Analog and Digital Communications
Final Exam
December 14, 2004

I pledge that I have neither given nor received any assistance on this exam.

____
(signed)

Name (print)

Student Number

1. (20 points) Short Answer

a. (5 points) Power Efficiency

Rank the following modulation schemes in terms of power efficiency from most efficient (1) to least efficient (4)

16-FSK	1
16-QAM	3
16-PSK	4
BPSK	2

(b) (5 points) Matched Filtering

Assuming that BPSK modulation and matched filtering are used, which of the following pulse shapes performs the *worst* in terms of BER performance?

square pulse
[✓ sinc pulse]
square root raised cosine roll-off factor 0.5
square root raised cosine roll-off factor 0

(c) (5 points) Spectral Efficiency

Rank the following modulation schemes in terms of bandwidth efficiency from most efficient (1) to least efficient (4)

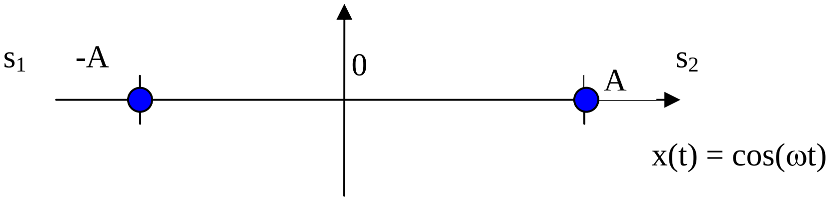
16-QAM with square root raised cosine pulse (roll-off factor 0.25)	1
16-PSK with square root raised cosine pulse (roll-off factor 0.5)	2
BPSK with square root raised cosine pulse (roll-off factor 0)	4
QPSK with square root raised cosine pulse (roll-off factor 0.75)	3

(d) (5 points) Why are square pulse shapes impractical?

Square pulses have infinite bandwidth and have sidelobes which fall off very slowly. Thus, they have very poor spectral properties.

2. (20 points) Modulation Performance

Consider the following constellation diagram



The received signal after matched filtering and sampling is

$$r = s + n$$

where n is a random noise sample with the following distribution

$$n = \begin{cases} 0 & p = 0.5 \\ B = \dfrac{\sigma_n}{2} & p = 0.5 \end{cases}$$

If the symbols $\{s_1, s_2\}$ are equally likely, determine the probability of error in terms of the signal power $S = E\left\{s^2\right\}$ and noise variance σ_n^2. (Assume that 0 is used as the threshold and that B > 0).

$$P_e = P(s_1)P(\varepsilon|s_1) + P(s_2)P(\varepsilon|s_2)$$

$$\begin{aligned} P(\varepsilon|s_1) &= P(-A+B>0) \\ &= \begin{cases} 0 & A>B \\ \dfrac{1}{2} & A \le B \end{cases} \end{aligned}$$

$$\begin{aligned} P(\varepsilon|s_2) &= P(A+B\le 0) \\ &= 0 \end{aligned}$$

If $A > B$

$$\begin{aligned} P_e &= P(s_1)P(\varepsilon|s_1)+P(s_2)P(\varepsilon|s_2) \\ &= \frac{1}{2}*0+\frac{1}{2}*0 \\ &= 0 \end{aligned}$$

If $A < B$

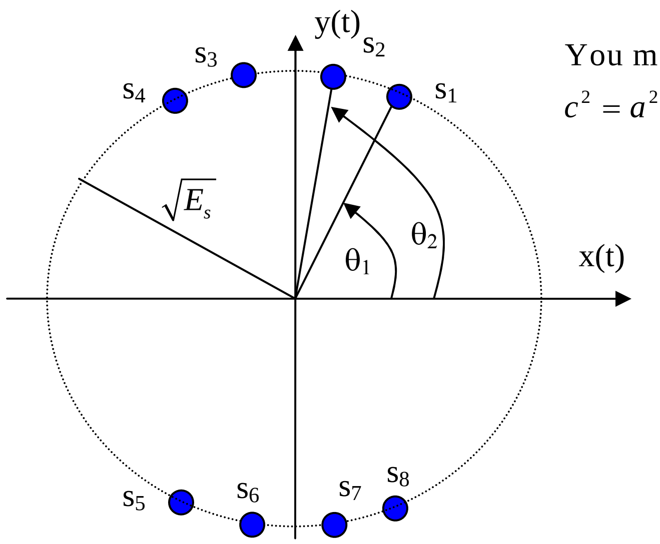
$$\begin{aligned} P_e &= P(s_1)P(\varepsilon|s_1)+P(s_2)P(\varepsilon|s_2) \\ &= \frac{1}{2}*\frac{1}{2}+\frac{1}{2}*0 \\ &= \frac{1}{4} \end{aligned}$$

Putting this in terms of A^2/σ^2*:*

$$P_e = \begin{cases} 0 & \dfrac{A^2}{\sigma^2} > \dfrac{1}{4} \\ \dfrac{1}{4} & \dfrac{A^2}{\sigma^2} \le \dfrac{1}{4} \end{cases}$$

3. (20 points) *M*-ary Modulation

Consider the following *M*-ary modulation scheme where $\theta_1=\pi/3$ and $\theta_2 = 4\pi/9$. Note that all four quadrants are symmetrical and Gray coding is used.



You may find the Law of Cosines Useful

$$c^2 = a^2 + b^2 - 2ab\cos C$$

a. (5 points) What is the spectral efficiency of this modulation scheme ?

There are eight symbols, thus the spectral efficiency is 3b/symbol.

b. (10 points) Determine the probability of bit error in an AWGN channel when matched filtering is used in terms of E_b/N_o (assume that all symbols are equally likely).

$$\begin{aligned} P_e &= \sum_i P(s_i)P(\varepsilon|s_i) \\ &= 4\left\{P(s_1)P(\varepsilon|s_1) + P(s_2)P(\varepsilon|s_2)\right\} \\ &= 4\left\{\frac{1}{8}P(\varepsilon|s_1) + \frac{1}{8}P(\varepsilon|s_2)\right\} \end{aligned}$$

Examining symbol s_1, we see that it is dominated by errors due to its nearest neighbor s_2. The probability of error for symbol 1 is then:

$$P(\varepsilon|s_1) = Q\left(\frac{d_{12}}{\sqrt{2N_o}}\right)$$

The distance between symbols 1 and 2 can be found using the law sines:

$$\begin{aligned} d_{12}^2 &= r^2 + r^2 - 2r^2\cos\theta \\ &= E_s + E_s - 2E_s\cos\frac{\pi}{9} \\ &= 2E_s\left(1-\cos\frac{\pi}{9}\right) \\ d_{12} &= \sqrt{2E_s\left(1-\cos\frac{\pi}{9}\right)} \end{aligned}$$

$$\begin{aligned} P\left(\varepsilon \middle| s_1\right) &= Q\left(\frac{\sqrt{2E_s\left(1-\cos\frac{\pi}{9}\right)}}{\sqrt{2N_o}}\right) \\ &= Q\left(\sqrt{\frac{0.06E_s}{N_o}}\right) \\ &= Q\left(\sqrt{\frac{0.18E_b}{N_o}}\right) \end{aligned}$$

For symbol s_2, we have two nearest neighbors, s_1 and s_3. Since the angle between s_2 and s_3 is also $\pi/9$, the distances are the same. Thus, we can simply double the error rate:

$$\begin{aligned} P\left(\varepsilon \middle| s_2\right) &= 2Q\left(\frac{\sqrt{2E_s\left(1-\cos\frac{\pi}{9}\right)}}{\sqrt{2N_o}}\right) \\ &= 2Q\left(\sqrt{\frac{0.18E_b}{N_o}}\right) \end{aligned}$$

$$P_e = 4\left\{\frac{1}{8}P\left(\varepsilon\middle|s_1\right) + \frac{1}{8}P\left(\varepsilon\middle|s_2\right)\right\}$$

$$= \frac{3}{2}Q\left(\sqrt{\frac{0.18E_b}{N_o}}\right)$$

$$P_b = \frac{1}{3}P_e$$

$$= \frac{1}{2}Q\left(\sqrt{\frac{0.18E_b}{N_o}}\right)$$

c. (5 points) If we wanted to minimize BER, what would be the optimal values of θ_1 and θ_2 ?

The best error rate performance would be obtained if the symbols were equally spaced. This is achieved if $\theta_1 = \pi/8$ *and* $\theta_2 = 3\pi/8$.

4. (20 points) Link Budgets and System Design

Consider a 10kbps mobile wireless system which has a mobile transceiver with a maximum transmit power of 200mW and a noise temperature of 600K. The base station transceiver has a maximum transmit power per channel of 2W and a noise temperature of 300K. The system uses BPSK and requires a BER of 10^{-3}in the presence of Rayleigh fading. The base station transmit and receive antennas have identical gains of 15dBi and the mobile transmit and receive antenna gains are 1.5dBi. The center frequency is 2GHz.

a. (5 points) Which link (link from the base station to the mobile station or the link from the mobile station to the base station) limits the range of the system?

Downlink: Pr = Pt + Gt + Gr – Lp = 3dBW + C
Eb/No = Pr – Rb – No = 3dBW + C – Rb – k – 27.8 = -24.8 + y

Uplink: Pr = Pt + Gt + Gr – Lp = -7dBW + C
Eb/No = Pr – Rb – No = -7dBW + C – Rb – k – 24.8 = -31.8 + y

Uplink is the limit.

b. (5 points) What is the maximum range of the system if a 6dB margin is required to combat shadowing and the path loss exponent is 3?

Pr = Pt + Gt – Lp +Gr = -7 +1.5 –Lp +15 = 9.5 – Lp
Eb = Pr – Rb = -30.5 – Lp
No = -228 + 24.8 = -203.2
Eb/No = 172.7 – Lp

$$P_e = \frac{1}{4 E_b/N_o}$$

$$10^{-3} = \frac{1}{4 E_b/N_o}$$

$$E_b/N_o = 250$$

$$\begin{aligned}\left(\frac{E_b}{N_o}\right)_{req} &= 24dB + 6dB \\ &= 30dB \\ L_p &= 172.7 - \left(\frac{E_b}{N_o}\right) \\ &= 142.7 \\ 30\log_{10}\left(\frac{4\pi R}{\lambda}\right) &= 142.7 \\ R &= \frac{\lambda}{4\pi}10^{142.7/30} \\ &= 681m\end{aligned}$$

c. (5 points) If coding can provide a 5dB gain in the link (i.e., 5dB less E_b/N_o is required) what is the maximum range obtainable?

$$\begin{aligned}30\log_{10}\left(\frac{4\pi R}{\lambda}\right) &= 147.7 \\ R &= \frac{\lambda}{4\pi}10^{147.7/30} \\ &= 1000m\end{aligned}$$

d. (5 points) If the mobile receiver noise temperature could be reduced to 200K, what would be the range of the system?

Range doesn't change since uplink is the limiting link.

5. (20 points) Communication System Design

Consider a satellite link that has a total bandwidth of 10MHz. The owners of the satellite would like to lease it for high speed data transfer with an error rate of 10^{-6}. The transmitter has a maximum power of 10W, the transmit antenna has a gain of 35dBi while the receiver has a noise temperature of 400K and an antenna gain of 25dBi. The link is at 28GHz and the range is 42,000km. The following modulation schemes are available: BPSK, QPSK, 8-PSK, 16-QAM, 64-QAM, and BFSK. Square root raised cosine pulses can be used with a roll-off factor between 0.25 and 1.0. (Assume that near infinitely long pulses are possible.) What is the maximum data rate achievable ?

Bandwidth

Choose α = 0.25 since it gives us the most channels.

$$\begin{aligned} 10MHz &= R_s(1+\alpha) \\ R_s &= 8Msps \\ R_b &= 8Mbps - 48Mbps \end{aligned}$$

Energy Limits

Pt	10dBW
Gt	35dBi
Lp	20log(4p*42000000 / (3/280)) = 213.8dB
Gr	25dB
Pr	-143.8dBW
Rb	
No	-228+26 = -202
Eb/No	59 - Rb

BPSK / QPSK:

$$Q\left(\sqrt{\frac{2E_b}{N_o}}\right) = 10^{-6}$$

$$\frac{E_b}{N_o} = 10.5dB$$

Rb = 59 – 10.5 = 48.5dB

Rb = 70.8kbps

Since BPSK/QPSK is the most energy efficient modulation schemes, and the system is severely energy limited, this is the maximum bit rate achievable.

The *Q*-function