Digital Communications
Midterm Exam II
November 5, 2007

SOLUTION

I pledge that I have neither given nor received any assistance on this exam.

____
(signed)

______________________________ ____________________
Name (print) Student Number

1. (20 points) Multiple Choice – Choose the answer that best completes the sentence.

(a) [5 points] Differential PSK modulation

(a) can be used with binary or *M*-ary modulation
(b) is based on modulating the phase difference between the transmit signal and some fixed reference signal
(c) requires a coherent reference
(d) All of the above
(e) None of the above

(b) [5 points] Random Variables

(a) are used to represent samples of signals in communication systems
(b) can be used to represent samples of random processes
(c) allow us to model events which are unknown in advance
(d) All of the above
(e) None of the above

(c) [5 points] Inter-symbol interference

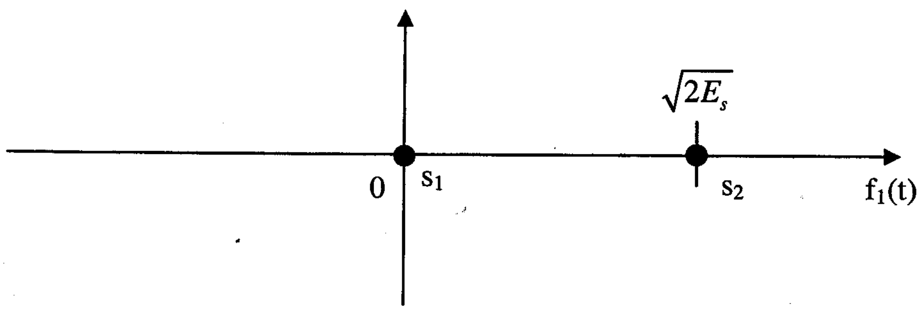
(a) is caused by AWGN
(b) is caused by bandwidth limitations in the channel
(c) is represented by an open eye-diagram
(d) All of the above
(e) None of the above

(d) [5 points] Increasing the number of symbols in a modulation scheme

(a) always increases bandwidth efficiency
(b) always decreases bandwidth efficiency
(c) always degrades energy efficiency
(d) both (a) and (c)
(e) None of the above

2. (30 points) Bandpass Modulation

Consider the following constellation diagram (or signal space representation)



where $f_1(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$, E_s is the average symbol energy and T is the symbol duration.

(a) [5 points] What type of modulation is this?

BASK

(b) [5 points] Assuming that this modulation scheme is used to transfer a bit stream which has a data rate of 200kbps, what is the 2nd null-to-null bandwidth?

$B_{null-to-null} = 2R_s$

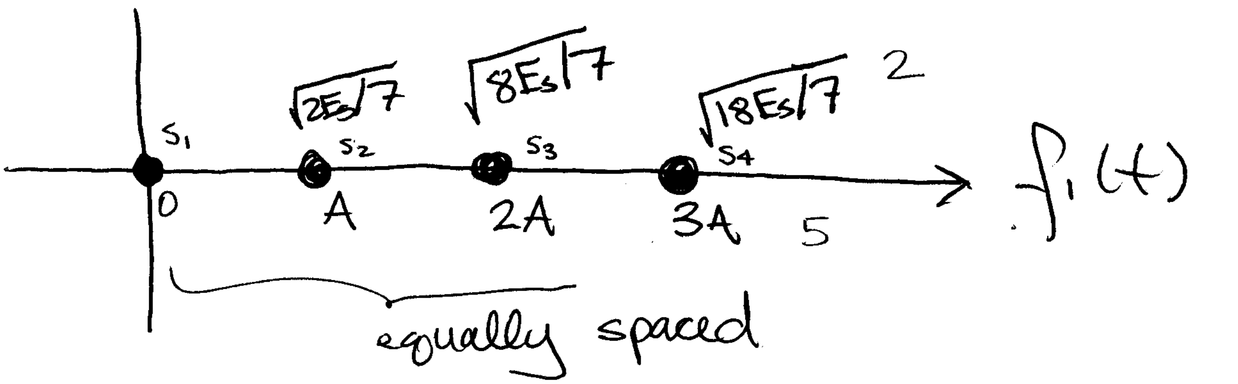
$B_{2nd\ null-to-null} = 4R_s$

binary modulation $\rightarrow R_s = R_b$

$B = 4R_b = 4 \cdot 200$ Kbps

$= 800$ KHz

(c) [10 points] Add two additional symbols to this modulation scheme without changing the type of modulation (i.e., amplitude, phase, frequency) and draw the constellation diagram below. Make sure that the average energy per symbol is E_s.



$$E_s = \frac{1}{4}\{0 + A^2 + 4A^2 + 9A^2\} = \frac{14A^2}{4} = \frac{7A^2}{2}$$

$$A = \sqrt{2E_s/7}$$ 3

(d) [10 points] If the basis function were changed to $f_1(t) = \sqrt{\frac{2}{T}}\, p(t)\cos(2\pi f_c t)$ where $p(t)$ is a unit energy raised cosine pulse with roll-off factor $\alpha = 0.35$, what is the absolute bandwidth if the 4-ary constellation from part (c) is used to deliver a data stream with a bit rate of 200kbps?

4 $B = (1+\alpha)R_s$

$= 1.35\, R_s$

3 4-ary ASK $\longrightarrow R_s = R_b/2$

$$B = 1.35 \cdot \frac{200\text{Kbps}}{2}$$

3 $= 135\,\text{KHz}$

3. (25 points) Bandpass representations

Consider the following bandpass signal:

$$s(t) = \sum_{i=-\infty}^{\infty} d_i rect\left(\frac{t-iT}{T}\right)\cos\left(2\pi f_c t + \pi \sum_{i=-\infty}^{\infty} b_i rect\left(\frac{t-iT}{T}\right)\right)$$

where T is the symbol duration, b_i and d_i depend on the information being sent with $b_i \in \{0,1\}$ and $d_i \in \{+1,+2\}$

(a) (5 points) Which of our three bandpass forms is this equation in?

Magnitude / Phase

$$s(t) = R(t)\cos(2\pi f_c t + \theta(t))$$

(b) (10 points) Find the complex baseband for this signal. Simplify as much as possible. (Note: I am looking for the complex baseband part of the complex baseband/envelope representation.)

2

$$g(t) = x(t) + j\,y(t)$$

$$= R(t)\cos(\theta(t)) + j\,R(t)\sin(\theta(t))$$ 3

$$= \left[\sum_{j=-\infty}^{\infty} d_j\, rect\left(\frac{t-jT}{T}\right)\right]\cos\left(\pi \cdot \sum_{i=-\infty}^{\infty} b_i\, rect\left(\frac{t-iT}{T}\right)\right)$$ 2

$$+j\left[\sum_{j=-\infty}^{\infty} d_j\, rect\left(\frac{t-jT}{T}\right)\right]\sin\left(\pi \cdot \sum_{i=-\infty}^{\infty} b_i\, rect\left(\frac{t-iT}{T}\right)\right)$$

since $b_i \in \{0,1\}$ $\sin\left(\pi \cdot \sum_{i=-\infty}^{\infty} b_i\, rect\left(\frac{t-iT}{T}\right)\right) = 0$

$$\cos\left(\pi \sum_{i=-\infty}^{\infty} b_i\, rect\left(\frac{t-iT}{T}\right)\right) = (-1)^{b_i}\, rect\left(\frac{t-iT}{T}\right)$$

Thus

$$g(t) = \sum_{i=-\infty}^{\infty} d_i (-1)^{b_i}\, rect\left(\frac{t-iT}{T}\right)$$ 3

(c) (10 points) Draw a constellation diagram (signal space) in terms of E_s (the average energy per symbol) assuming that the basis functions are $f_1(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $f_2(t)=\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$

Four symbols

$b_i=0 \quad d_i=1 \qquad s_1(t)=\cos(2\pi f_c t)\Big|_0^T$

$b_i=0 \quad d_i=2 \qquad s_2(t)=2\cos(2\pi f_c t)\Big|_0^T$

$b_i=1 \quad d_i=1 \qquad s_3(t)=-\cos(2\pi f_c t)\Big|_0^T$

$b_i=1 \quad d_i=2 \qquad s_4(t)=-2\cos(2\pi f_c t)\Big|_0^T$

Basis function $f_2(t)$ not needed

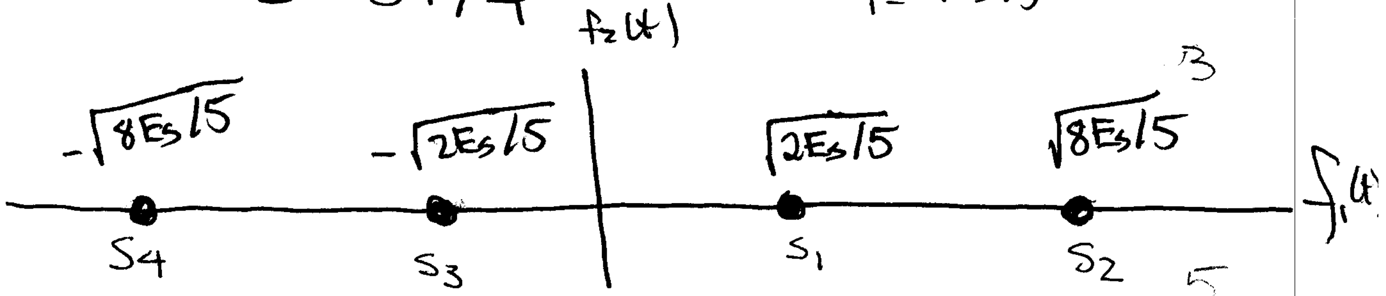
In vector notation

$s_1 = [\sqrt{T/2}, 0] \quad s_3 = [-\sqrt{T/2}, 0]$

$s_2 = [2\sqrt{T/2}, 0] \quad s_4 = [-2\sqrt{T/2}, 0]$

$$E_s = \{T/2 + 4\cdot T/2 + T/2 + 4T/2\}\cdot\frac{1}{4}$$

$$= 5T/4 \qquad T = 4E_s/5$$



4. (25 points) M-ary Modulation

Consider two transmission schemes. The first uses 16-QAM with square pulses and the second uses BPSK with raised cosine pulses with $\alpha = 0.25$. One of the two modulation schemes is going to be used to transmit the bits created by sampling and quantizing a voice signal which has a bandwidth of 4kHz. Assuming that 7 bits of quantization are used, determine which of the two schemes ~~as~~ has the lower bandwidth requirements and state what that minimum bandwidth is. Assume Nyquist sampling and for QAM with square pulses use the 25dB bandwidth (approximated to be $10R_s$) as the absolute bandwidth.

(1)

$$B = 10R_s \qquad R_s = R_b/4$$

$$= \frac{10R_b}{4} \qquad R_b = 8000 \cdot 7 = 56\text{Kbps}$$

$$= \frac{10 \cdot 56{,}000}{4}$$

$$= 140\text{KHz}$$

10

(2)

$$B = (1+\alpha)R_s \qquad R_b = R_s$$

$$= 1.25 \cdot 56{,}000 \qquad R_b = 56\text{ Kbps}$$

$$= 70\text{KHz}$$

10

BPSK w/ Raised Cosine is best and B = 70KHz

5

Rectangular Pulse	$\text{rect}\left(\frac{t}{T}\right)$	$T[\text{sinc}(fT)]$
Triangular Pulse	$\text{tri}\left(\frac{t}{T}\right)$	$T[\text{sinc}(fT)]^2$
Unit Step	$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
exponential	$e^{-at}u(t)$	$\frac{1}{a+j2\pi f}$
Constant	1	$\delta(f)$
Impulse at t_o	$\delta(t-t_o)$	$e^{-j2\pi f t_o}$
Sinc	$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
Phasor	$e^{j\omega_o t+\varphi}$	$e^{j\varphi}\delta(f-f_o)$
Sinusoid	$\cos(2\pi ft+\varphi)$	$\frac{1}{2}e^{j\varphi}\delta(f-f_o)+\frac{1}{2}e^{-j\varphi}\delta(f+f_o)$
Gaussian	$e^{-\pi(t/t_o)^2}$	$t_o e^{-\pi(f t_o)^2}$