

EE 5654 - Digital Communications - Spring 2004
Homework 5
Due Tuesday 4/22/04

- Write an expression for the probability of error for a Golay code when used in conjunction with BPSK in terms of E_b/N_o . Plot the performance versus E_b/N_o .

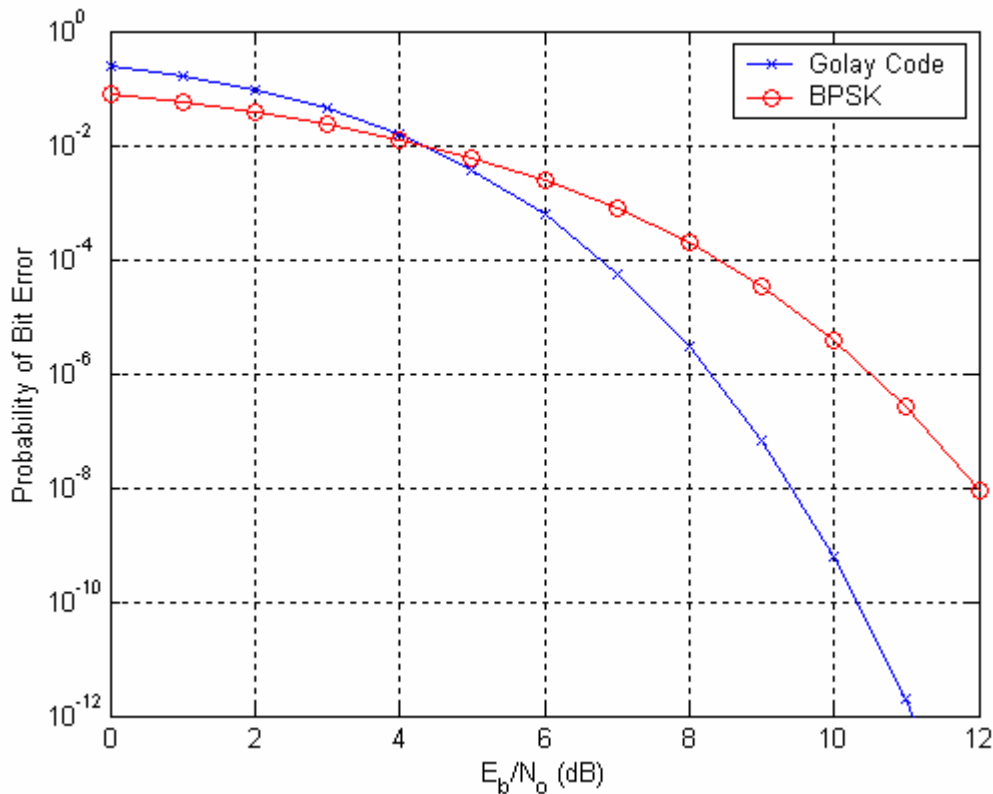


Figure 1: Performance of BPSK and Golay Code with Hard Decision Decoding

Since the Golay code is a perfect code, the exact error probability for hard decision decoding can be calculated from (8.1-92) as

$$P_M = 1 - \sum_{m=0}^3 \binom{23}{m} p^m (1-p)^{23-m}$$

where p is the probability of bit error

$$p = Q\left(\sqrt{\frac{2E_b}{N_o} r}\right)$$

where r is the code rate, $r=12/23$. The plots are given in Figure 1.

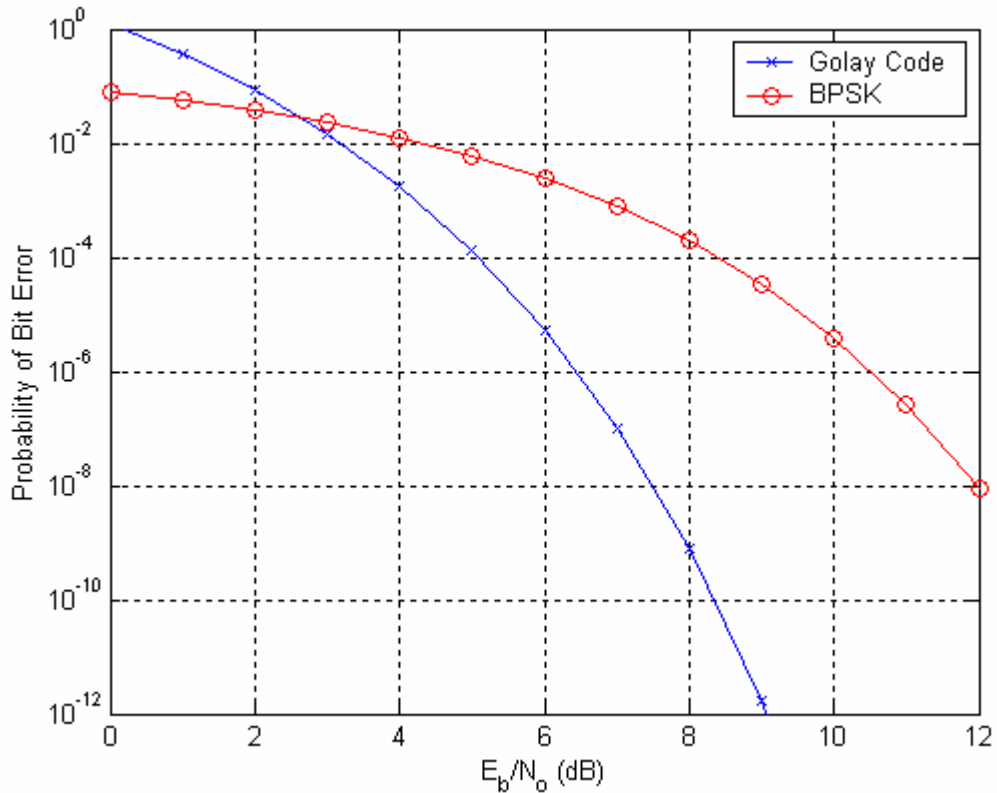


Figure 2: Performance of BPSK and Golay Code with Soft Decision Decoding

If we use soft decisions, the performance is more dependent on the weight distribution. That is the union bound for soft decisions is:

$$P_M \leq \sum_{m=2}^M Q\left(\frac{2E_b}{N_o} r w_m\right)$$

Using the weight distributions given in Table 8.1-1 we can calculate the codeword error probability as shown in Figure 2.

2. You are in charge of a modem design project at Fred and Brothers Aerospace Corp. Your task is to find the most efficient design for the modulation and coding. Specifically, your team has developed Raised Cosine pulse filters with $\alpha = 0.33$. Further, you have 100kHz of bandwidth in which you must transmit 150kbps. You may choose any modulation scheme you wish, but must choose from BCH codes. Design the modem to meet your requirements with the lowest required E_b/N_o . Provide plots of various options to verify your design.

We must deliver 150kbps in a bandwidth of 100kHz using RC pulses with $\alpha = 0.33$. Thus, we have the following relationship between bandwidth and symbol rate

$$B = (1 + \alpha)R_s$$

$$100kHz = 1.33R_s$$

Further, the data rate is limited by the symbol rate, modulation scheme and code rate that is used:

$$R_b = R_s kr$$

$$150kbps = R_s kr$$

Combining the two equations results in the requirement:

$$100kHz = 1.33 \frac{150kbps}{kr}$$

$$kr = 2$$

In other words, we must obtain 2b/s/Hz regardless of coding and modulation scheme. To satisfy this requirement, we could simply use QPSK uncoded. We would like to try to improve performance by adding coding. The options with PSK are listed in Table 1. The bit error performance of PSK is given by:

$$P_b = \frac{2}{k} Q \left(\sqrt{\frac{2kE_b}{N_o}} r \sin \left(\frac{\pi}{M} \right) \right)$$

while the coded error performance for hard decision decoding of the BCH code is

$$P_M = 1 - \sum_{m=0}^t \binom{n}{m} P_b^m (1 - P_b)^{n-m}$$

and n and $r = m/n$ will depend on the BCH code chosen. We choose to use $n=255$ since it will provide the best performance (of those given in the text) and choose k to match the

rate specified in Table 1. The resulting performance of each option with PSK is given in Figure 4.

Table 1: Options for Obtaining 2 b/s/Hz using PSK

Modulation	Coding	k	r	kr
QPSK	None	2	1	2
8-PSK	BCH (255,171)	3	$\sim 2/3$	2
16-PSK	BCH (255,131)	4	$\sim 1/2$	2
32-PSK	BCH (255,107)	5	$\sim 2/5$	2
64-PSK	BCH (255,87)	6	$\sim 1/3$	2

We can see that in order to accommodate coding we must increase the order of the modulation scheme. However, PSK degrades quickly as modulation order increases due to the $\sin(\pi/M)$ term in the bit error probability. Also, increasing the code rate reduces the energy per information symbol due to the term r in the bit error probability. Thus, the coding gains typically cannot keep up with that degradation (particularly when using the BCH code with hard decisions). The best option will depend on the target BER for the system. If the error rate is fairly high, (greater than 10^{-4}) uncoded QPSK is actually the best scheme. However if the target error rate is relatively low (less than 10^{-5}) coded 8-PSK is the best scheme

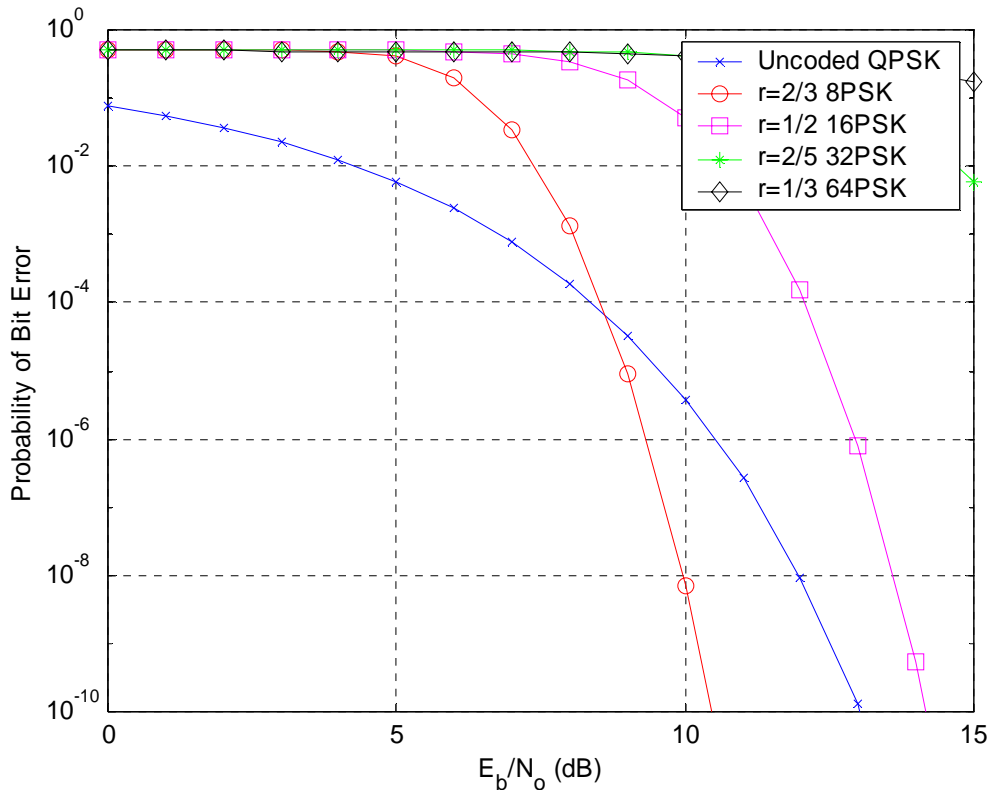


Figure 4: Performance of Various PSK schemes which achieve 2 b/s/Hz

We know that QAM provides better performance than PSK, so we would like to also consider 8-QAM, 16-QAM and 64-QAM as shown in Table 2. For a coded QAM system with code rate r the BER performance of 8-QAM can be shown to be

$$P_b = \frac{4}{3} Q \left(\sqrt{\frac{9E_b}{7N_o}} r \right)$$

while the BER performance of 16-QAM can be shown to be

$$P_b = \frac{3}{4} Q \left(\sqrt{\frac{4E_b}{5N_o}} r \right) + \frac{1}{2} Q \left(3 \sqrt{\frac{4E_b}{5N_o}} r \right)$$

Finally, the BER performance of 64-QAM can be shown to be

$$\begin{aligned} P_b = & \frac{7}{12} Q \left(\sqrt{\frac{6E_b}{21N_o}} r \right) + \frac{1}{12} Q \left(3 \sqrt{\frac{6E_b}{21N_o}} r \right) - \dots \\ & \frac{1}{12} Q \left(5 \sqrt{\frac{6E_b}{21N_o}} r \right) + \frac{1}{6} Q \left(9 \sqrt{\frac{6E_b}{21N_o}} r \right) + \dots \\ & \frac{1}{12} Q \left(11 \sqrt{\frac{6E_b}{21N_o}} r \right) - \frac{1}{12} Q \left(13 \sqrt{\frac{6E_b}{21N_o}} r \right) \end{aligned}$$

The resulting performance of QAM schemes is shown in Figure 5. Now we see a slightly different story. QAM provides approximately 1.5dB improvement over PSK for $M=8$ and about 4dB improvement for $M=16$. The best scheme appears to be 8-QAM with $\frac{1}{2}$ rate BCH coding, unless high error rates are to be tolerated in which case uncoded QPSK is better.

Table 2: Options for Obtaining 2 b/s/Hz using QAM

Modulation	Coding	k	r	kr
QPSK	None	2	1	2
8-QAM	BCH (255,171)	3	$\sim 2/3$	2
16-QAM	BCH (255,131)	4	$\sim 1/2$	2
64-QAM	BCH (255,87)	6	$\sim 1/3$	2

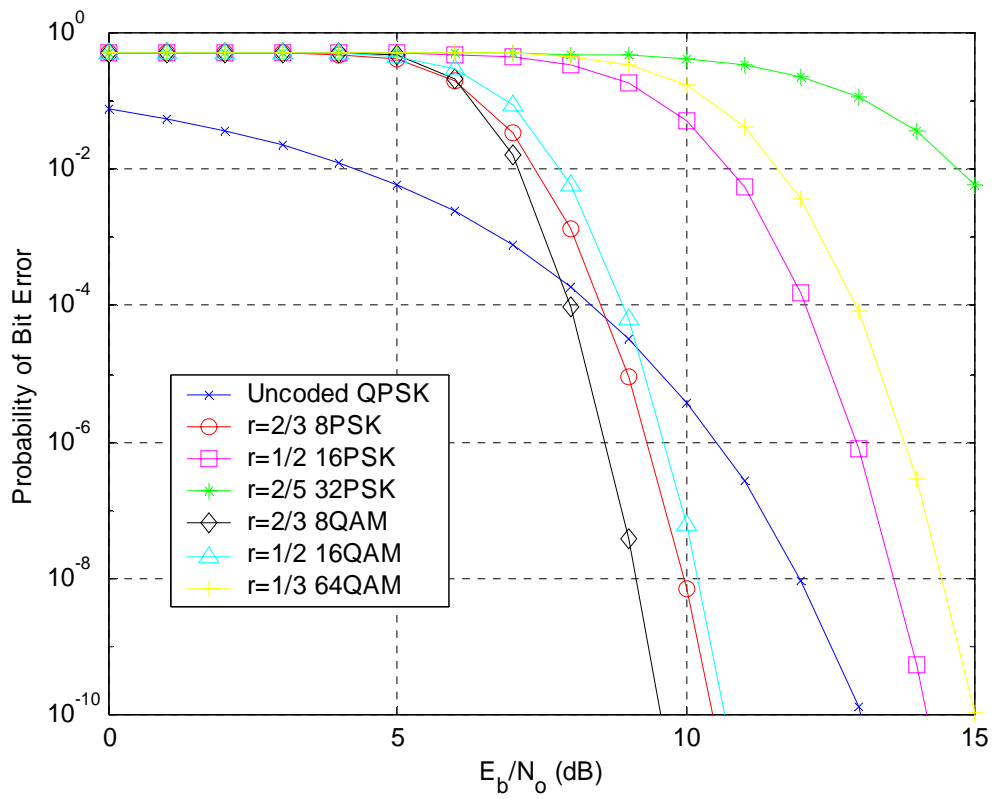
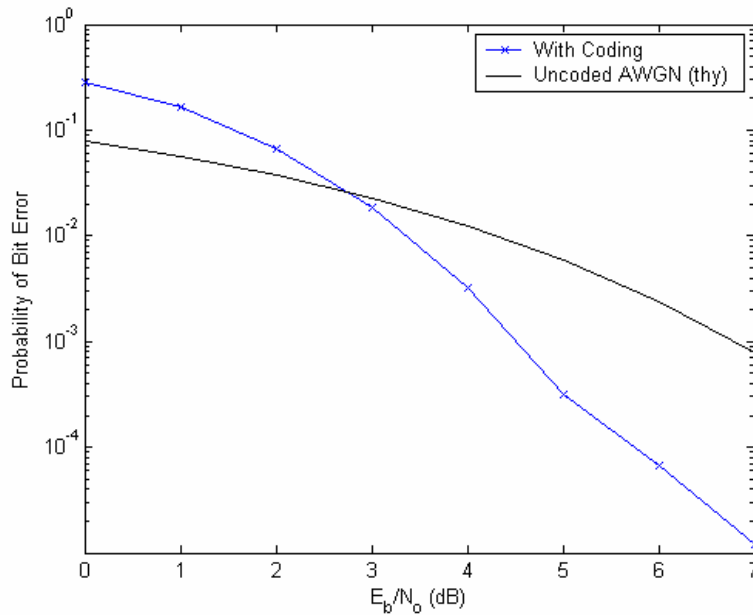


Figure 5: Performance of Various QAM schemes which achieve 2 b/s/Hz

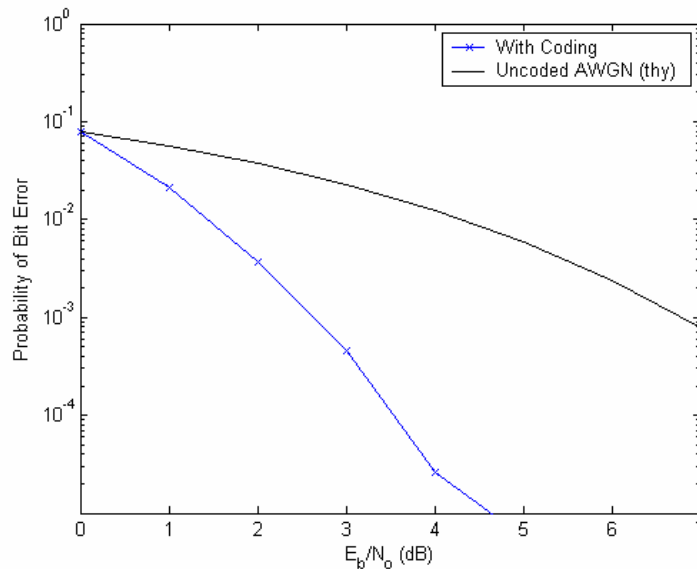
3. Using Matlab, simulate the performance of BPSK with rate 1/6, K=8 convolutional coding. Use both hard and soft decision decoding and assume an E_b/N_0 range of 0-7dB.

Some commands used:

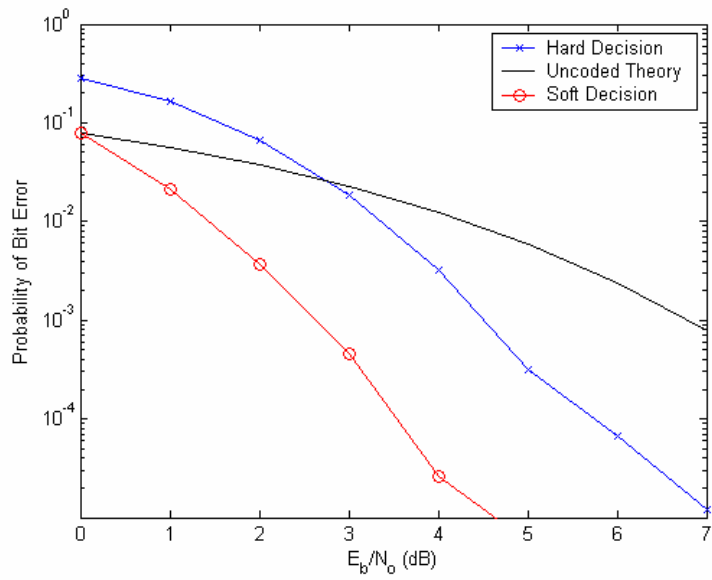
```
g = [253,235,275,313,331,357];  
trellis = poly2trellis(K,g);  
coded = convenc(b,trellis);  
b_hat = vitdec(s_hard,trellis,12,'trunc','hard');  
b_hat = vitdec(r_noisy,trellis,12,'trunc','unquant');
```



Performance with Hard Decision Decoding



Performance with Soft Decision Decoding



Comparison of Hard and Soft Decisions