text{ Kbps}$ 2

$SNR = 6.02n + 4.77 - 20\log_{10}\left(\frac{V_{peak}}{V_{rms}}\right)$

$R_b = n \cdot f_s \qquad f_s \geq 30\text{KHz}$ 3

$n \leq \frac{300\text{Kbps}}{30\text{KHz}} = 10\text{ b/sample}$ 2

$SNR = 6.02 \cdot 10 + 4.77 - 20\log_{10}(25)$ 3

$= 37.0\text{ dB}$

(b) [10 points] Can we improve the performance using a compander with $\mu = 255$?

$$SNR = 6.02n + 4.77 - 20\log_{10}(\ln(1+\mu)) \quad 4$$

$$= 50.1 \quad 4$$

<u>Yes!</u> 2

(c) [10 points] Can we improve the quantization SNR with a delta modulation as opposed to PCM? Assume that Δ is chosen to avoid slope overload.

$$SNR = 10\log_{10}\left(\frac{3f_s^3}{4\pi^2 B^3} \cdot \left(\frac{V_{rms}}{V_{peak}}\right)^2\right) \quad 4$$

<u>Assume $B_f = B$</u>

$B = 15\text{KHZ}$

$$f_s \leq \frac{300\text{Kbps}}{1\text{ b/s}} = 300\text{Ksps} \quad 3$$

$$SNR = 10\log_{10}\left(\underbrace{\frac{3\cdot(300K)^3}{4\pi^2(15K)^3} \cdot \frac{1}{25^2}}_{0.973}\right)$$

$$= -0.12\text{ dB} \quad 3$$

<u>No</u>